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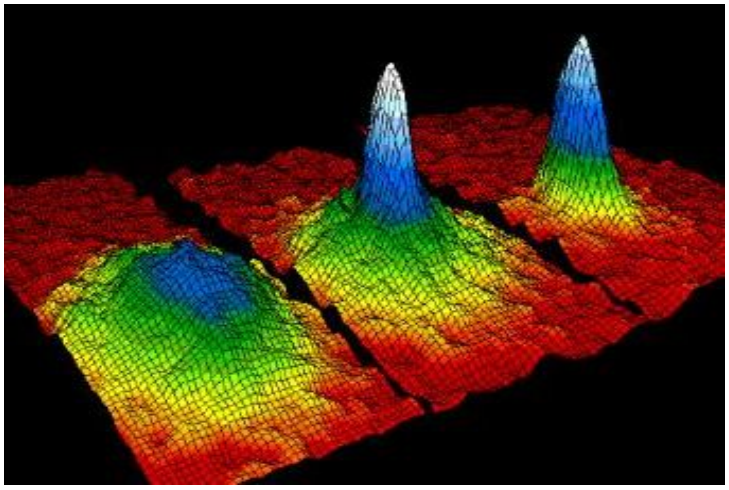
Introduction to Combinatorial Testing

Rick Kuhn
National Institute of
Standards and Technology
Gaithersburg, MD

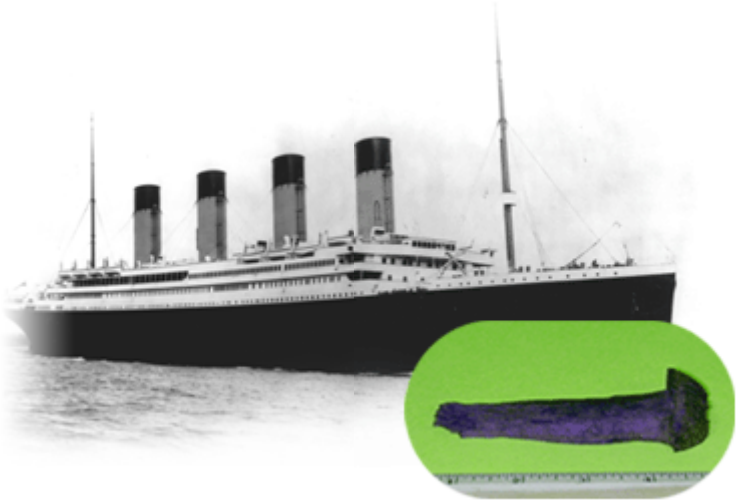
Carnegie-Mellon University, 7 June 2011

What is NIST and why are we doing this?

- A US Government agency
- The nation's measurement and testing laboratory – 3,000 scientists, engineers, and support staff including 3 Nobel laureates



Research in physics, chemistry, materials, manufacturing, computer science



Analysis of engineering failures, including buildings, materials, **and ...**

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Software Failure Analysis

- We studied software failures in a variety of fields including 15 years of FDA medical device recall data
- What causes software failures?
  - logic errors?
  - calculation errors?
  - interaction faults?
  - inadequate input checking? Etc.
- What testing and analysis would have prevented failures?
- Would statement coverage, branch coverage, all-values, all-pairs etc. testing find the errors?

Interaction faults: e.g., failure occurs if

pressure < 10 (1-way interaction <= all-values testing catches)

pressure < 10 & volume > 300 (2-way interaction <= all-pairs testing catches)

Software Failure Internals

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- How does an interaction fault manifest itself in code?

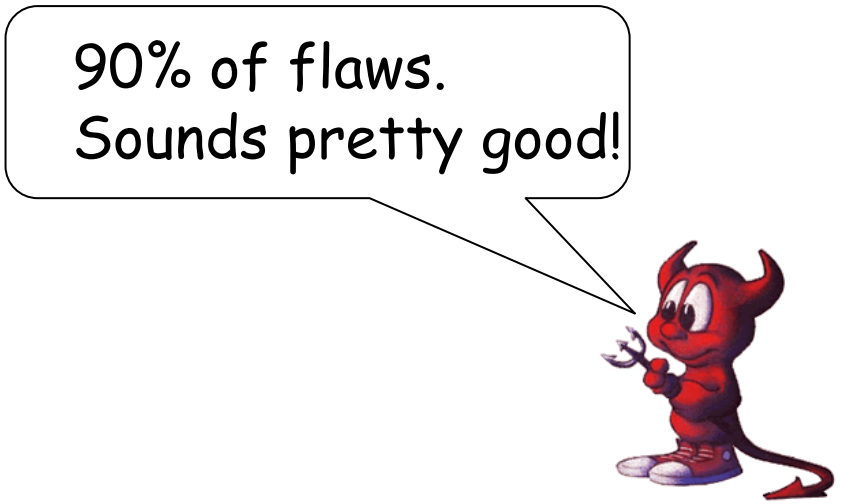
Example: pressure < 10 & volume > 300 (2-way interaction)

```
if (pressure < 10) {
    // do something
    if (volume > 300)  { faulty code!  BOOM! }
    else { good code, no problem}
} else {
    // do something else
}
```

A test that included pressure = 5 and volume = 400 would trigger this failure

Pairwise testing is popular, but is it enough?

- Pairwise testing commonly applied to software
- Intuition: some problems only occur as the result of an interaction between parameters/components
- Tests all pairs (2-way combinations) of variable values
- Pairwise testing finds about 50% to 90% of flaws



Finding 90% of flaws is pretty good, right?

"Relax, our engineers found 90 percent of the flaws."

How about hard-to-find flaws?

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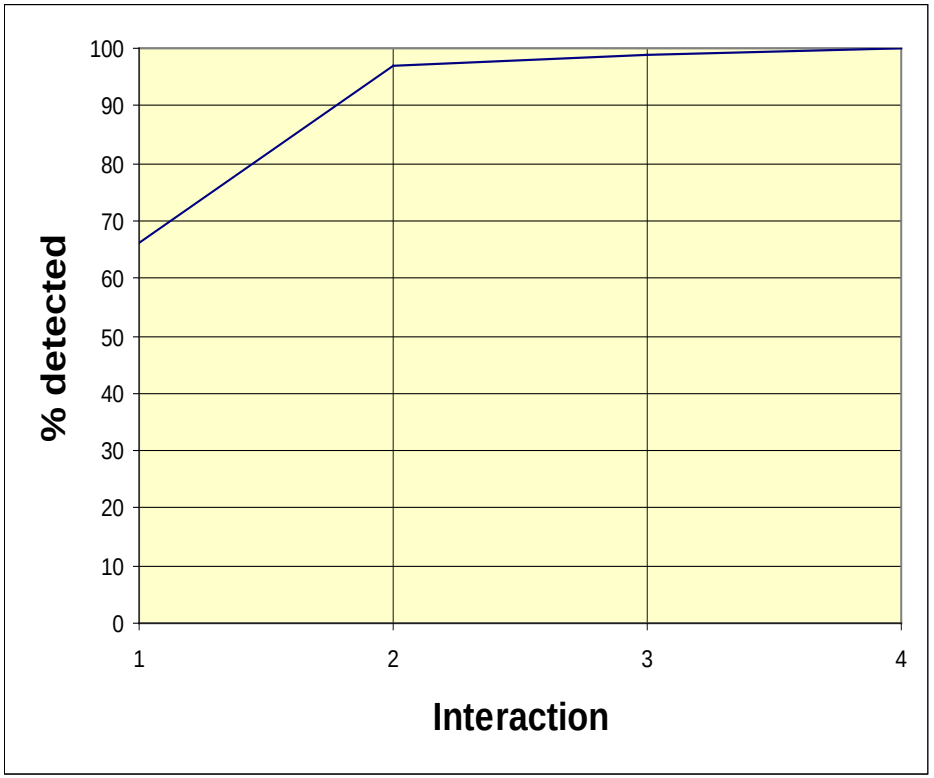
- Interactions e.g., failure occurs if
- pressure < 10 (1-way interaction)
- pressure < 10 & volume > 300 (2-way interaction)
- pressure < 10 & volume > 300 & velocity = 5 (3-way interaction)
- The most complex failure reported required 4-way interaction to trigger



NIST study of 15 years of FDA medical device recall data

Interesting, but that's just one kind of application.

How about other applications?

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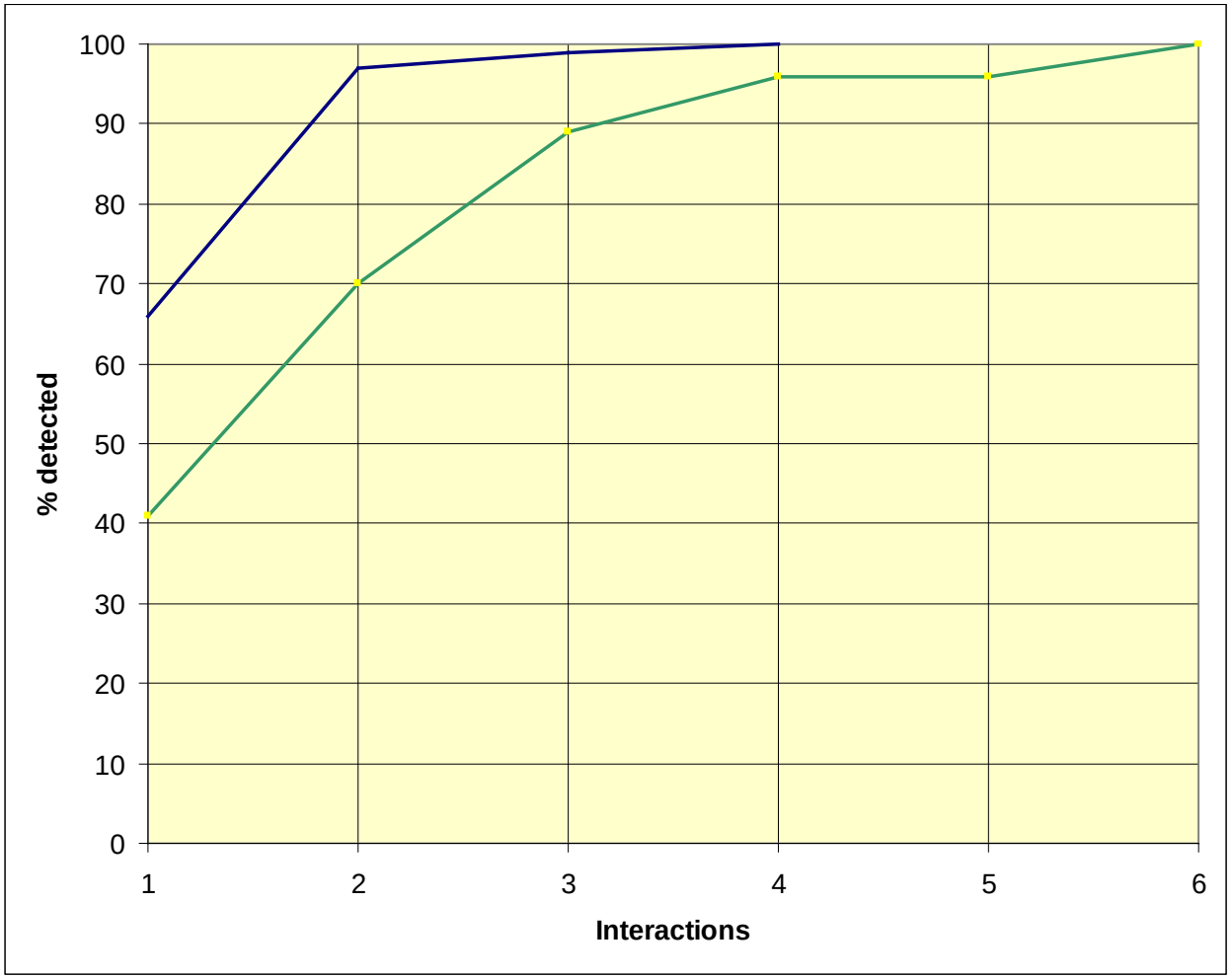
Browser (green)



These faults more complex than medical device software!!

Why?

And other applications?

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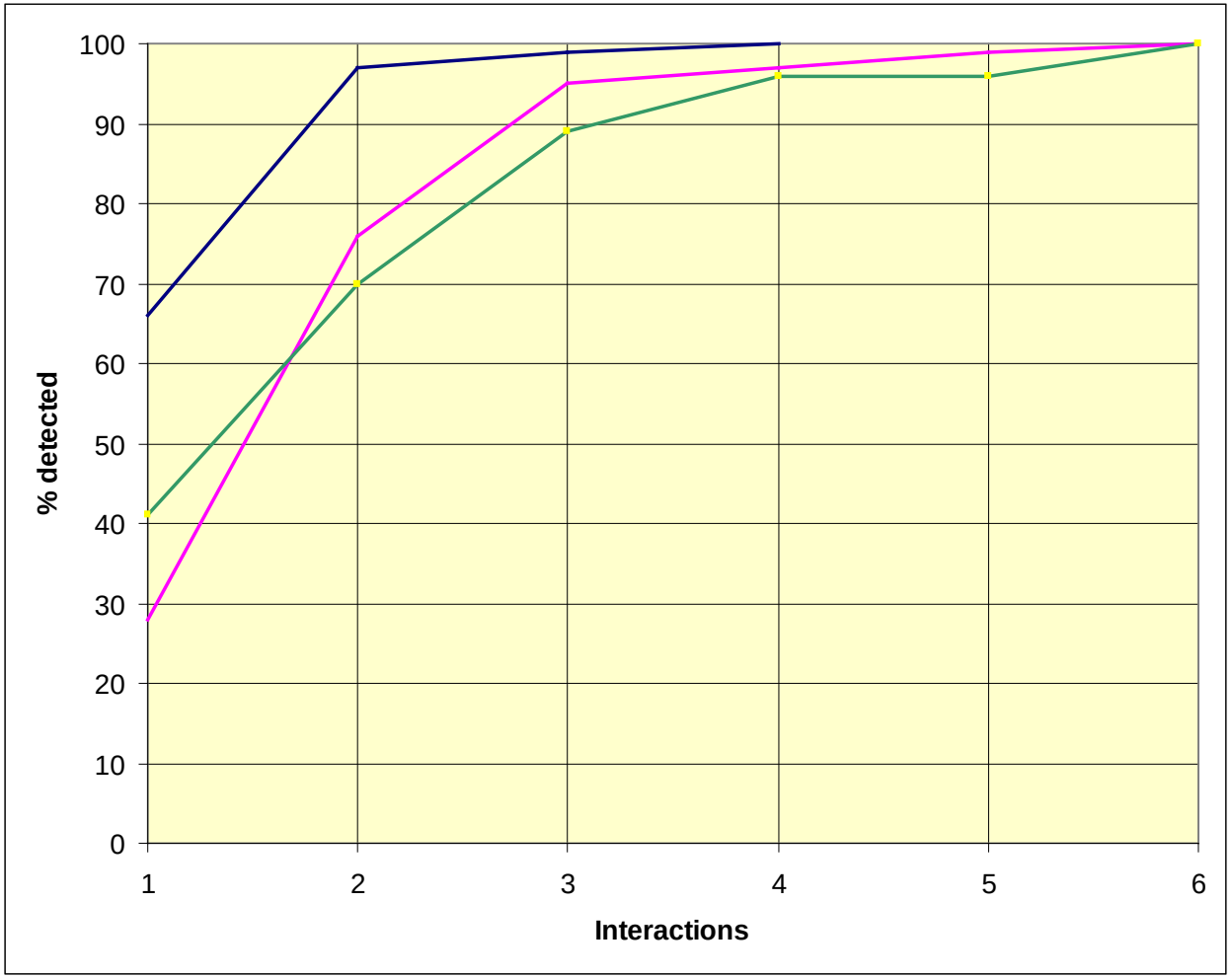
Server (magenta)



Still more?

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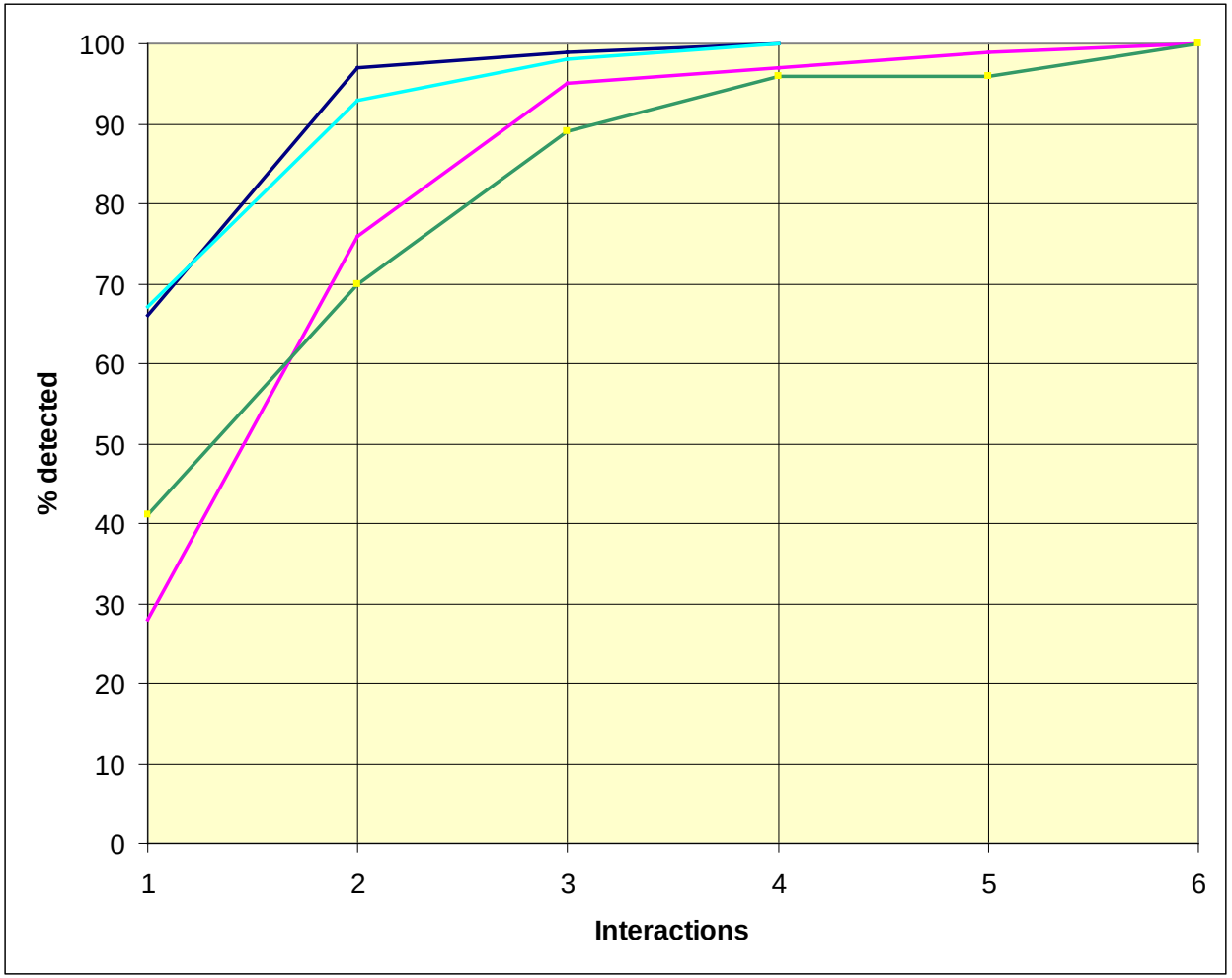
NASA distributed database

(light blue)



Even more?

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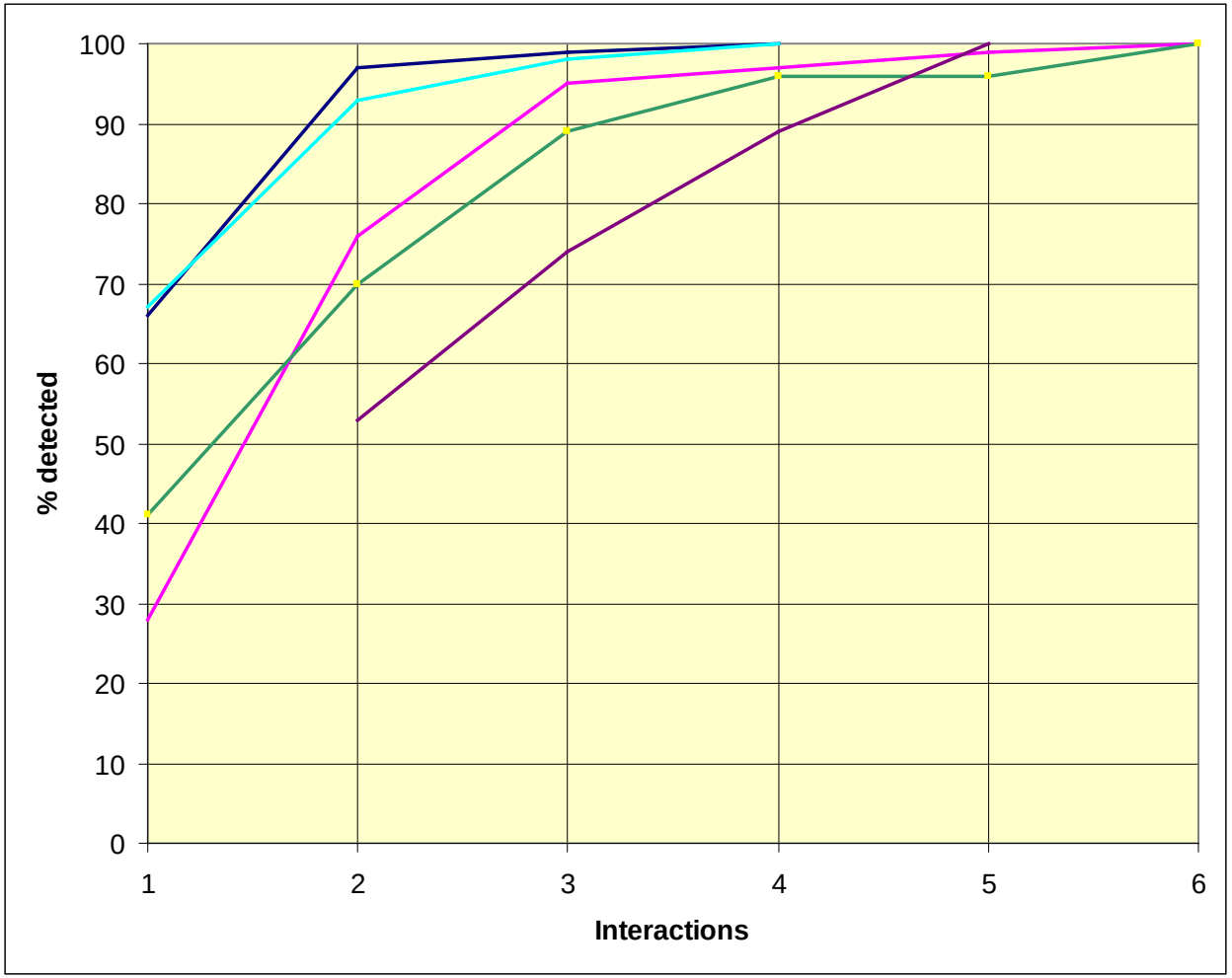
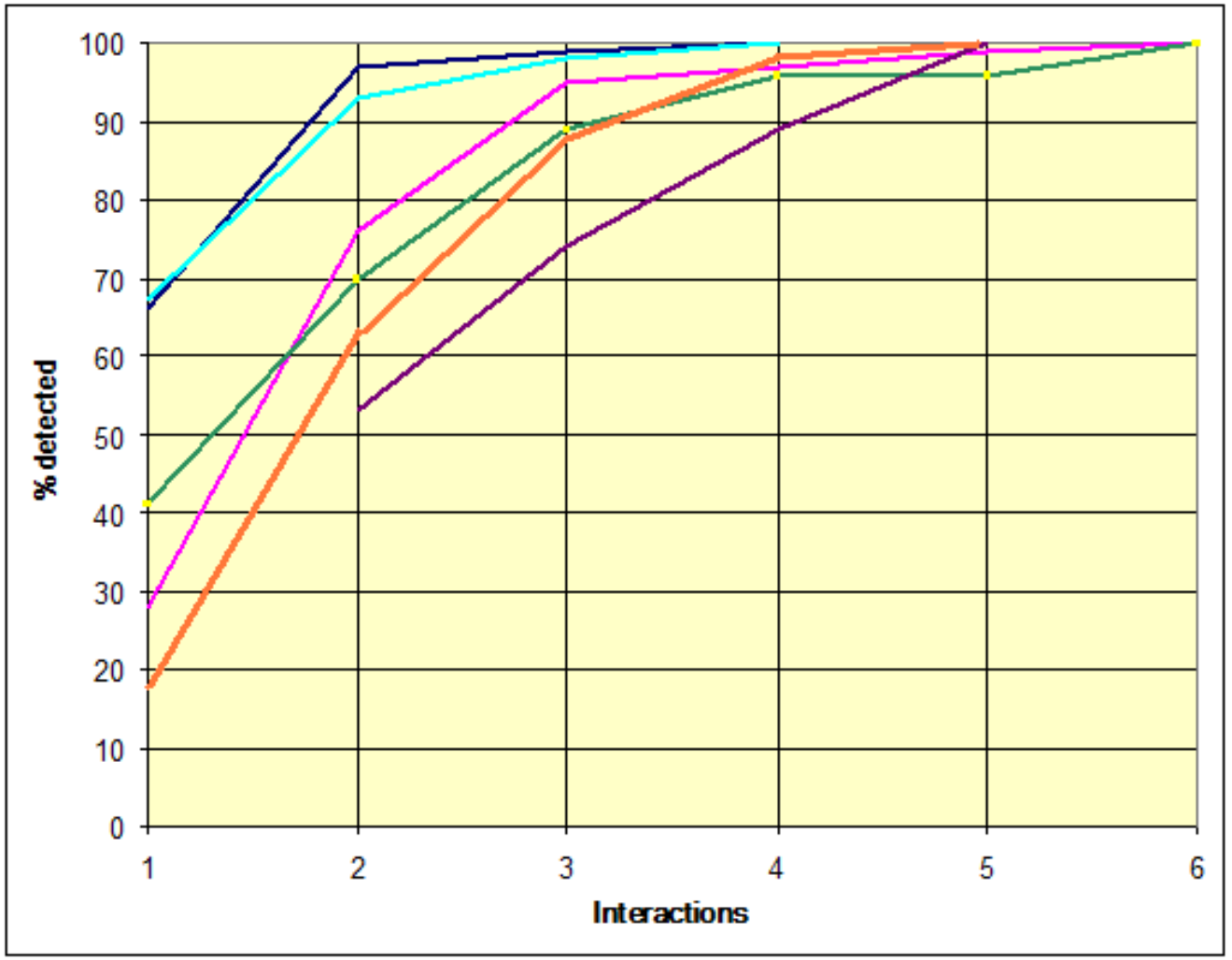
Traffic Collision Avoidance System module (seeded errors) (purple)



Finally

Network security (Bell, 2006)

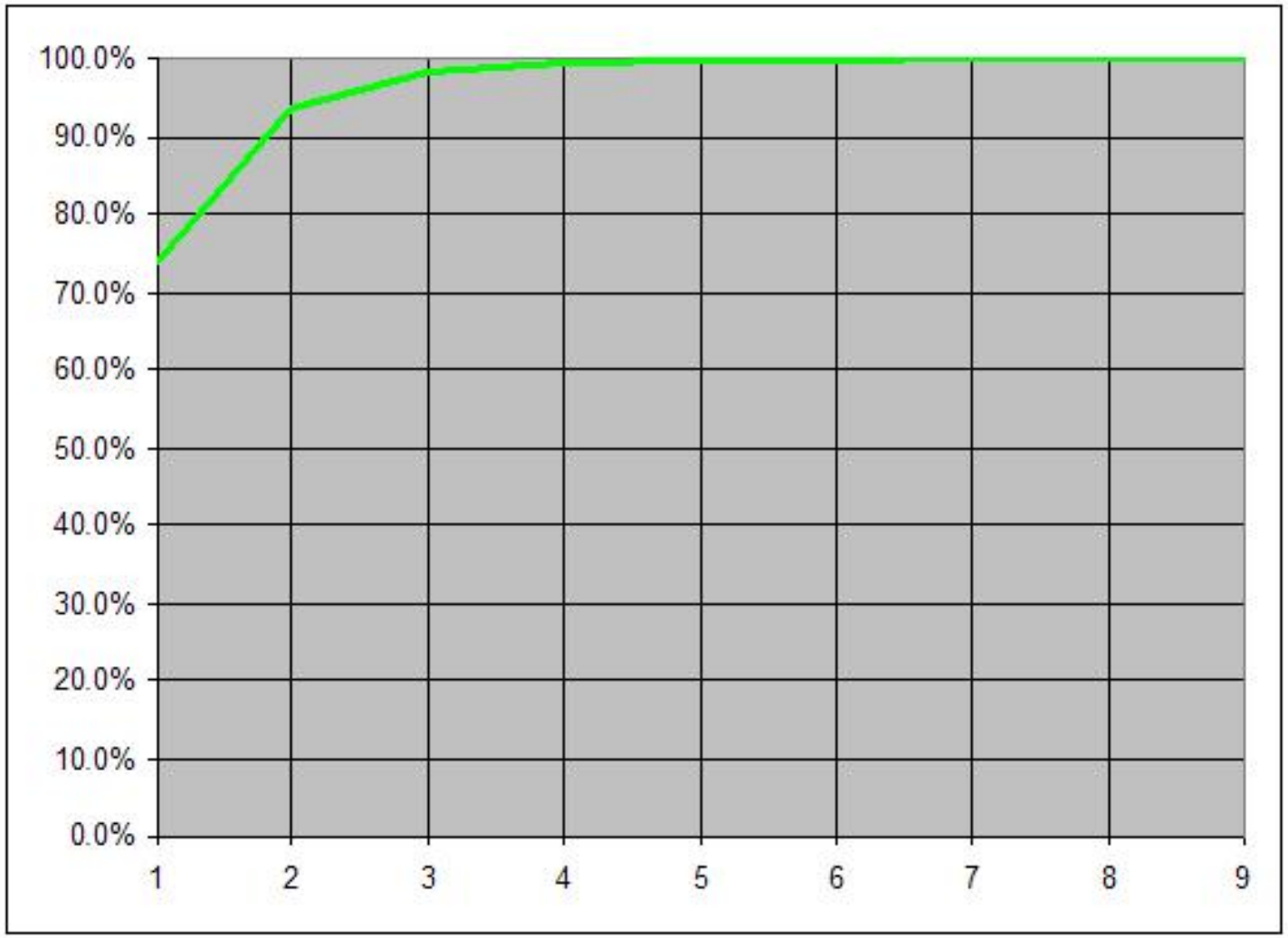
(orange)



Curves appear to be similar across a variety of application domains.

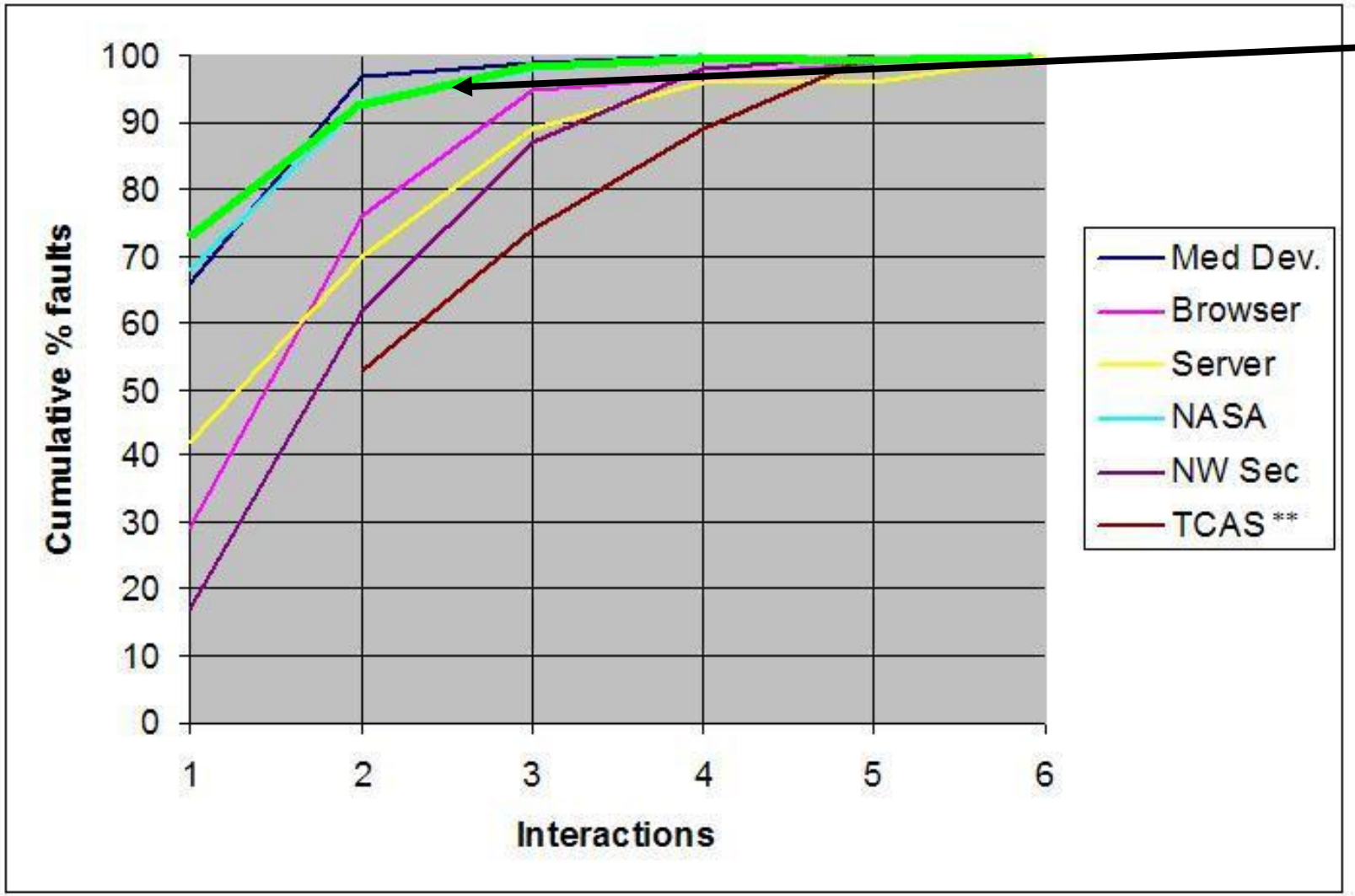
Why this distribution?

What causes this distribution?



One clue: branches in avionics software.
7,685 expressions from *if* and *while* statements

Comparing with Failure Data



Branch statements

So, how many parameters are involved in really tricky faults?

- Maximum interactions for fault triggering for these applications was 6
- Much more empirical work needed
- Reasonable evidence that maximum interaction strength for fault triggering is relatively small

How does this knowledge help?

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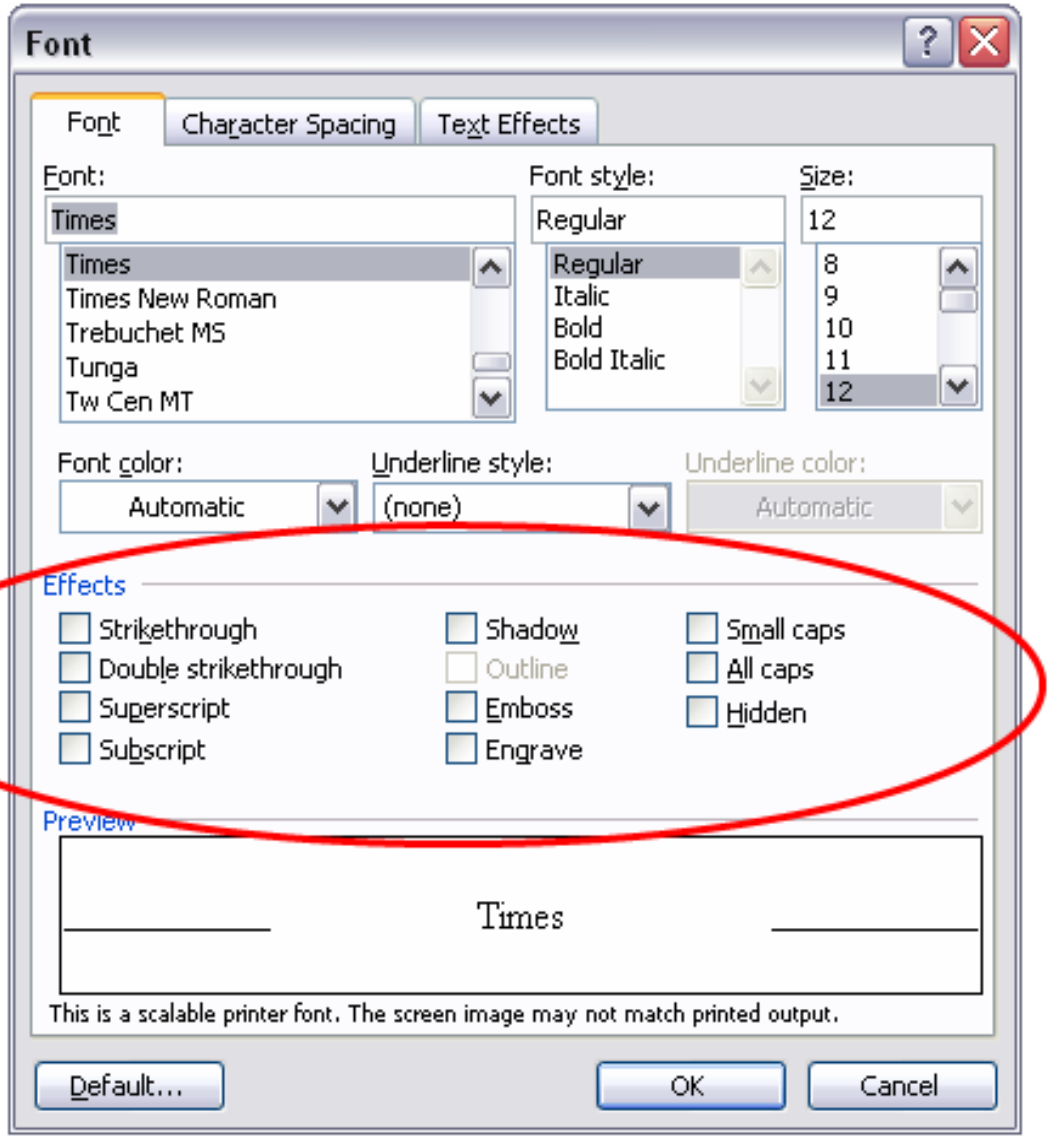
Biologists have a “central dogma”, and so do we:

If all faults are triggered by the interaction of *t* or fewer variables, then testing all *t*-way combinations can provide strong assurance

(taking into account: value propagation issues, equivalence partitioning, timing issues, more complex interactions, . . .)

What is combinatorial testing?
A simple example



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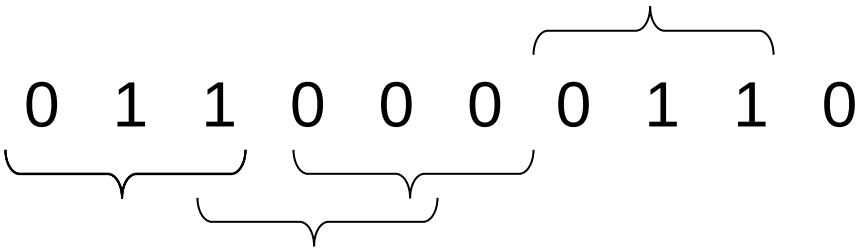
How Many Tests Would It Take?

- There are 10 effects, each can be on or off
- All combinations is 2^{10} = 1,024 tests
- What if our budget is too limited for these tests?
- Instead, let's look at all 3-way interactions …

Now How Many Would It Take?

- There are $\binom{10}{3}$ = 120 3-way interactions.
- Naively 120 x 2^3 = 960 tests.
- Since we can pack 3 triples into each test, we need no more than 320 tests.
- Each test exercises many triples:

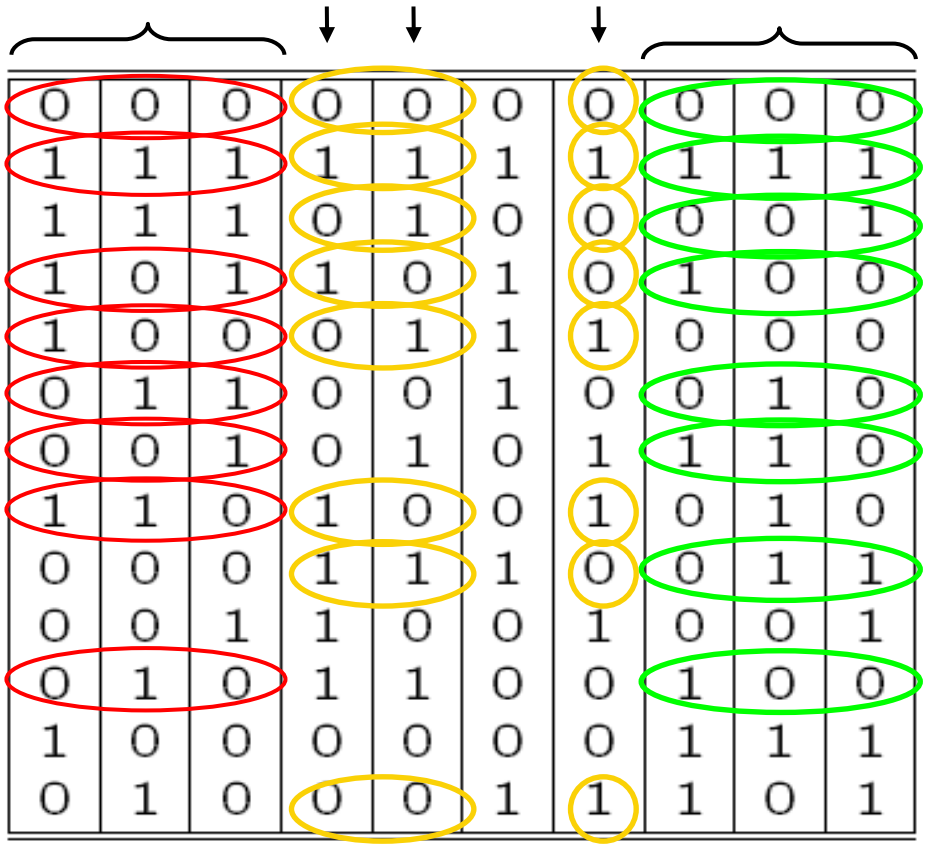


We can pack a lot into one test, so what's the smallest number of tests we need?

A covering array

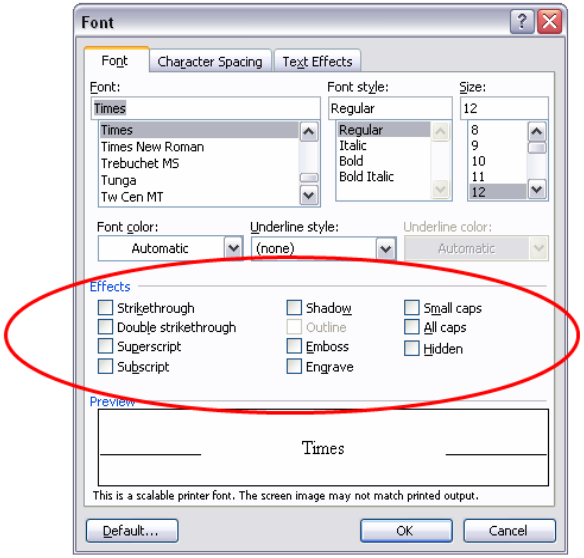
All triples in only 13 tests, covering $\binom{10}{3}2^3 = 960$ combinations

Each row is a test:



0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1
1	1	1	0	1	0	0	0	0	1
1	0	1	1	0	1	0	1	0	0
1	0	0	0	1	1	1	0	0	0
0	1	1	0	0	1	0	0	1	0
0	0	1	0	1	0	1	1	1	0
1	1	0	1	0	0	1	0	1	0
0	0	0	1	1	1	0	0	1	1
0	0	1	1	0	0	1	0	0	1
0	1	0	1	1	0	0	1	0	0
1	0	0	0	0	0	0	1	1	1
0	1	0	0	0	1	1	1	0	1

Each column is a parameter:



Each test covers $\binom{10}{3}$ = 120 3-way combinations

Finding covering arrays is NP hard

Ordering Pizza

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Step 1 *Select your favorite size and pizza crust.*

Large Original Crust

Step 2

Select your favorite pizza toppings from the pull down. Whole toppings cover the entire pizza. First ½ and second ½ toppings cover half the pizza. For a regular cheese pizza, do not add toppings.

☑ I want to add or remove toppings on this pizza -- add on whole or half pizza.

Extra Cheese Remove | Bacon Remove | Black Olives Remove

Add toppings whole pizza

Add toppings 1st half

Add toppings 2nd half

$6x2^{17}x2^{17}x2^{17}x4x3x2x2x5x2$
= WAY TOO MUCH TO TEST

Simplified pizza ordering:

6x4x4x4x4x3x2x2x5x2
= 184,320 possibilities

Step 3 *Select your pizza instructions.*

☑ I want to add special instructions for this pizza -- light, extra or no sauce; light or no cheese; well done bake

Regular Sauce | Normal Cheese | Normal Bake | Normal Cut

Step 4 *Add to order.*

Quantity 1

Add To Order | Add To Order & Checkout

Ordering Pizza Combinatorially

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Simplified pizza ordering:

6x4x4x4x4x3x2x2x5x2
= 184,320 possibilities

2-way tests:	32
3-way tests:	150
4-way tests:	570
5-way tests:	2,413
6-way tests:	8,330

If all failures involve 5 or fewer parameters, then we can have confidence after running all 5-way tests.

A larger example

- Suppose we have a system with on-off switches:

How do we test this?

- 34 switches = 2^{34} = 1.7 x 10^{10} possible inputs = 1.7 x 10^{10} tests

What if we knew no failure involves more than 3 switch settings interacting?

- 34 switches = 2^{34} = 1.7 x 10^{10} possible inputs = **1.7 x 10^{10}** tests
- If only 3-way interactions, need only **33** tests
- For 4-way interactions, need only **85** tests

Two ways of using combinatorial testing

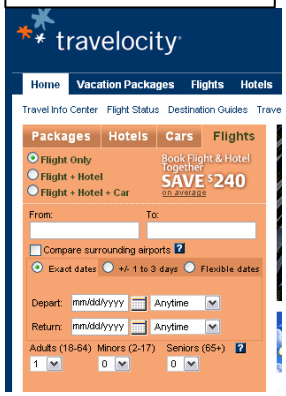
Use combinations here

or here

Test case	OS	CPU	Protocol
1	Windows	Intel	IPv4
2	Windows	AMD	IPv6
3	Linux	Intel	IPv6
4	Linux	AMD	IPv4

Configuration

Test data inputs



System under test

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Testing Configurations

- Example: app must run on any configuration of OS, browser, protocol, CPU, and DBMS
- Very effective for interoperability testing

Test	OS	Browser	Protocol	CPU	DBMS
1	XP	IE	IPv4	Intel	MySQL
2	XP	Firefox	IPv6	AMD	Sybase
3	XP	IE	IPv6	Intel	Oracle
4	OS X	Firefox	IPv4	AMD	MySQL
5	OS X	IE	IPv4	Intel	Sybase
6	OS X	Firefox	IPv4	Intel	Oracle
7	RHL	IE	IPv6	AMD	MySQL
8	RHL	Firefox	IPv4	Intel	Sybase
9	RHL	Firefox	IPv4	AMD	Oracle
10	OS X	Firefox	IPv6	AMD	Oracle

Configurations to Test

Degree of interaction coverage: 2
Number of parameters: 5
Maximum number of values per parameter: 3
Number of configurations: 10

Configuration #1:
1 = OS=XP
2 = Browser=IE
3 = Protocol=IPv4
4 = CPU=Intel
5 = DBMS=MySQL

Configuration #2:
1 = OS=XP
2 = Browser=Firefox
3 = Protocol=IPv6
4 = CPU=AMD
5 = DBMS=Sybase

Configuration #3:
1 = OS=XP
2 = Browser=IE
3 = Protocol=IPv6
4 = CPU=Intel
5 = DBMS=Oracle
. . . etc.

t	# Configs	% of Exhaustive
2	10	14
3	18	25
4	36	50
5	72	100

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Testing Smartphone Configurations

Android configuration options:

int HARDKEYBOARDHIDDEN_NO;
int HARDKEYBOARDHIDDEN_UNDEFINED;
int HARDKEYBOARDHIDDEN_YES;
int KEYBOARDHIDDEN_NO;
int KEYBOARDHIDDEN_UNDEFINED;
int KEYBOARDHIDDEN_YES;
int KEYBOARD_12KEY;
int KEYBOARD_NOKEYS;
int KEYBOARD_QWERTY;
int KEYBOARD_UNDEFINED;
int NAVIGATIONHIDDEN_NO;
int NAVIGATIONHIDDEN_UNDEFINED;
int NAVIGATIONHIDDEN_YES;
int NAVIGATION_DPAD;
int NAVIGATION_NONAV;
int NAVIGATION_TRACKBALL;
int NAVIGATION_UNDEFINED;
int NAVIGATION_WHEEL;
int ORIENTATION_LANDSCAPE;
int ORIENTATION_PORTRAIT;
int ORIENTATION_SQUARE;
int ORIENTATION_UNDEFINED;
int SCREENLAYOUT_LONG_MASK;
int SCREENLAYOUT_LONG_NO;
int SCREENLAYOUT_LONG_UNDEFINED;
int SCREENLAYOUT_LONG_YES;
int SCREENLAYOUT_SIZE_LARGE;
int SCREENLAYOUT_SIZE_MASK;
int SCREENLAYOUT_SIZE_NORMAL;
int SCREENLAYOUT_SIZE_SMALL;
int SCREENLAYOUT_SIZE_UNDEFINED;
int TOUCHSCREEN_FINGER;
int TOUCHSCREEN_NOTOUCH;
int TOUCHSCREEN_STYLUS;
int TOUCHSCREEN_UNDEFINED;

Configuration option values

Parameter Name	Values	# Values
HARDKEYBOARDHIDDEN	NO, UNDEFINED, YES	3
KEYBOARDHIDDEN	NO, UNDEFINED, YES	3
KEYBOARD	12KEY, NOKEYS, QWERTY, UNDEFINED	4
NAVIGATIONHIDDEN	NO, UNDEFINED, YES	3
NAVIGATION	DPAD, NONAV, TRACKBALL, UNDEFINED, WHEEL	5
ORIENTATION	LANDSCAPE, PORTRAIT, SQUARE, UNDEFINED	4
SCREENLAYOUT_LONG	MASK, NO, UNDEFINED, YES	4
SCREENLAYOUT_SIZE	LARGE, MASK, NORMAL, SMALL, UNDEFINED	5
TOUCHSCREEN	FINGER, NOTOUCH, STYLUS, UNDEFINED	4

Total possible configurations:

3 x 3 x 4 x 3 x 5 x 4 x 4 x 5 x 4 = 172,800

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Number of configurations generated

t	# Configs	% of Exhaustive
2	29	0.02
3	137	0.08
4	625	0.4
5	2532	1.5
6	9168	5.3

New algorithms

- Smaller test sets faster, with a more advanced user interface
- First parallelized covering array algorithm
- More information per test

T-Way	IPOG		ITCH (IBM)		Jenny (Open Source)		TConfig (U. of Ottawa)		TVG (Open Source)	
	Size	Time	Size	Time	Size	Time	Size	Time	Size	Time
2	100	0.8	120	0.73	108	0.001	108	>1 hour	101	2.75
3	400	0.36	2388	1020	413	0.71	472	>12 hour	9158	3.07
4	1363	3.05	1484	5400	1536	3.54	1476	>21 hour	64696	127
5	4226	18s	NA	>1 day	4580	43.54	NA	>1 day	313056	1549
6	10941	65.03	NA	>1 day	11625	470	NA	>1 day	1070048	12600

Traffic Collision Avoidance System (TCAS): $2^7 3^2 4^1 10^2$

Times in seconds

That's fast!

Unlike diet plans,
results ARE typical!

ACTS Tool

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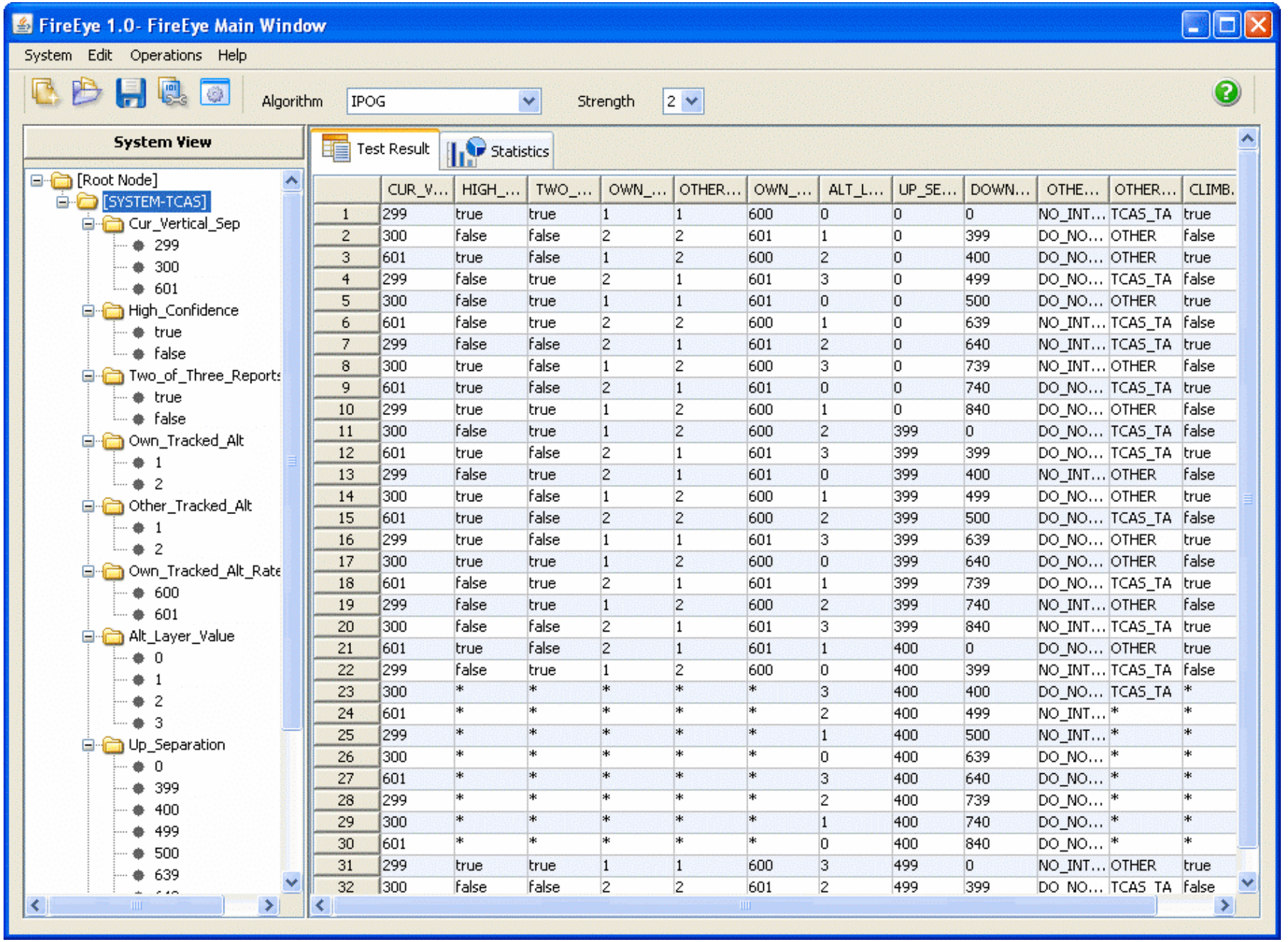
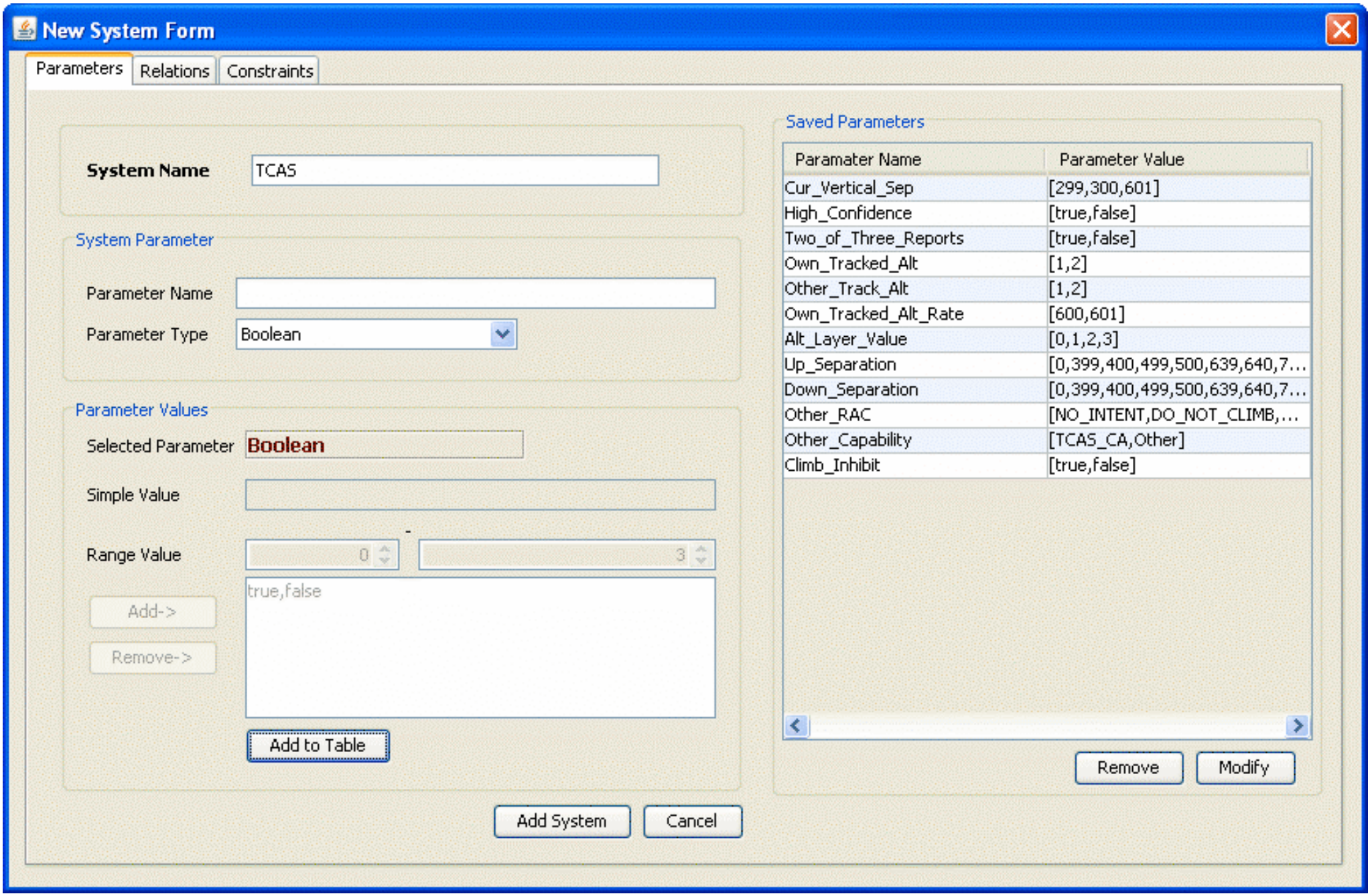
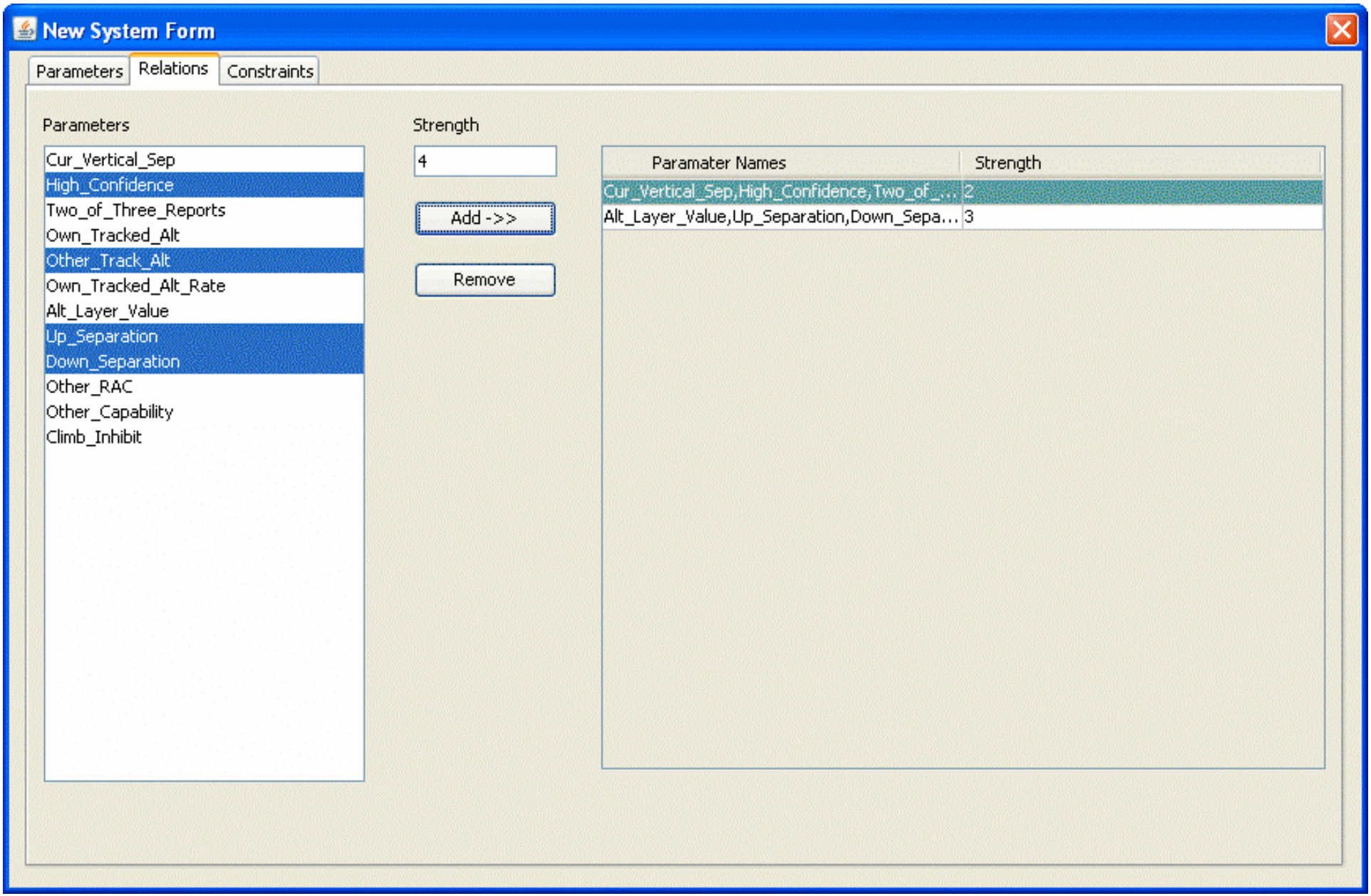


	CUR_V...	HIGH_...	TWO_...	OWN_...	OTHER...	OWN_...	ALT_L...	UP_SE...	DOWN...	OTHE...	OTHER...	CLIMB.
1	299	true	true	1	1	600	0	0	0	NO_INT...	TCAS_TA	true
2	300	false	false	2	2	601	1	0	399	DO_NO...	OTHER	false
3	601	true	false	1	2	600	2	0	400	DO_NO...	OTHER	true
4	299	false	true	2	1	601	3	0	499	DO_NO...	TCAS_TA	false
5	300	false	true	1	1	601	0	0	500	DO_NO...	OTHER	true
6	601	false	true	2	2	600	1	0	639	NO_INT...	TCAS_TA	false
7	299	false	false	2	1	601	2	0	640	NO_INT...	TCAS_TA	true
8	300	true	false	1	2	600	3	0	739	NO_INT...	OTHER	false
9	601	true	false	2	1	601	0	0	740	DO_NO...	TCAS_TA	true
10	299	true	true	1	2	600	1	0	840	DO_NO...	OTHER	false
11	300	false	true	1	2	600	2	399	0	DO_NO...	TCAS_TA	false
12	601	true	false	2	1	601	3	399	399	DO_NO...	TCAS_TA	true
13	299	false	true	2	1	601	0	399	400	NO_INT...	OTHER	false
14	300	true	false	1	2	600	1	399	499	DO_NO...	OTHER	true
15	601	true	false	2	2	600	2	399	500	DO_NO...	TCAS_TA	false
16	299	true	false	1	1	601	3	399	639	DO_NO...	OTHER	true
17	300	true	true	1	2	600	0	399	640	DO_NO...	OTHER	false
18	601	false	true	2	1	601	1	399	739	DO_NO...	TCAS_TA	true
19	299	false	true	1	2	600	2	399	740	NO_INT...	OTHER	false
20	300	false	false	2	1	601	3	399	840	NO_INT...	TCAS_TA	true
21	601	true	false	2	1	601	1	400	0	DO_NO...	OTHER	true
22	299	false	true	1	2	600	0	400	399	NO_INT...	TCAS_TA	false
23	300	*	*	*	*	*	3	400	400	DO_NO...	TCAS_TA	*
24	601	*	*	*	*	*	2	400	499	NO_INT...	*	*
25	299	*	*	*	*	*	1	400	500	NO_INT...	*	*
26	300	*	*	*	*	*	0	400	639	DO_NO...	*	*
27	601	*	*	*	*	*	3	400	640	DO_NO...	*	*
28	299	*	*	*	*	*	2	400	739	DO_NO...	*	*
29	300	*	*	*	*	*	1	400	740	DO_NO...	*	*
30	601	*	*	*	*	*	0	400	840	DO_NO...	*	*
31	299	true	true	1	1	600	3	499	0	NO_INT...	OTHER	true
32	300	false	false	2	2	601	2	499	399	DO_NO...	TCAS_TA	false

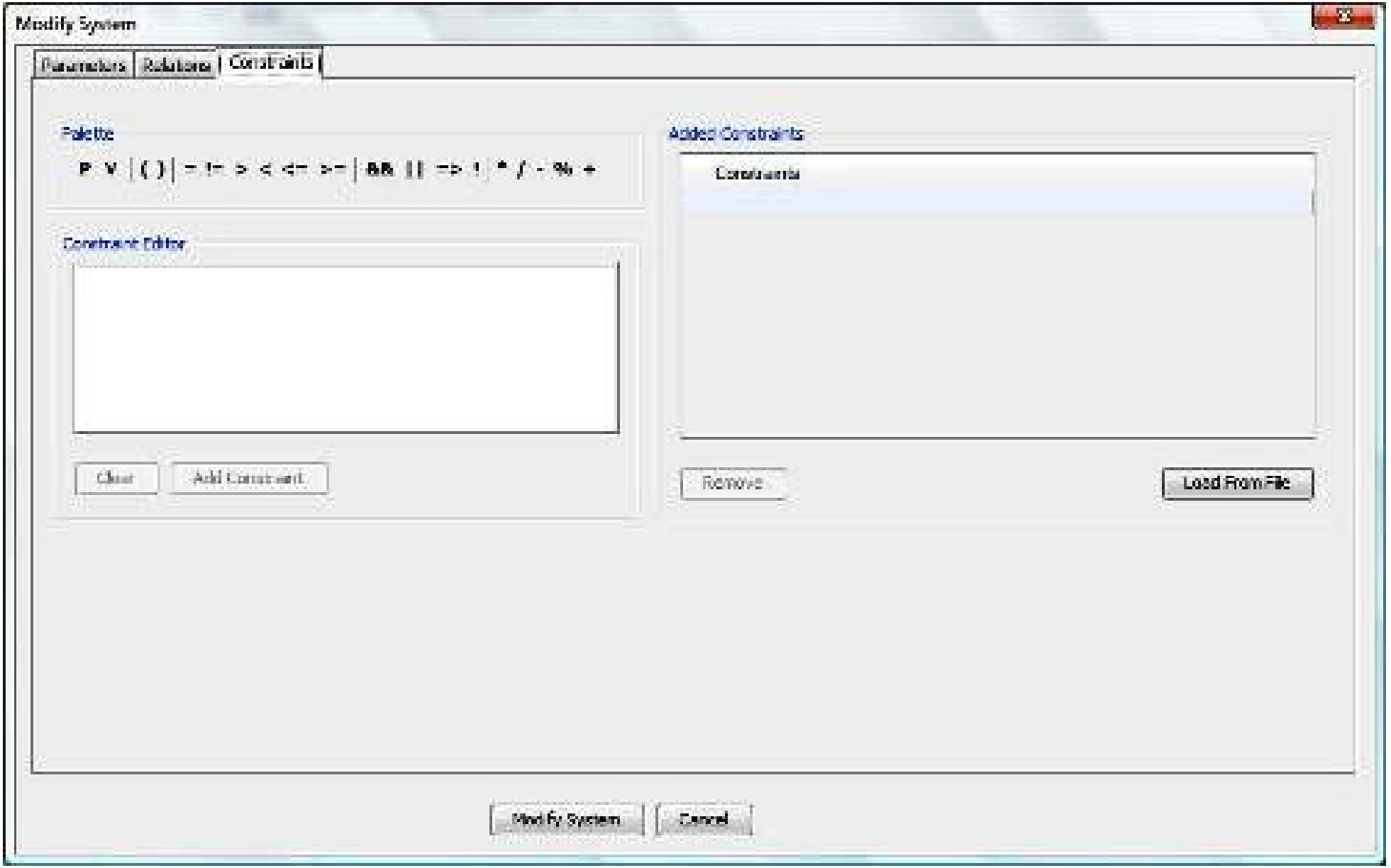
Defining a new system



Variable interaction strength



Constraints



Covering array output

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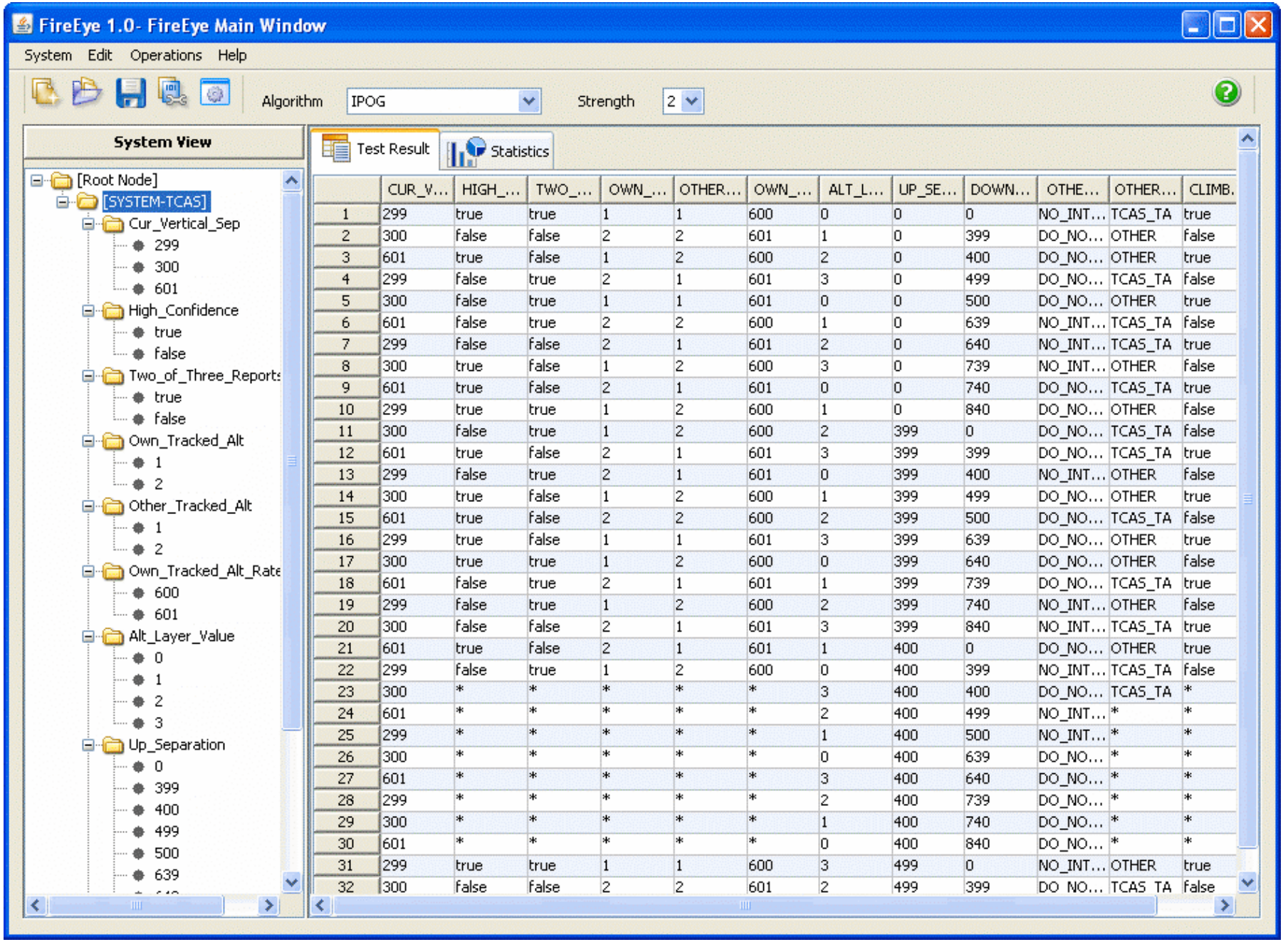

	CUR_V...	HIGH_...	TWO_...	OWN_...	OTHER...	OWN_...	ALT_L...	UP_SE...	DOWN...	OTHE...	OTHER...	CLIMB.
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2	300	false	false	2	2	601	1	0	399	DO_NO...	OTHER	false
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4	299	false	true	2	1	601	3	0	499	DO_NO...	TCAS_TA	false
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7	299	false	false	2	1	601	2	0	640	NO_INT...	TCAS_TA	true
8	300	true	false	1	2	600	3	0	739	NO_INT...	OTHER	false
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10	299	true	true	1	2	600	1	0	840	DO_NO...	OTHER	false
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13	299	false	true	2	1	601	0	399	400	NO_INT...	OTHER	false
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17	300	true	true	1	2	600	0	399	640	DO_NO...	OTHER	false
18	601	false	true	2	1	601	1	399	739	DO_NO...	TCAS_TA	true
19	299	false	true	1	2	600	2	399	740	NO_INT...	OTHER	false
20	300	false	false	2	1	601	3	399	840	NO_INT...	TCAS_TA	true
21	601	true	false	2	1	601	1	400	0	DO_NO...	OTHER	true
22	299	false	true	1	2	600	0	400	399	NO_INT...	TCAS_TA	false
23	300	*	*	*	*	*	3	400	400	DO_NO...	TCAS_TA	*
24	601	*	*	*	*	*	2	400	499	NO_INT...	*	*
25	299	*	*	*	*	*	1	400	500	NO_INT...	*	*
26	300	*	*	*	*	*	0	400	639	DO_NO...	*	*
27	601	*	*	*	*	*	3	400	640	DO_NO...	*	*
28	299	*	*	*	*	*	2	400	739	DO_NO...	*	*
29	300	*	*	*	*	*	1	400	740	DO_NO...	*	*
30	601	*	*	*	*	*	0	400	840	DO_NO...	*	*
31	299	true	true	1	1	600	3	499	0	NO_INT...	OTHER	true
32	300	false	false	2	2	601	2	499	399	DO_NO...	TCAS_TA	false

Output

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- Variety of output formats:
  - XML
  - Numeric
  - CSV
  - Excel
- Separate tool to generate .NET configuration files from ACTS output
- Post-process output using Perl scripts, etc.

Output options

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Mappable values

```
Degree of interaction
coverage: 2
Number of parameters: 12
Number of tests: 100

-----------------------------

0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 1 1 1 1 0 1 1 1 1
2 0 1 0 1 0 2 0 2 2 1 0
0 1 0 1 0 1 3 0 3 1 0 1
1 1 0 0 0 1 0 0 4 2 1 0
2 1 0 1 1 0 1 0 5 0 0 1
0 1 1 1 0 1 2 0 6 0 0 0
1 0 1 0 1 0 3 0 7 0 1 1
2 0 1 1 0 1 0 0 8 1 0 0
0 0 0 0 1 0 1 0 9 2 1 1
1 1 0 0 1 0 2 1 0 1 0 1
```

Etc.

Human readable

```
Degree of interaction coverage: 2
Number of parameters: 12
Maximum number of values per
parameter: 10
Number of configurations: 100
-----------------------------------
Configuration #1:

1 = Cur_Vertical_Sep=299
2 = High_Confidence=true
3 = Two_of_Three_Reports=true
4 = Own_Tracked_Alt=1
5 = Other_Tracked_Alt=1
6 = Own_Tracked_Alt_Rate=600
7 = Alt_Layer_Value=0
8 = Up_Separation=0
9 = Down_Separation=0
10 = Other_RAC=NO_INTENT
11 = Other_Capability=TCAS_CA
12 = Climb_Inhibit=true
```

Using ACTS

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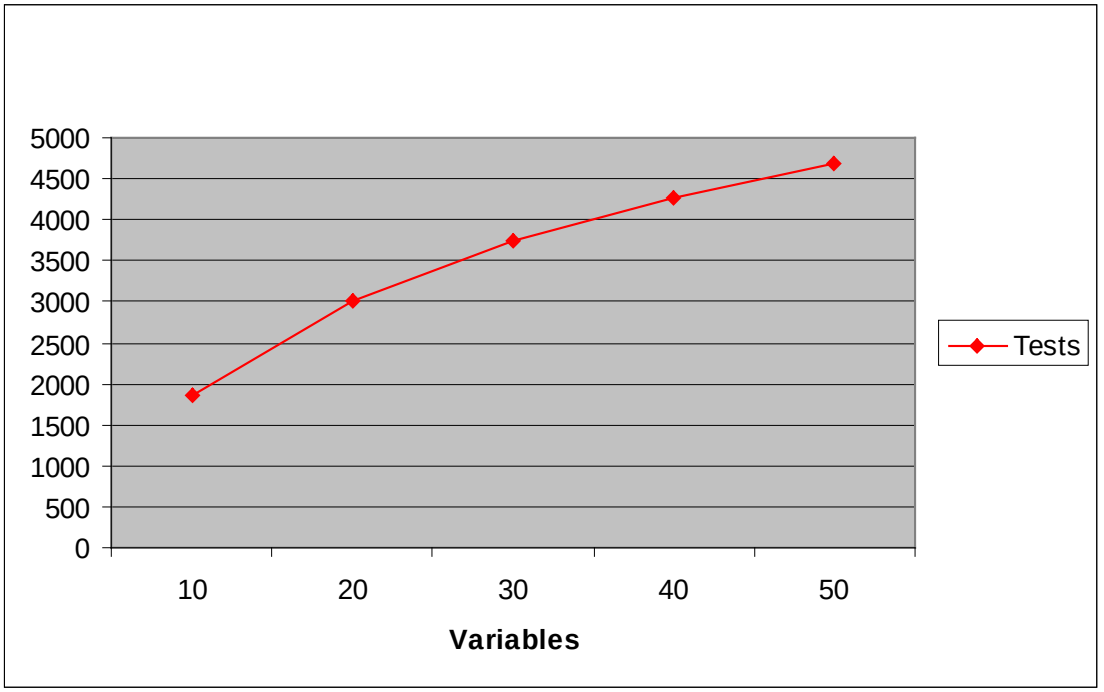
Cost and Volume of Tests

- Number of tests: proportional to $v^t \log n$
  for v values, n variables, t-way interactions
- *Thus:*
  - Tests increase exponentially with interaction strength t : BAD, but unavoidable
  - But only logarithmically with the number of parameters : GOOD!
- Example: suppose we want all 4-way combinations of n parameters, 5 values each:

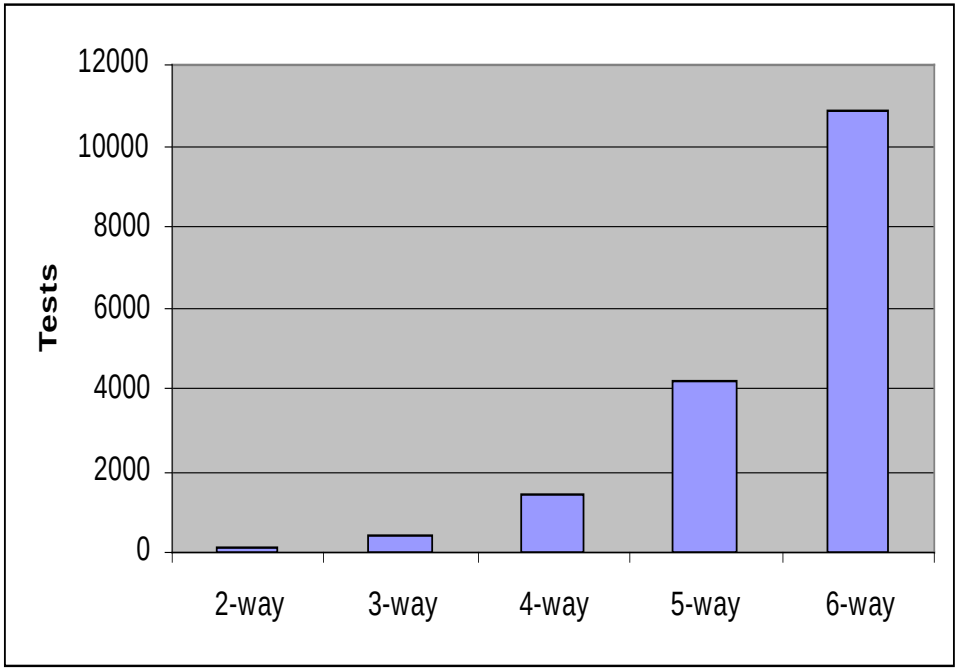


Example 1: Traffic Collision Avoidance System (TCAS) module

- Used in previous testing research
- 41 versions seeded with errors
- 12 variables: 7 boolean, two 3-value, one 4-value, two 10-value
- All flaws found with 5-way coverage
- Thousands of tests - generated by model checker in a few minutes

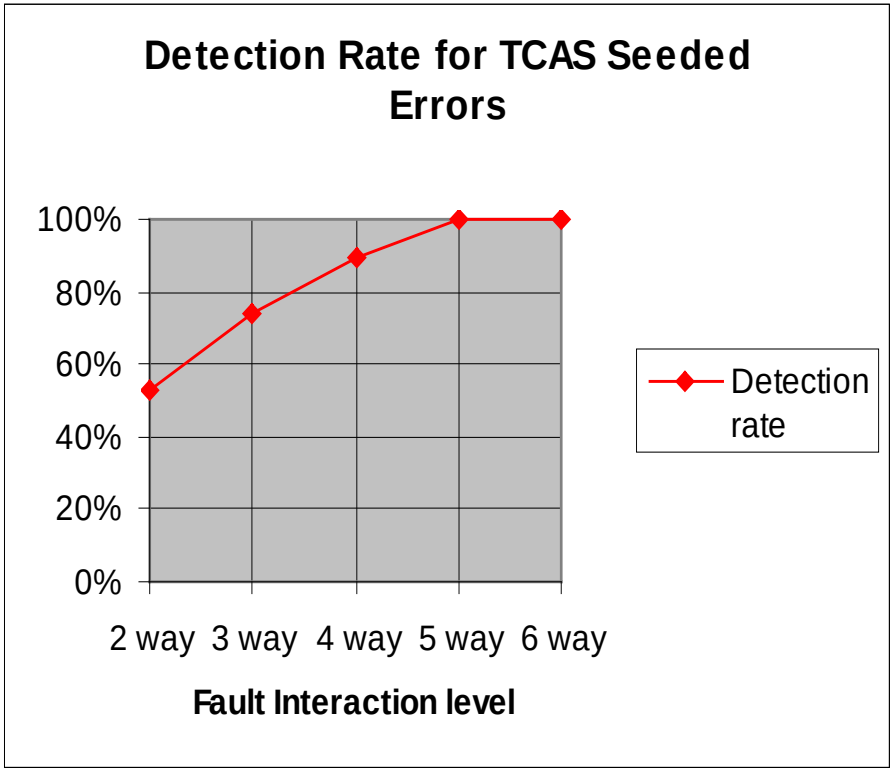
Tests generated

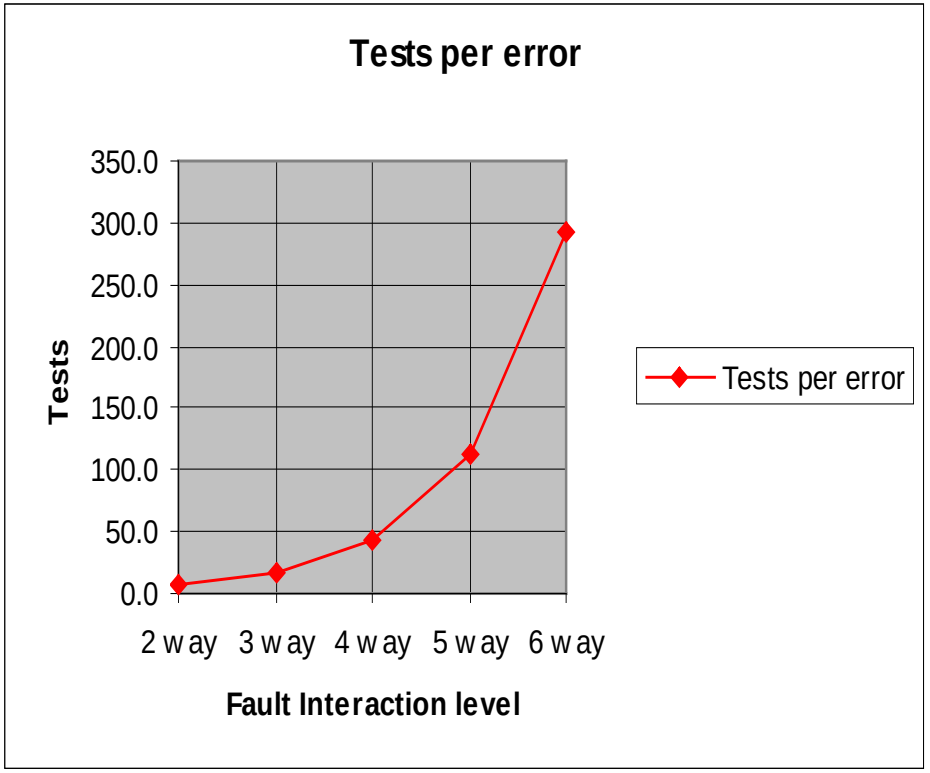
t	Test cases
2-way:	156
3-way:	461
4-way:	1,450
5-way:	4,309
6-way:	11,094



Results

- Roughly consistent with data on large systems
- But errors harder to detect than real-world examples





Bottom line for model checking based combinatorial testing:
Expensive but can be highly effective

EXAMPLE 2: Document Object Model Events

- DOM is a World Wide Web Consortium standard incorporated into web browsers

- NIST Systems and Software division develops tests for standards such as DOM

- DOM testing problem:
  - large number of events handled by separate functions
  - functions have 3 to 15 parameters
  - parameters have many, often continuous, values
  - verification requires human interaction (viewing screen)
  - testing takes a *long* time

DOM Functions

Event Name	Param.	Tests
Abort	3	12
Blur	5	24
Click	15	4352
Change	3	12
dblClick	15	4352
DOMActivate	5	24
DOMAttrModified	8	16
DOMCharacterDataModified	8	64
DOMElementNameChanged	6	8
DOMFocusIn	5	24
DOMFocusOut	5	24
DOMNodeInserted	8	128
DOMNodeInsertedIntoDocument	8	128
DOMNodeRemoved	8	128
DOMNodeRemovedFromDocument	8	128
DOMSubTreeModified	8	64
Error	3	12
Focus	5	24
KeyDown	1	17
KeyUp	1	17
Load	3	24
MouseDown	15	4352
MouseMove	15	4352
MouseOut	15	4352
MouseOver	15	4352
MouseUp	15	4352
MouseWheel	14	1024
Reset	3	12
Resize	5	48
Scroll	5	48
Select	3	12
Submit	3	12
TextInput	5	8
Unload	3	24
Wheel	15	4096
Total Tests		36626

Exhaustive testing of equivalence class values

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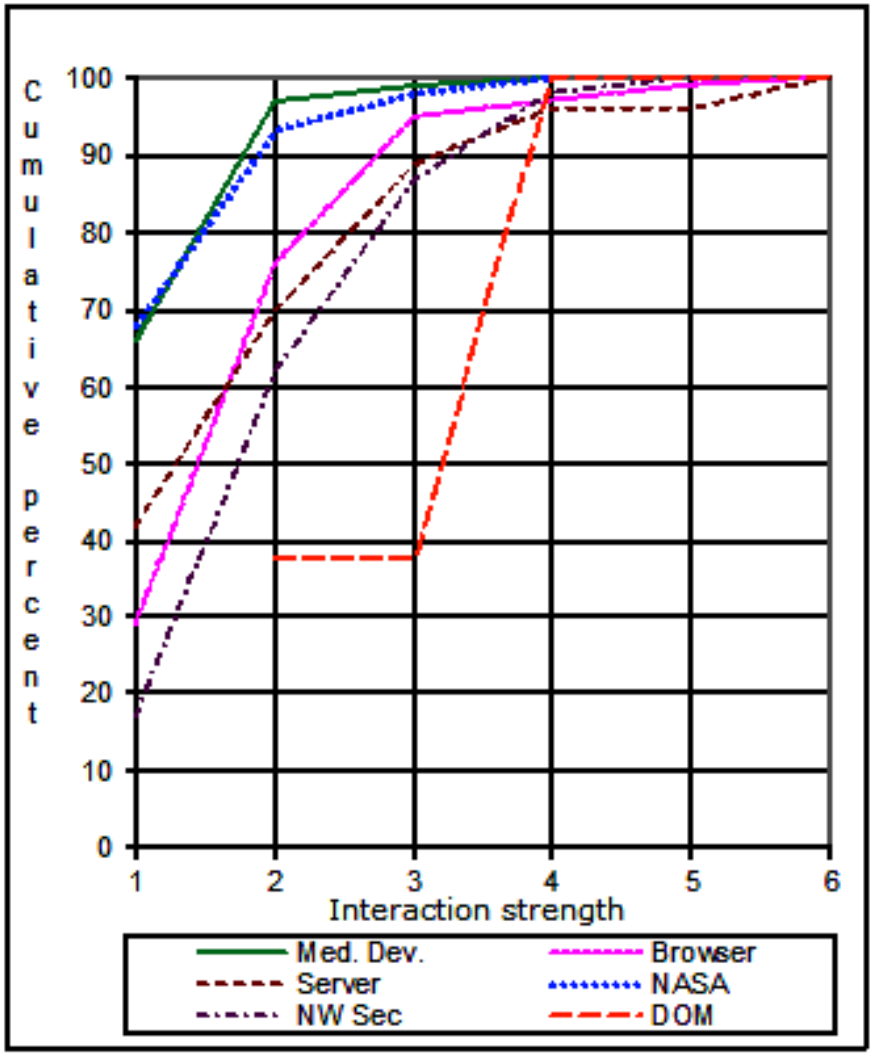
World Wide Web Consortium Document Object Model Events

t	Tests	% of Orig.	Test Results		
			Pass	Fail	Not Run
2	702	1.92%	202	27	473
3	1342	3.67%	786	27	529
4	1818	4.96%	437	72	1309
5	2742	7.49%	908	72	1762
6	4227	11.54 %	1803	72	2352

All failures found using < 5% of original exhaustive discretized test set



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SUMMARY

- Combinatorial testing is now a practical approach that produces high quality testing at lower cost
- Good algorithms and user-friendly tools are available – no cost tools from NIST, Microsoft, others
- Basic combinatorial testing can be used in two ways:
  - combinations of configuration values
  - combinations of input values
  - these can be used separately or at the same time
- Case studies are beginning to appear
- All tools and materials available at NIST web site csrc.nist.gov/acts

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