Efficient Actively Secure OT Extension: 5 Years Later[1] (Part I)

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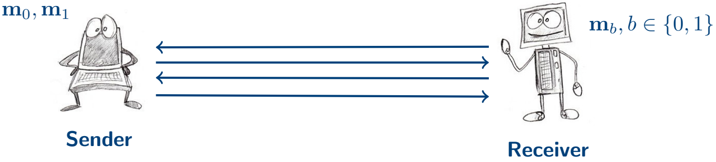
[1] Based on the paper *Efficient Actively Secure OT Extension*, M. Keller, E. Orsini, P. Scholl CRYPTO 2015

Oblivious transfer - Definition

Oblivious Transfer (OT) is a ubiquitous cryptographic primitive designed to transfer specific data based on the receiver's choice.

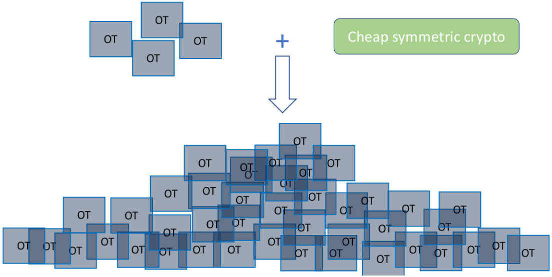


No further information should be learned by any party

Relevant to this workshop: distribution of keys for GC, Threshold ECDSA, etc..

Extending oblivious transfer - Motivation

- Impagliazzo, Rudich [IR98]
  Black-box separation result $\rightarrow$ OT is impossible without public-key primitives (?)
- Beaver [Beaver96]: OT can be extended



OT-extension: 2003-2020

- **Y. Ishai, J. Kilian, K. Nissim, E. Petrank**
  "Extending oblivious transfers efficiently", CRYPTO 2003
- G. Asharov, Y. Lindell, T. Schneider, and M. Zohner
  More Efficient Oblivious Transfer and Extensions for Faster Secure Computation, ACM CCS 2013
- V. Kolesnikov, R. Kumaresan
  Improved OT extension for transferring short secrets, CRYPTO 2013

+ J. B. Nielsen, P. S. Nordholt, C. Orlandi, and S. S. Burra.
  A new approach to practical active-secure two-party computation, CRYPTO 2012
+ G. Asharov, Y. Lindell, T. Schneider, and M. Zohner
  More efficient oblivious transfer extensions with security for malicious adversaries, EUROCRYPT 2015
+ **M. Keller, E. Orsini, P. Scholl**
  Actively Secure OT Extension with Optimal Overhead, CRYPTO 2015
+ **M. Orrù, E. Orsini, P. Scholl**
  Actively Secure 1-out-of-N OT Extension with Application to Private Set Intersection, CT-RSA 2017

x D. Masny, P. Rindal
  Endemic Oblivious Transfer, CCS 2019

x C. Guo, J. Katz, X. Wang, Y. Yu
  Efficient and Secure Multiparty Computation from Fixed-Key Block Ciphers, IEEE S&P 2020

* **E. Boyle, G. Couteau, N. Gilboa, Y. Ishai, L. Kohl, P. Scholl Efficient Pseudorandom Correlation Generators: Silent OT Extension and More, CRYPTO 2019**

OT, Correlated OT and Random OT

Standard OT and COT functionality (Sender chosen message)

OT and COT with uniform message security

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Endemic security [MR19]

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IKNP OT-extension

	Receiver 	Sender 
INPUT.	$(x_1, \ldots, x_m) \in \{0,1\}^m$	$\mathbf{m}_{0,i}, \mathbf{m}_{1,i} \in \{0,1\}^k$ $i \in [m], k \ll m$
1. m COT	$\mathbf{t}_i, \mathbf{x}$ $\mathbf{t}_i \in \{0,1\}^k, i \in [m]$	$\mathbf{q}_i, \Delta$ $\mathbf{t}_i = \mathbf{q}_i + x_i \cdot \Delta$
2. RO	$\mathbf{m}_{x_i,i} = H(\mathbf{t}_i, i) + \mathbf{c}_{x_i,i}$	Send: $\mathbf{c}_{0,i} = H(\mathbf{q}_i, i) + \mathbf{m}_{0,i}$ $\mathbf{c}_{1,i} = H(\mathbf{q}_i + \Delta, i) + \mathbf{m}_{1,i}$

IKNP OT extension - Security

- Assuming that Phase 1. of the protocol is passively/actively secure then
  - IKNP is passively/actively secure when H is a random oracle
  - For passive security it is enough for H to be a correlation robust hash function [IKNP03]
  - For active security H has to be a tweakable correlation robust hash function
- To achieve active security we need:
  - Prove that Phase 1 is secure
    1. Achieve security against a malicious receiver
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Proctecting against a malicious receiver - Attack

$$\begin{array}{c} \mathbf{q}_1 = \mathbf{t}_1 + x_1 \cdot \Delta \\ \mathbf{q}_2 = \mathbf{t}_2 + x_2 \cdot \Delta \\ \mathbf{q}_3 = \mathbf{t}_3 + x_3 \cdot \Delta \\ \\ \vdots \\ \\ \mathbf{q}_m = \mathbf{t}_m + x_m \cdot \Delta \end{array} \begin{pmatrix} t_{1,1} + x_1 \cdot \Delta_1 & \dots & t^{\kappa}_{1,k} + \cdot x_1 \cdot \Delta_k \\ t_{2,1} + x_2 \cdot \Delta_1 & \dots & t^{\kappa}_{2,k} + x_2 \cdot \Delta_k \\ t_{3,1} + x_3 \cdot \Delta_1 & \dots & t^{\kappa}_{3,k} + tx_3 \cdot \Delta_k \\ \vdots & \dots & \vdots \\ \vdots & \ddots & \vdots \\ \vdots & \dots & \vdots \\ t_{m,1} + x_m \cdot \Delta_1 & \dots & t_{m,k} + x_m \cdot \Delta_k \end{pmatrix}$$

Protecting against a malicious receiver - Attack

$$\mathbf{q}_1 = \mathbf{t}_1 + (\Delta_1, 0, \ldots, 0)$$

$$\mathbf{q}_2 = \mathbf{t}_2 + (0, \Delta_2, 0, \ldots, 0)$$

$$\mathbf{q}_3 = \mathbf{t}_3 + (0, 0, \Delta_3, 0, \ldots, 0)$$

$$\vdots$$

$$\begin{pmatrix} t_{1,1} + \Delta_1 & \ldots & \ldots & t^{\kappa}_{1,k} \\ t_{2,1} & t_{2,2} + \Delta_2 & \ldots & t^{\kappa}_{2,k} \\ t_{3,1} & \ldots & \ldots & t^{\kappa}_{3,k} \\ \vdots & \vdots & \ldots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ldots & \vdots \\ t_{m,1} & \ldots & \ldots & t_{m,k} + \Delta_k \end{pmatrix}$$

- $\mathbf{c}_{0,1} = H(\mathbf{q}_1, 1) + \mathbf{m}_{0,1} = H(\mathbf{t}_1 + (\Delta_1, 0, \ldots, 0), 1) + \mathbf{m}_{0,1}$, can extract Δ_1
- Repeating the attack can recover the entire Δ and hence all the messages

Protecting against a malicious receiver - Consistency check

	Receiver 	**Sender** 
INPUT	$(x_1, \dots, x_m) \in \{0,1\}^m$ $(x_{m+1}, \dots, x_{m'}) \in \{0,1\}^{m'-m}$, $m' - m = k + s$	$\mathbf{m}_{0,i}, \mathbf{m}_{1,i} \in \{0,1\}^k$ $i \in [m'], k \ll m'$
1. m COT$^-$	$\mathbf{t}_i, \mathbf{x}$ $\mathbf{t}_i \in \{0,1\}^k, i \in [m']$	$\mathbf{q}_i, \Delta$ $\mathbf{q}_i + \mathbf{t}_i = x_i \cdot \Delta$
2. CHECK	Receive $\chi_1, \dots, \chi_{m'} \in \mathbb{F}_{2^k}$ Send $t = \sum_i \chi_i t_i$ and $x = \sum_i \chi_i x_i$	Compute $q = \sum_i \chi_i q_i$ and check that $t = q + x \cdot \Delta$
3. RO	$\mathbf{m}_{x_i,i} = H(\mathbf{t}_i, i) + \mathbf{c}_{x_i,i}$	$\mathbf{c}_{0,i} = H(\mathbf{q}_i, i) + \mathbf{m}_{0,i}$ $\mathbf{c}_{1,i} = H(\mathbf{q}_i + \Delta, i) + \mathbf{m}_{1,i}$

Part II: Instantiating the Primitives; and Silent OT Extension

Instantiating the Base OTs [Masny-Rindal 19]

Some instantiations allow corrupt parties to bias random-OT outputs

- $(\mathsf{OT} \text{ or } \mathsf{OT}^-) \xrightarrow{\text{OT-ext}} (\mathsf{COT}^-, \mathsf{ROT}^- \text{ or } \mathsf{OT})$
- $(\mathsf{OT} \text{ or } \mathsf{OT}^-) \not\xrightarrow{\text{OT-ext}} \mathsf{ROT}$

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- (OT or OT$^-$) $\xrightarrow{\text{OT-ext}}$ (COT$^-$, ROT$^-$ or OT)
- (OT or OT$^-$) $\overset{\text{OT-ext}}{\not\longrightarrow}$ ROT

		Receiver 	Sender 
	Input	$x_1 \in \{0,1\}$	
1.	m COT	$\mathbf{t}, x_1 \in \{0,1\}^k$ $\mathbf{t} \in \{0,1\}^k$	$\mathbf{q}, \Delta$ $\mathbf{q} + \mathbf{t} = x_1 \cdot \Delta$
2.	Check		
3.	RO	$\mathbf{m}_{x_1} \; = H(\mathbf{t}, 1)$	$\mathbf{m}_0 = H(\mathbf{q}, 1)$ $\mathbf{m}_1 = H(\mathbf{q} + \Delta, 1)$

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		Receiver 	Sender 
	INPUT	$x_1 \in \{0,1\}$	
1.	m COT	$\mathbf{0} \in \{0,1\}^k$ $\mathbf{0}, x_1 = 1$	$\mathbf{q}, \Delta$ $\mathbf{q} = \Delta$
2.	CHECK		
3.	RO	$\mathbf{m}_1 = H(\mathbf{0}, 1)$	$\mathbf{m}_0 = H(\mathbf{q}, 1)$ $\mathbf{m}_1 = H(\mathbf{0}, 1)$

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- COT^- or ROT^- enough for OT and most applications
  - But not always: e.g. be careful with ROT^- and some PSI protocols

- If true ROT needed, protocols can be modified:

$$\mathsf{OT}^- \xrightarrow{\text{OT-ext}} \mathsf{COT}^- \xrightarrow{\text{coin}} \mathsf{ROT}$$

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Instantiating the hash function $H(x, i)$ [GKWY 20]

Security requirement: form of *correlation robustness*

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- SHA 256: straightforward, but slow
- Fixed-key block cipher, e.g. AES
  - ≈ 10x faster
  - Incorporating index i: can be done with one extra AES call [GKWY20]

Instantiating the hash function $H(x, i)$ [GKWY 20]

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- SHA 256: straightforward, but slow
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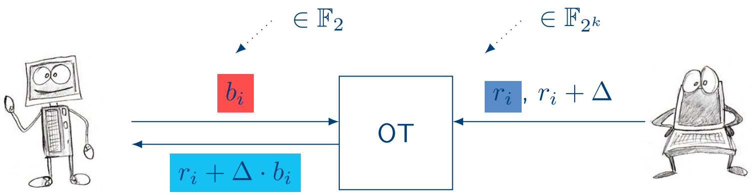
  - Incorporating index i: can be done with one extra AES call [GKWY20]
- What if i is omitted?
  - Can lead to attack, depending on base OTs [MR19]

Silent OT Extension: a Different Approach to Correlated OT [BCGIKS19]



As vectors: variant of vector-OLE with $b_i \in \mathbb{F}_2$

$$\Delta \cdot \mathbf{b} = \mathbf{r} + \Delta \cdot \mathbf{b} + \mathbf{r}$$

Silent OT: compress vector-OLE with a pseudorandom correlation generator (PCG)

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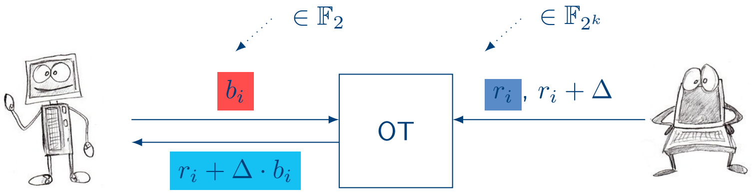

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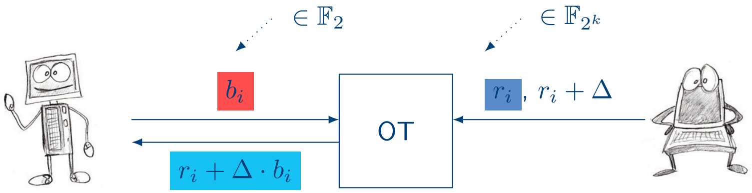

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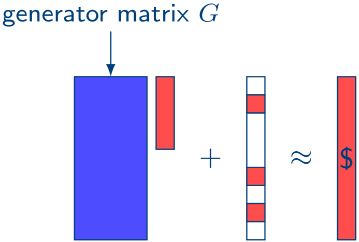
From a PCG to Silent OT Extension

1. Setup protocol for generating keys [BCGIKRS19, SGRR19]
   - 2-round setup for puncturable PRF
2. Malicious security [BCGIKRS19,YWLZW20]
   - Consistency check (similar to [KOS15]), $< 10\%$ overhead

Security of Silent OT: variants of Learning Parity with Noise

Primal-LPN:



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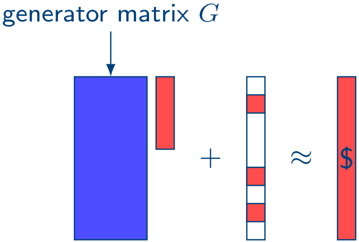

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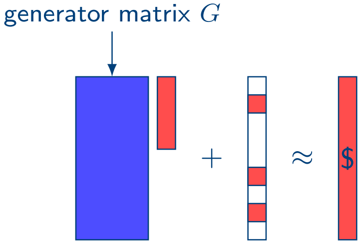

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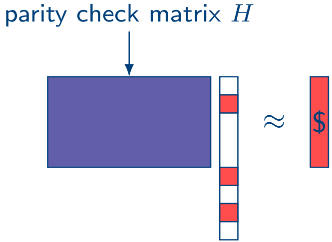

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H must be dense; use *quasi-cyclic codes*

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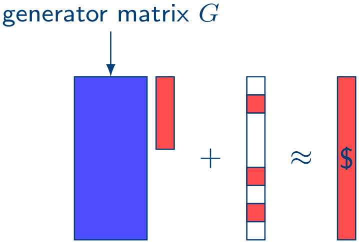

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Security as in [Ale03]

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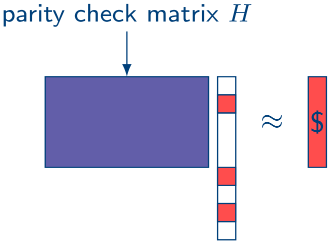

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Security as in BIKE, HQC schemes

Comparing practical, actively secure OT extension protocols

128-bit security; estimates for 10 million random OTs

Reference	Silent	Rounds	Communication	Computation	Based on
[KOS15]	✗	$3/5^*$	160 MB	$\approx 0.2s$	crh
[BCGIKRS19]	✓	$2/4^*$	80 kB	$\approx 2.0s$	QC-reg-LPN, crh
[YWLZW20]	✓	$O(1)$	2.4 MB	≈ 0.3s	sparse-reg-LPN, crh
[YWLZW20]	✓	$O(1)$	2.1 MB	≈ 0.2s	sparse-LPN, crh

* passive/active; crh = correlation robust hash function

Conclusion

- Pitfalls when implementing OT extension
  - Take care with hashing, and security of random OT

- Many flavours of OT extension to choose from:
  - Correlated OT, random OT
  - 1-out-of-2, 1-out-of-N
  - IKNP-style, silent