A Multiparty Computation Approach to Threshold ECDSA

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Based on work in papers from IEEE S&P 2018 and IEEE S&P 2019

Threshold Signature

$$\mathsf{sk}\ , \mathsf{pk} \leftarrow \mathsf{Gen}(1^\kappa)$$

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$$\{\mathsf{sk}_{\mathsf{A}}, \mathsf{sk}_{\mathsf{B}}, \mathsf{sk}_{C}\} \leftarrow \mathrm{Share}(\mathsf{sk}), \mathsf{pk} \leftarrow \mathsf{Gen}(1^{\kappa})$$

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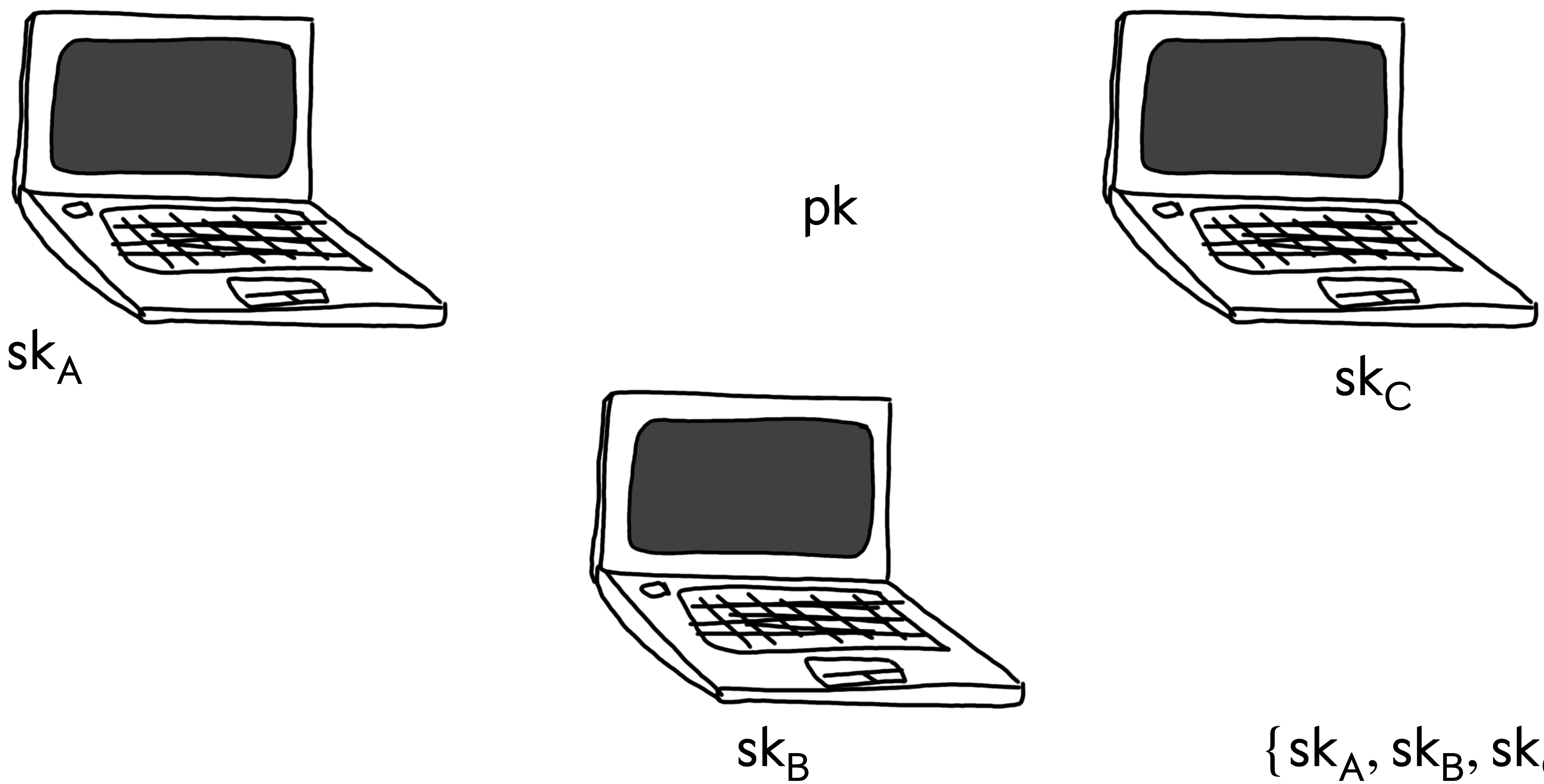

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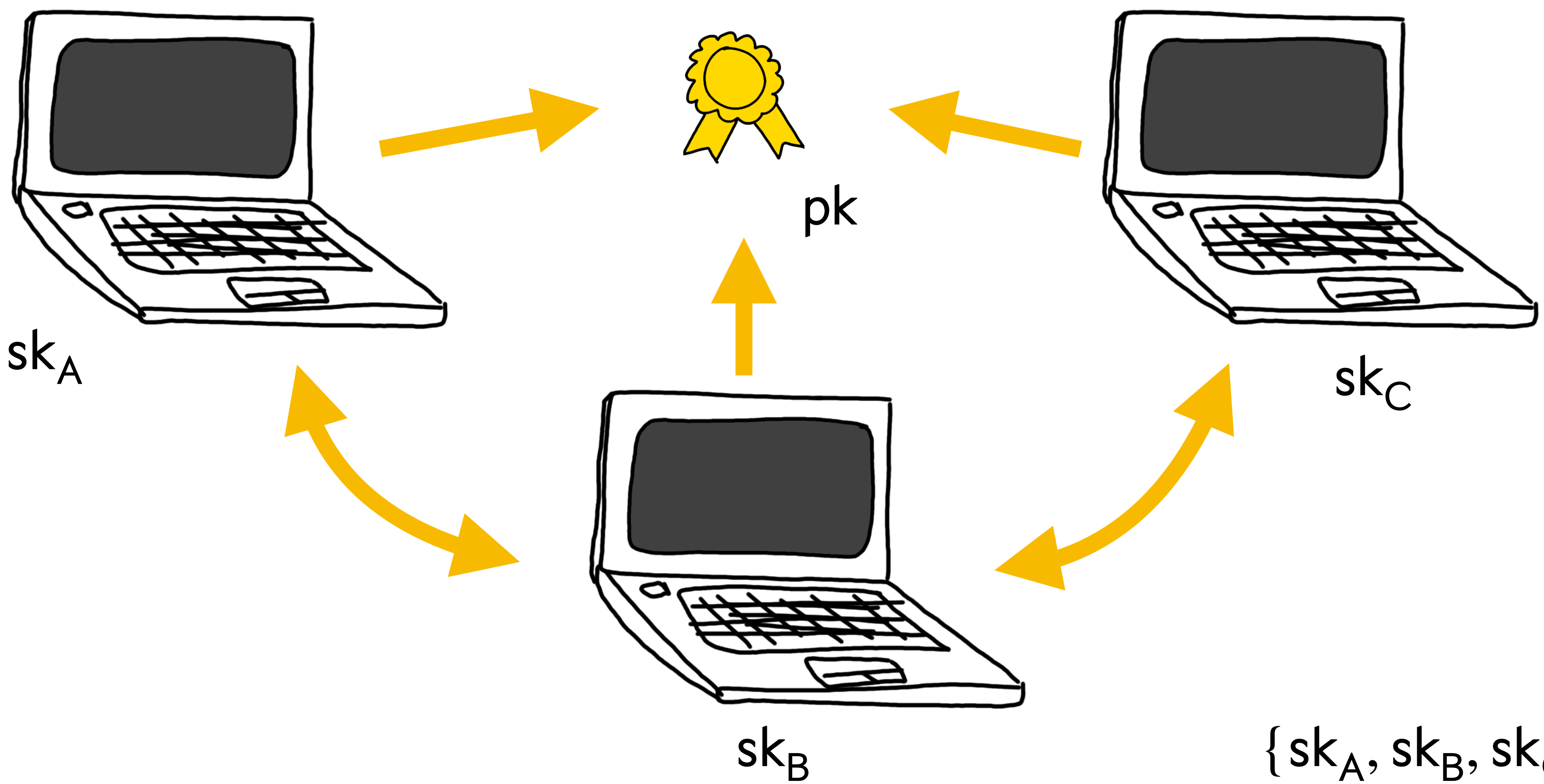

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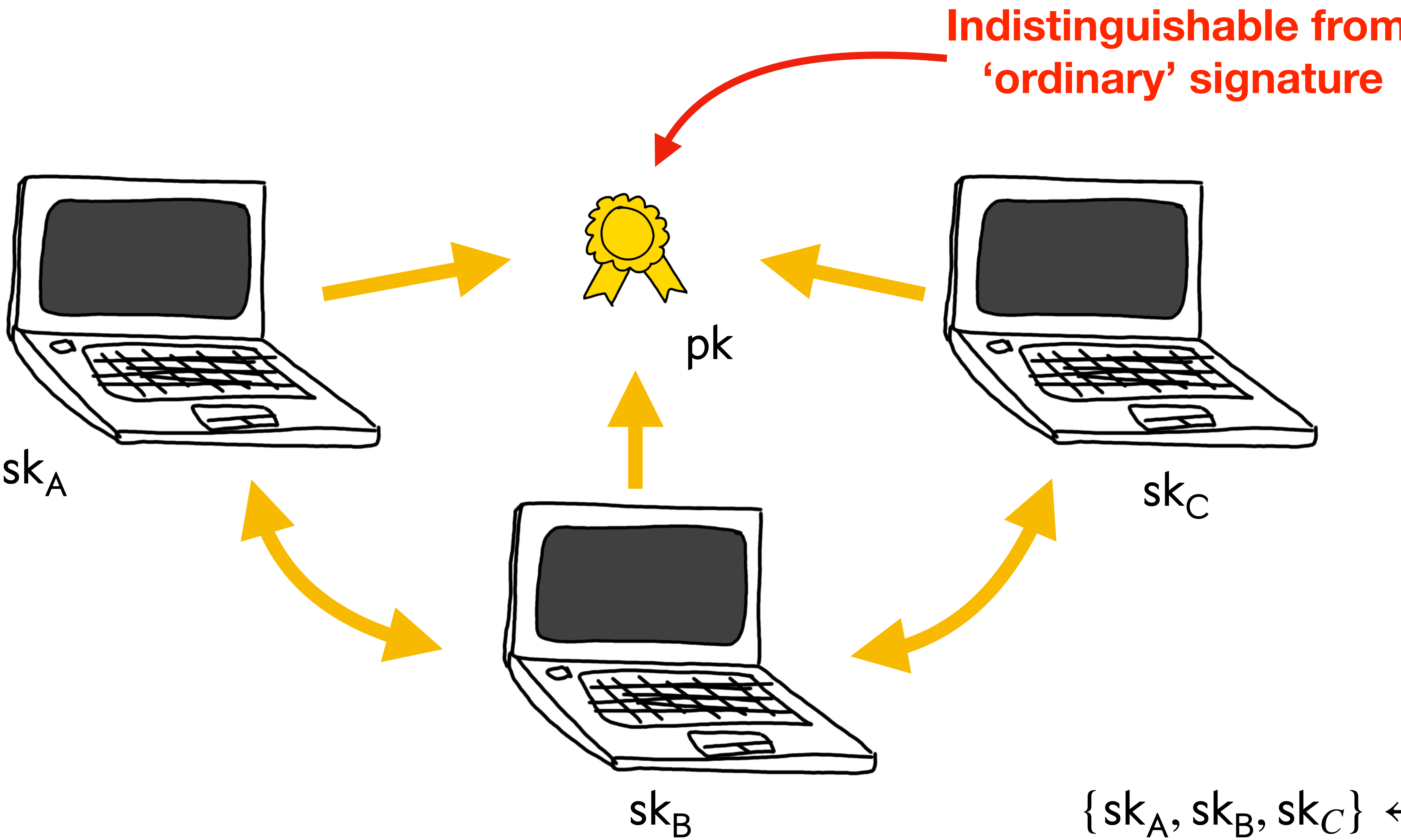

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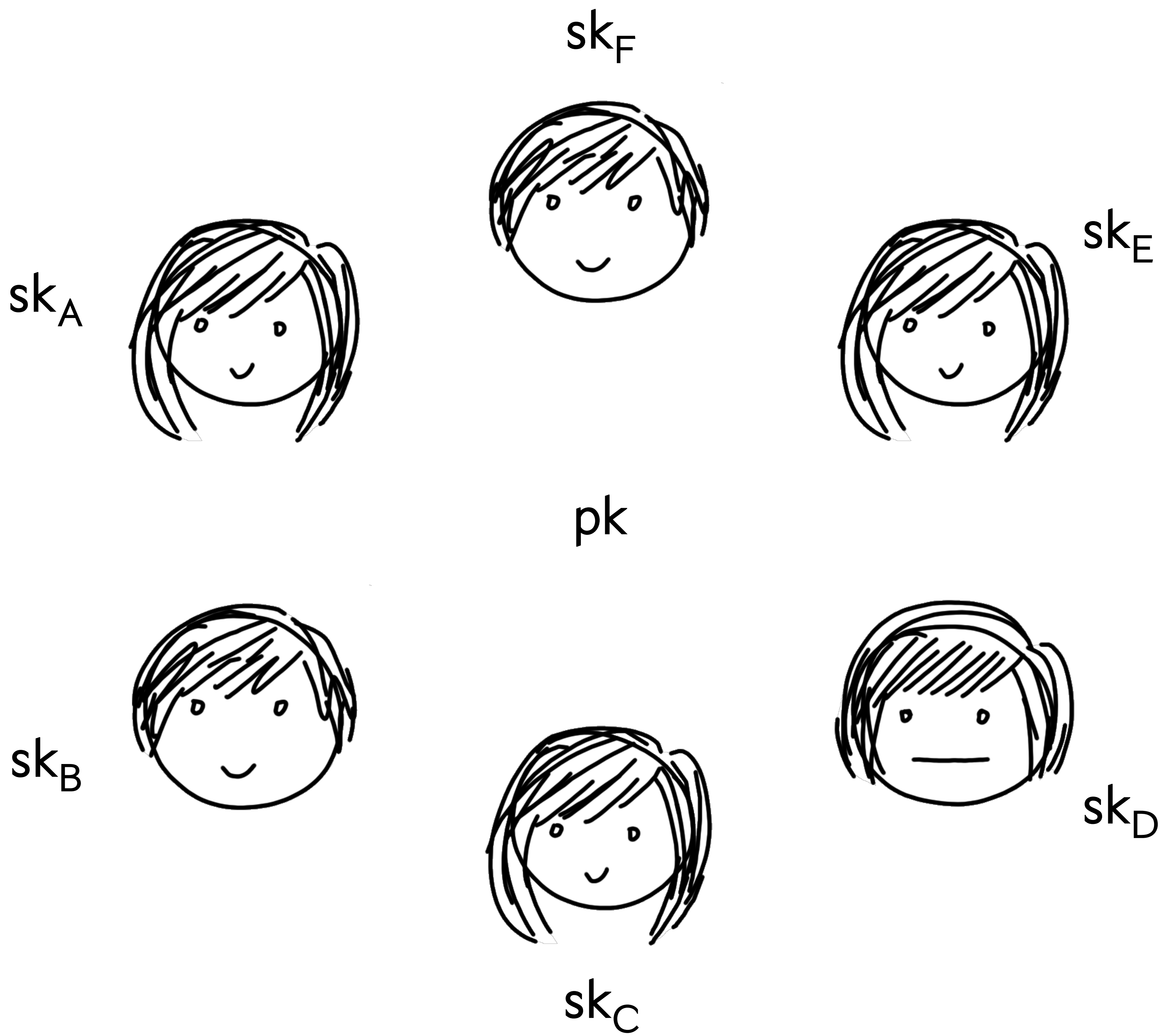

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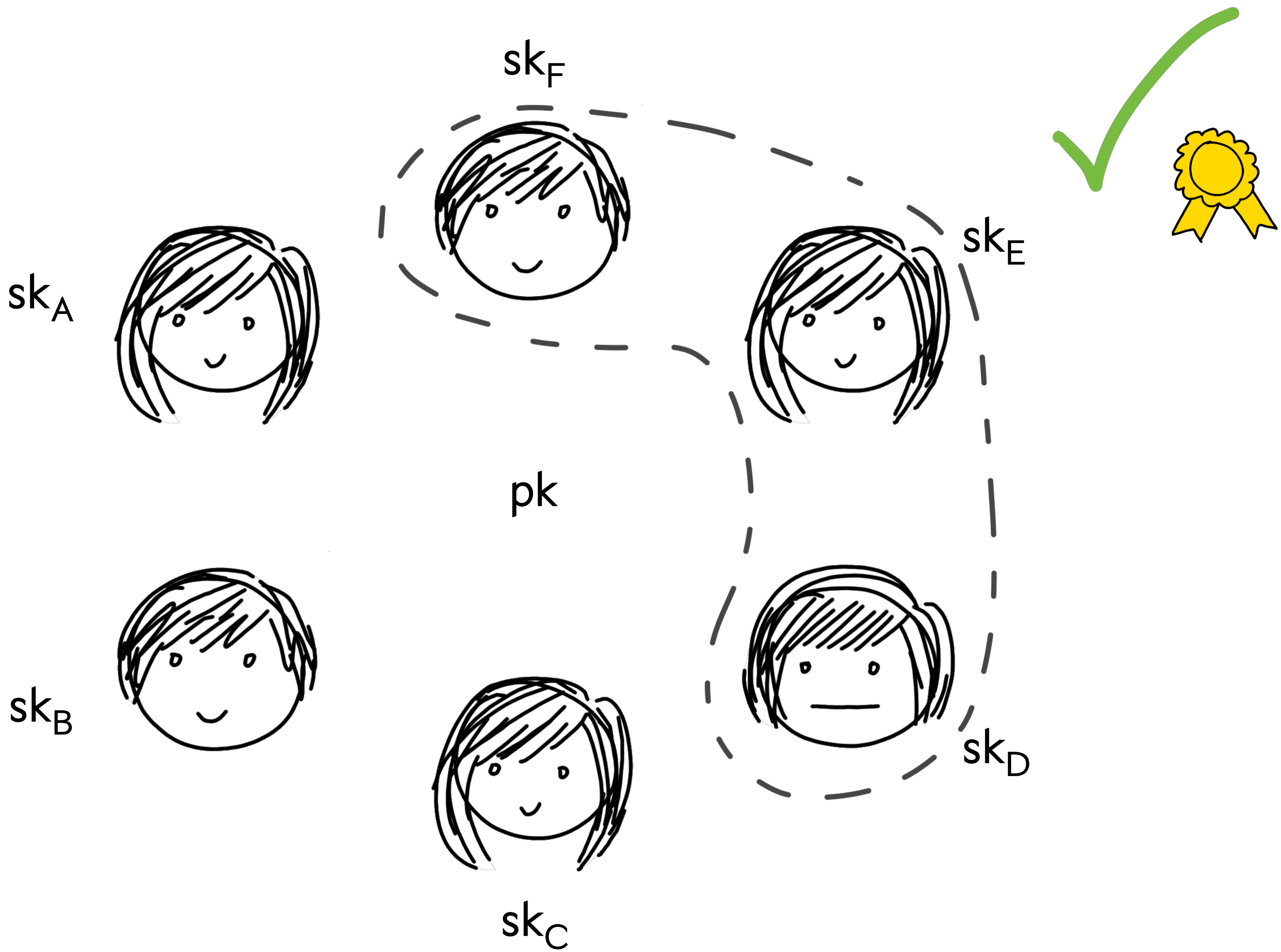

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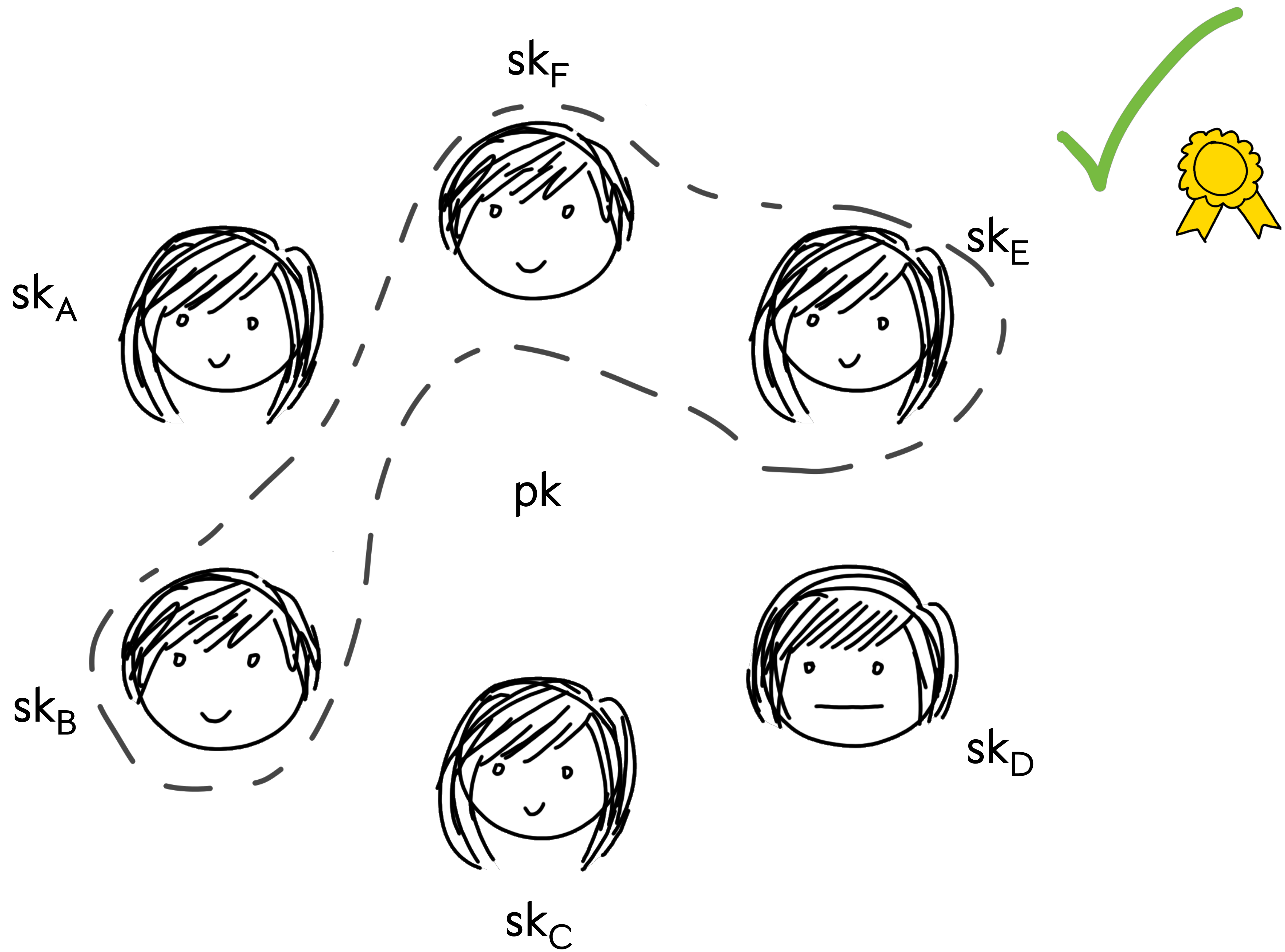

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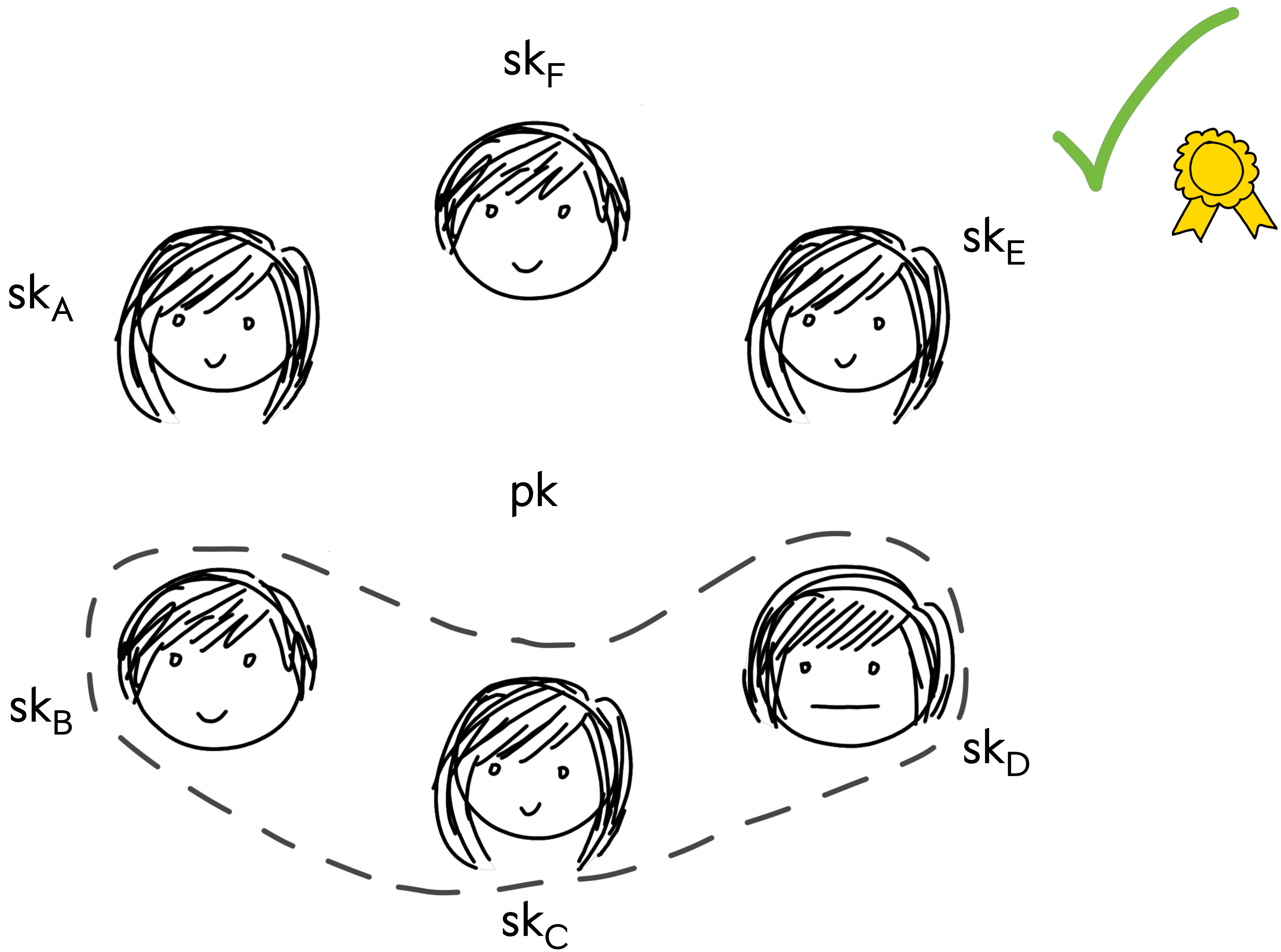

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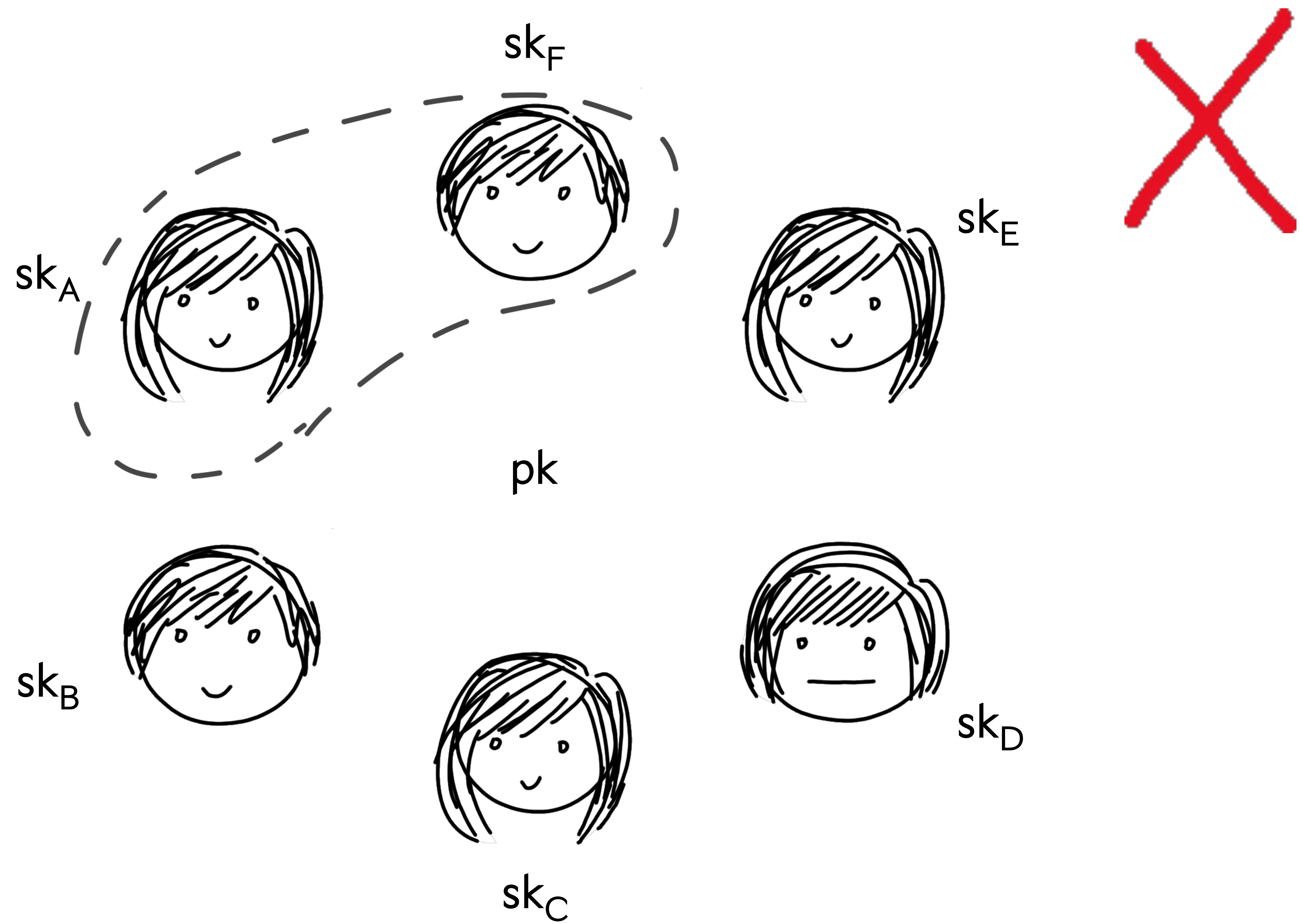

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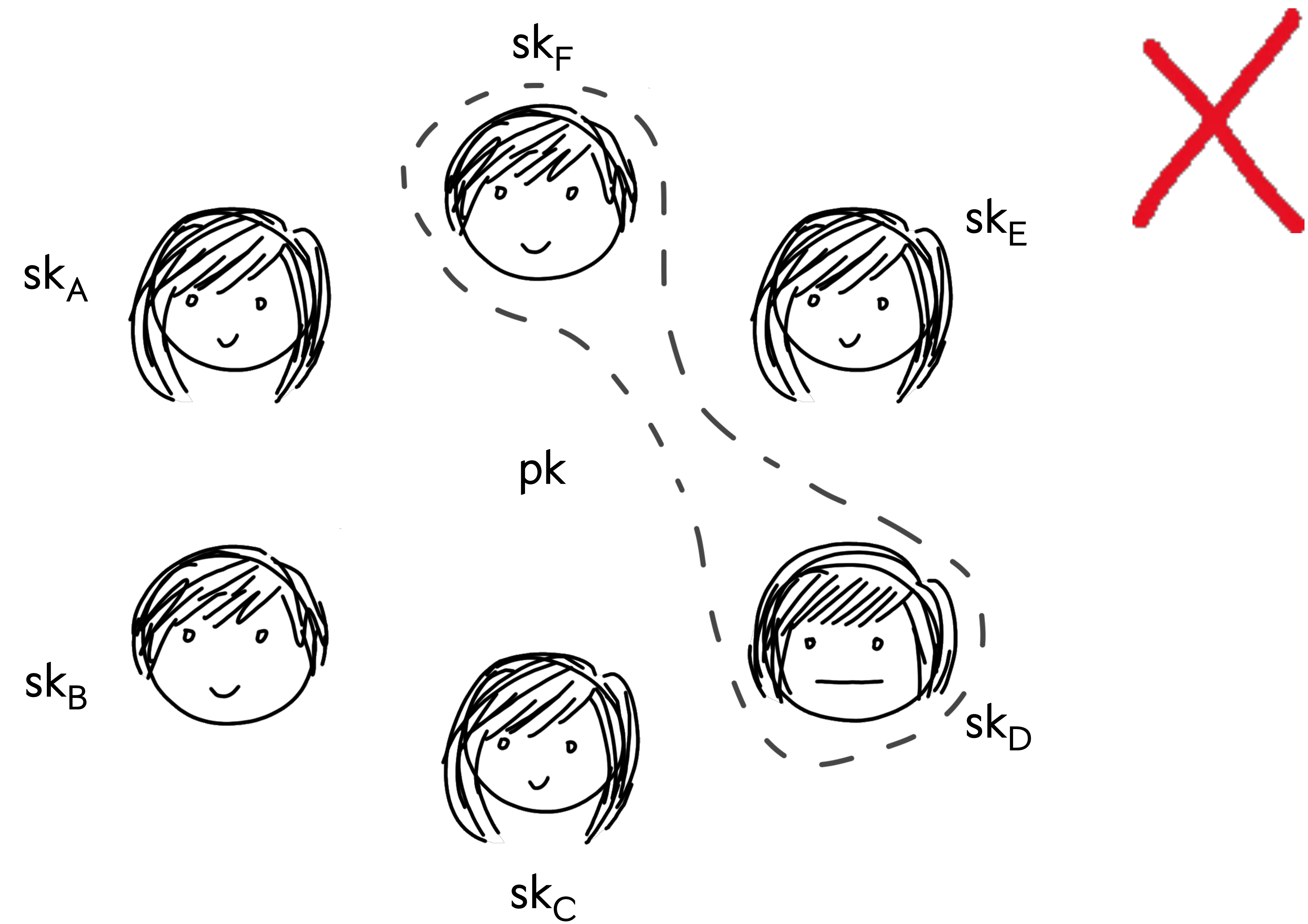

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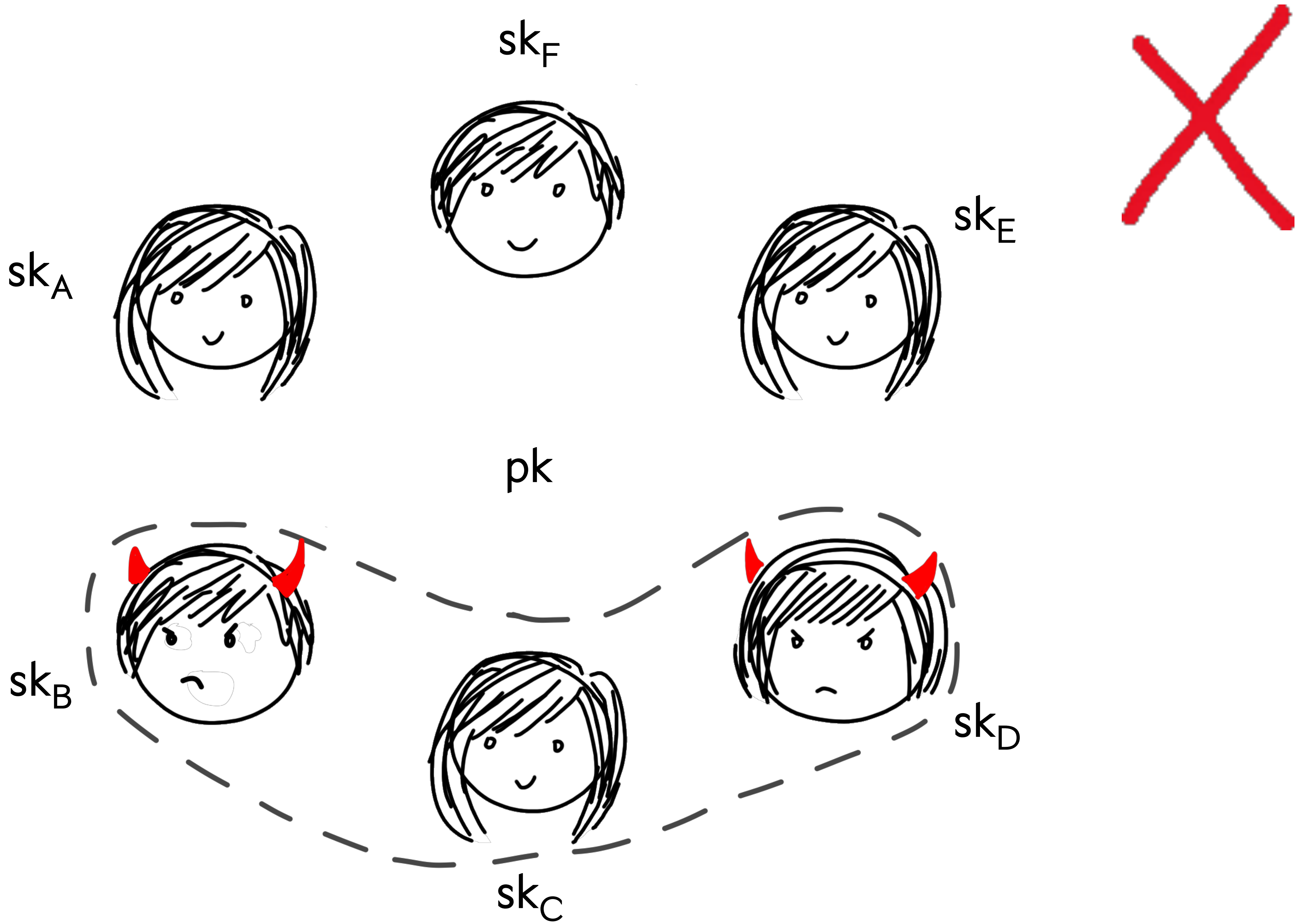

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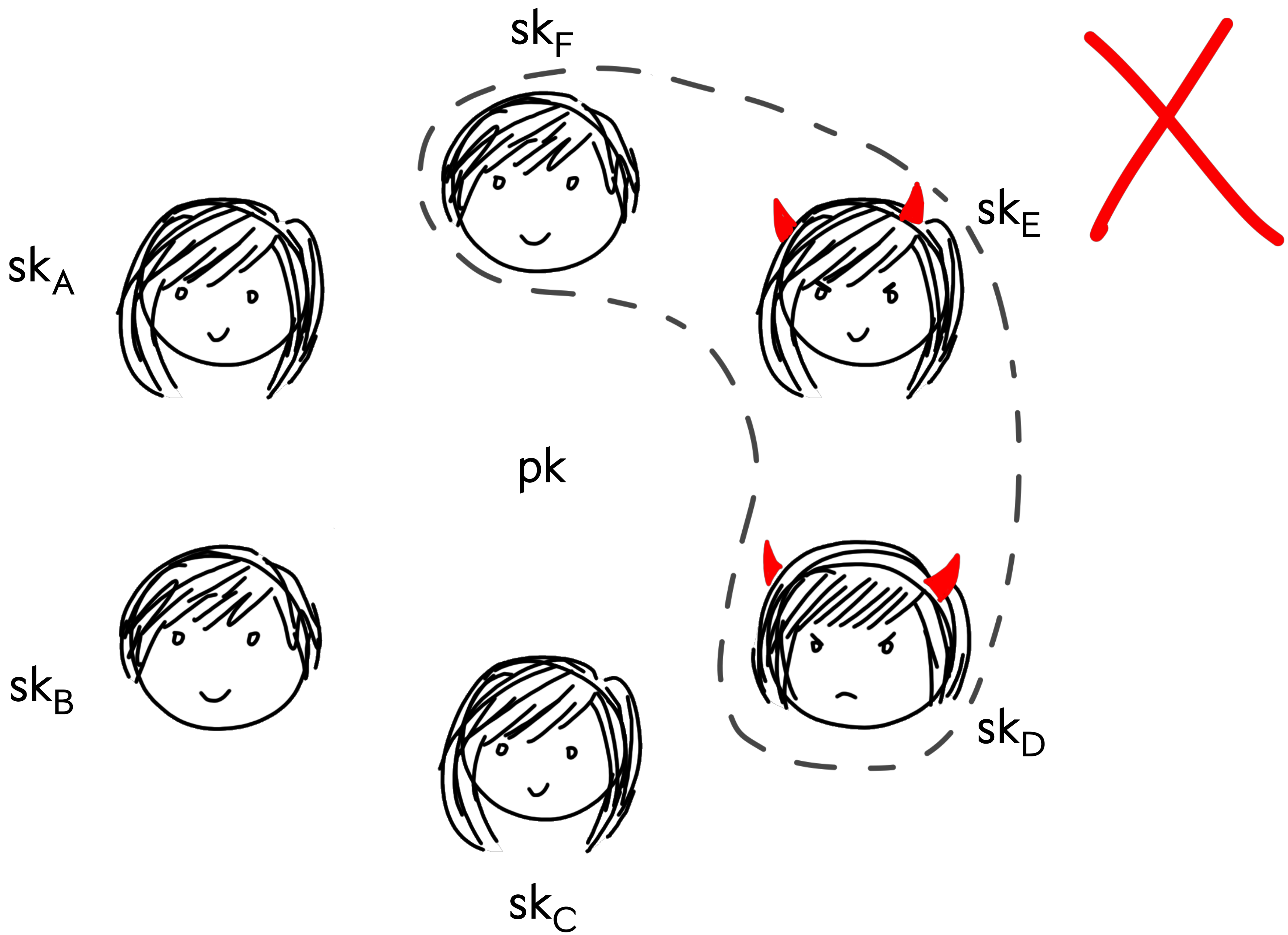

Full Threshold

- Scheme can be instantiated with any $t \leq n$
- Adversary can corrupt up to t-1 parties

Notation

Elliptic curve parameters	G	q
Secret values	sk	k
Public values	pk	R

Schnorr Signatures

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Linear function of *k*, sk

Threshold friendly w. linear secret sharing

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x – **coordinate of** R

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Bottleneck for threshold setting

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  - Benchmarks: order of magnitude better wall-clock time

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  - Verify in the exponent that parties' shares are on the same polynomial

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- **Alternative**: [Bar-Ilan&Beaver '89] approach yields constant round protocol (work in progress)

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- We harden to **full malicious security** in the RO model

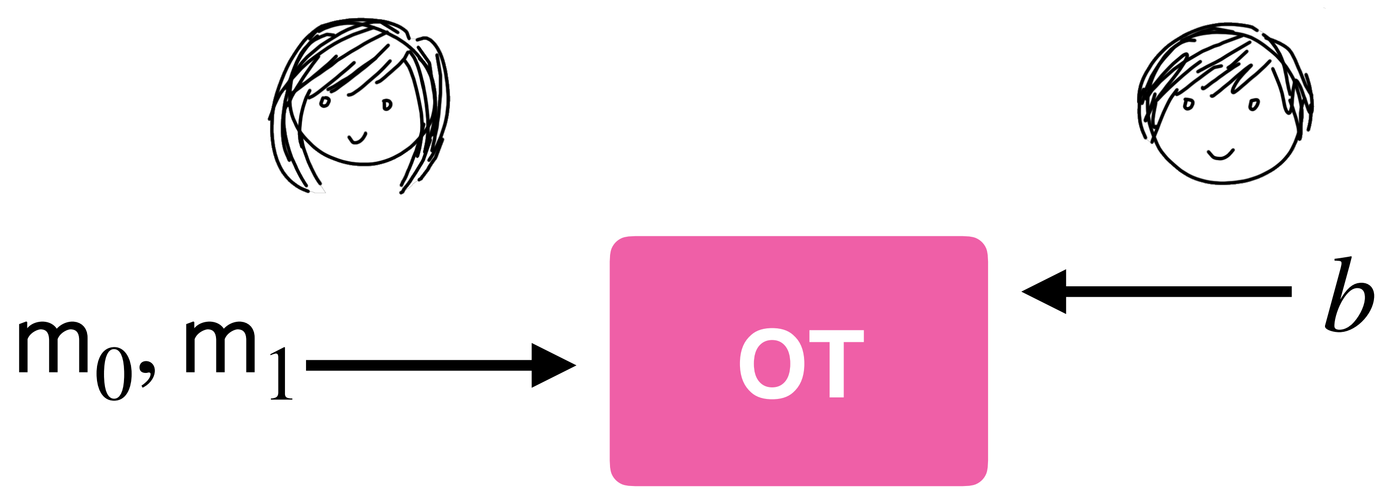
Oblivious Transfer

m_0, m_1

OT

b

Oblivious Transfer



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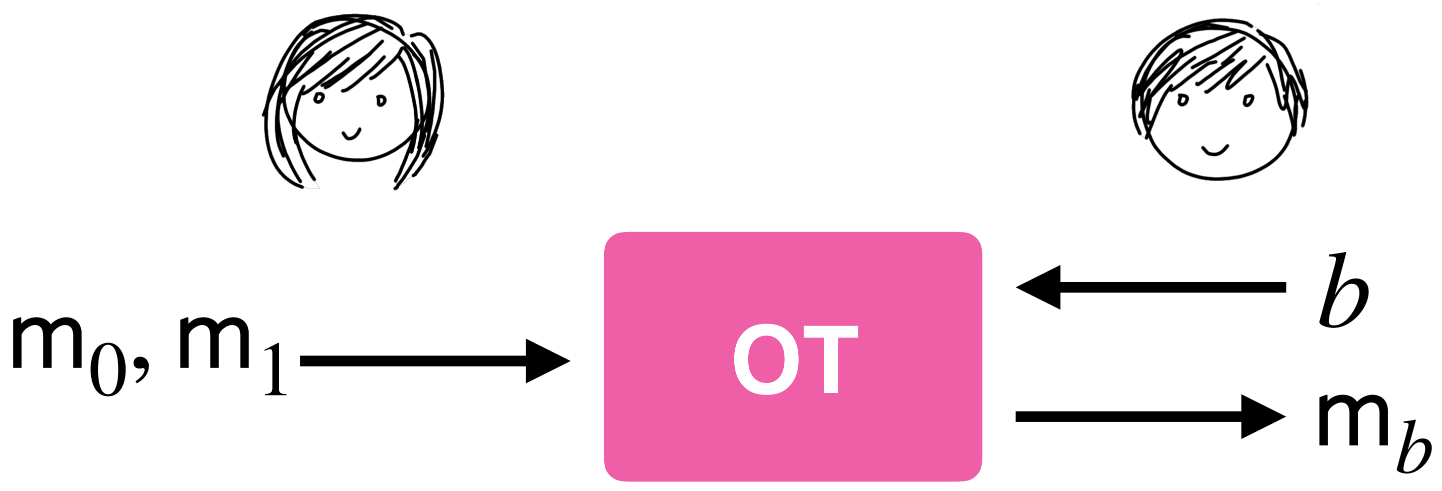

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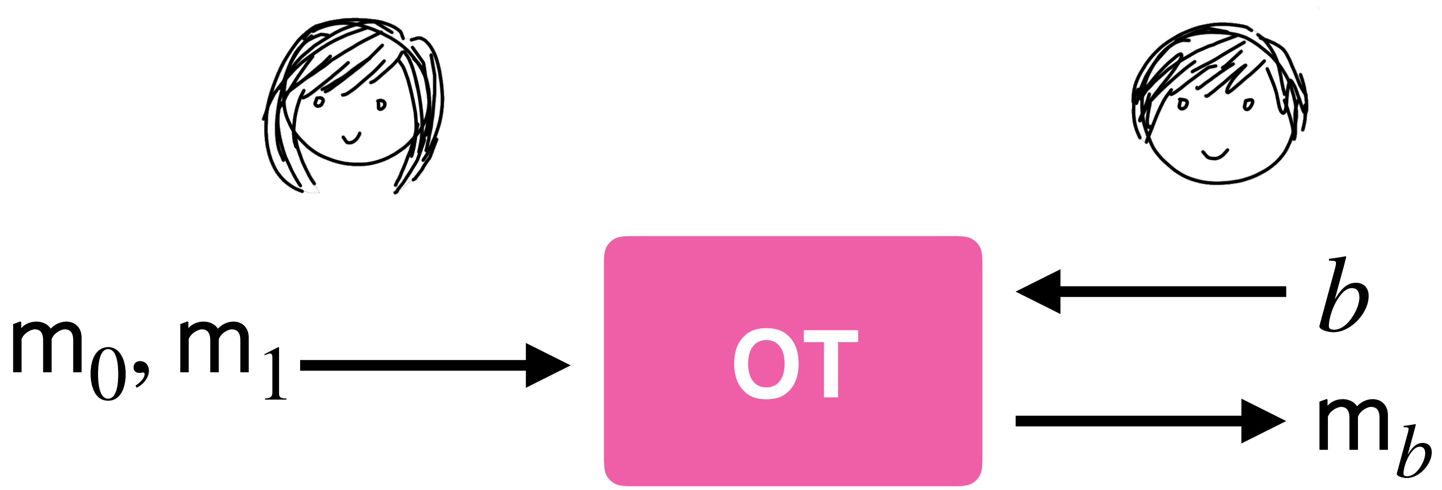

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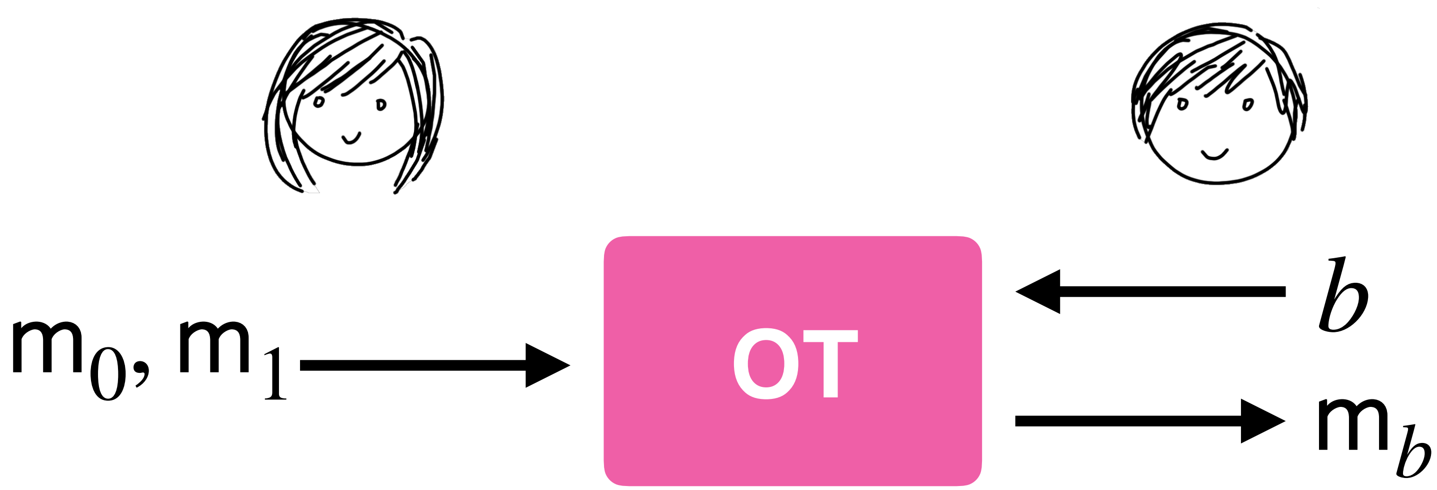

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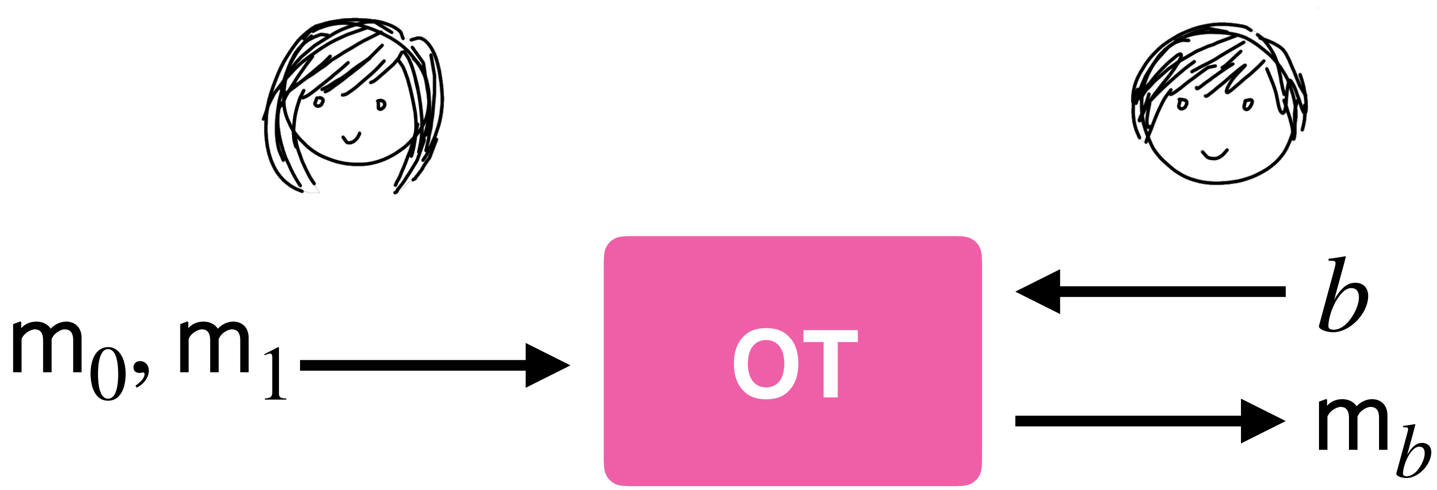

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- **OT Extension**: [Keller Orsini Scholl '15]

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- **Minimally interactive**: two messages
- **Overhead**: ~6x, room for improvement

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- **Cost**: $5+t$ exponentiations, 5 group elements per party

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$$
\begin{tikzcd}
{[k]} \arrow[r, bend left, "R"] & \left[\dfrac{1}{k}\right] & \left[\dfrac{\mathsf{sk}}{k}\right]
\end{tikzcd}
$$

Check in Exponent

- **Task:** verify relationship between [*k*] and [1/*k*]

- **Auxiliary information**: '*k*' in the exponent, ie. $R=k \cdot G$

- **Idea**: verify $\left[\frac{1}{k}\right][k] = 1$ by verifying $\left[\frac{1}{k}\right][k] \cdot G = G$

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Attempt at a solution:

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Verify $$\sum_{i \in [n]} \Gamma_i = G$$

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Adversary's contribution

Attempt at a solution:

Honest Party's contribution

Public $$R = k_A k_h \cdot G$$

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$$\Gamma_i = \left[\left(\frac{1}{k_A} + \epsilon \right) \frac{1}{k_h} \right]_i \cdot R$$

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Easy for Adv. to offset

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- Define $\phi = \prod_{i \in [n]} \phi_i$
- Randomize inversion: compute $\left[\dfrac{\phi}{k}\right]$ instead of $\left[\dfrac{1}{k}\right]$
- Reveal ϕ only after *every* other value is committed

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Public $R = k_A k_h \cdot G$

Broadcast $\Gamma_i = \left[\frac{\phi_A}{k_A} \frac{\phi_h}{k_h} \right]_i \cdot R$

Verify $\sum_{i \in [n]} \Gamma_i = \Phi$

Check in Exponent

Attempt at a solution:

Adversary's contribution

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Completely unpredictable

Check in Exponent

- There are **three** relations that have to be verified

$$[k] \qquad \left[\frac{1}{k}\right] \qquad \left[\frac{\mathsf{sk}}{k}\right]$$

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Our Approach

- **Setup**: MUL setup, VSS for [sk]
- **Signing**:
  1. Get candidate shares [k], [$1/k$], and $R=k\cdot G$
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  4. Reconstruct sig = [$1/k$]$\cdot H(m)$+[sk/k]

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Broadcast linear combination of shares

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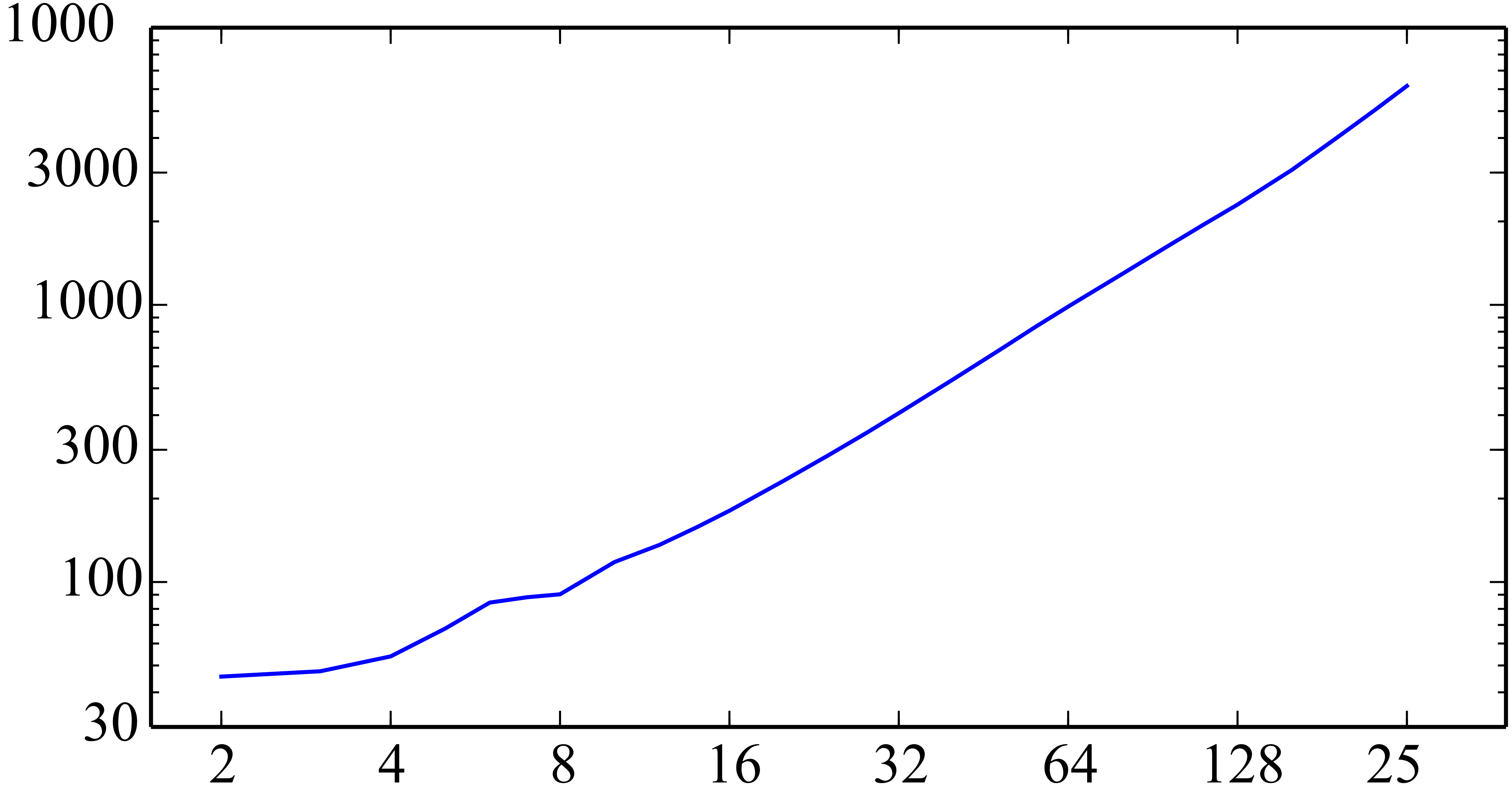
- **Low Power Friendliness**: Raspberry Pi benchmark (~60ms for 2-of-2)

LAN Setup



Broadcast PoK (DLog), **Pairwise**: 128 OTs

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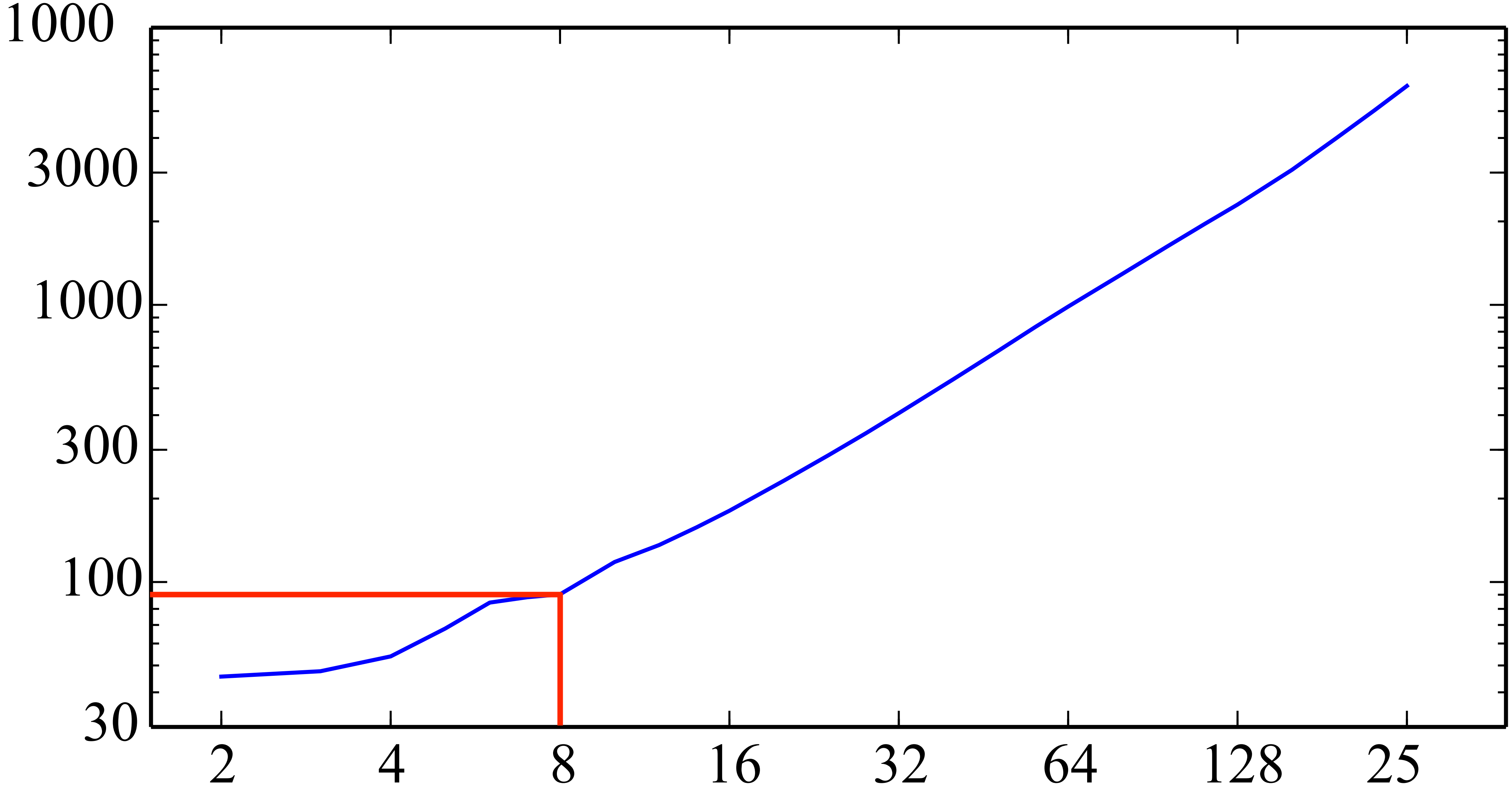

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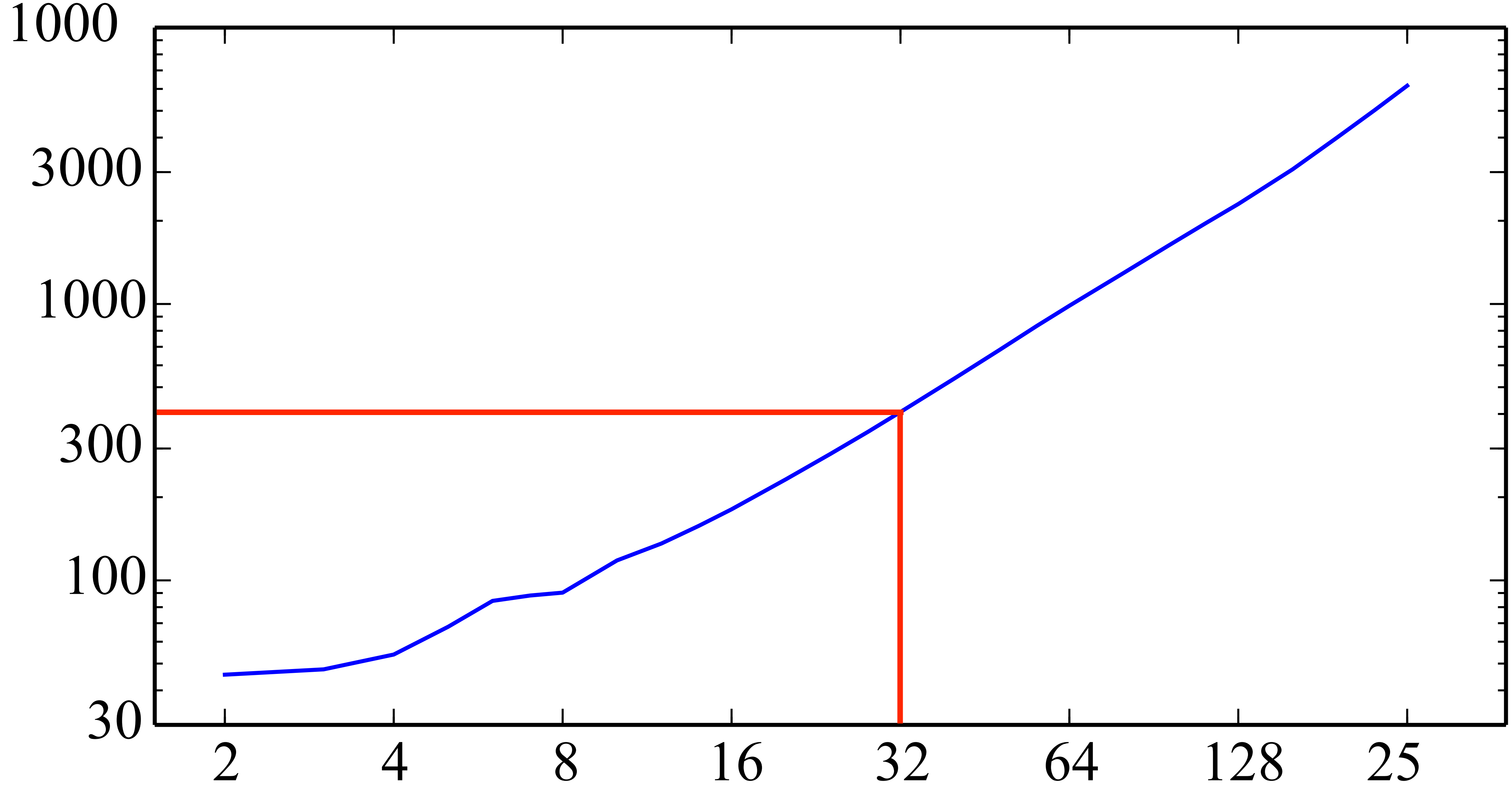

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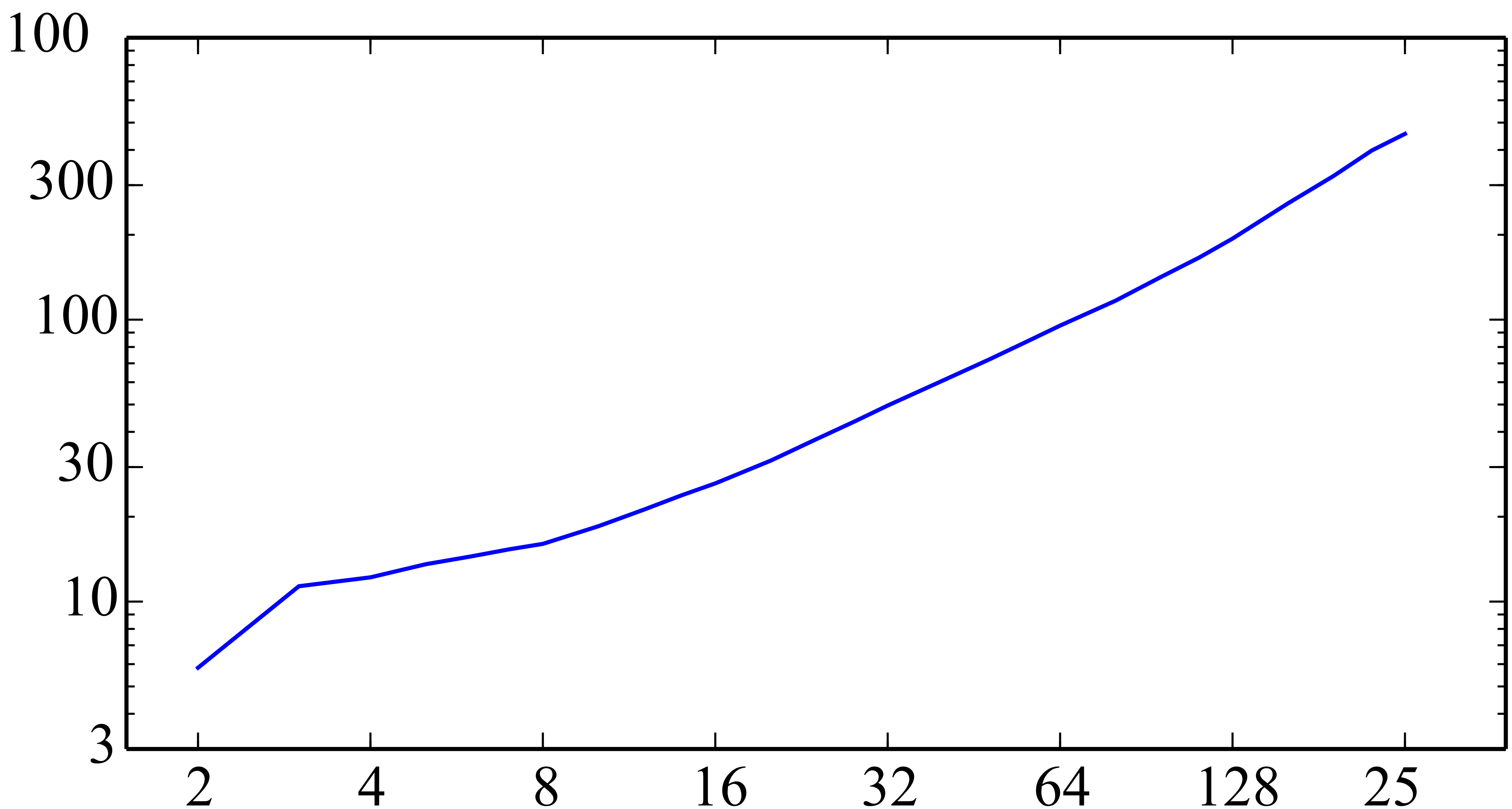

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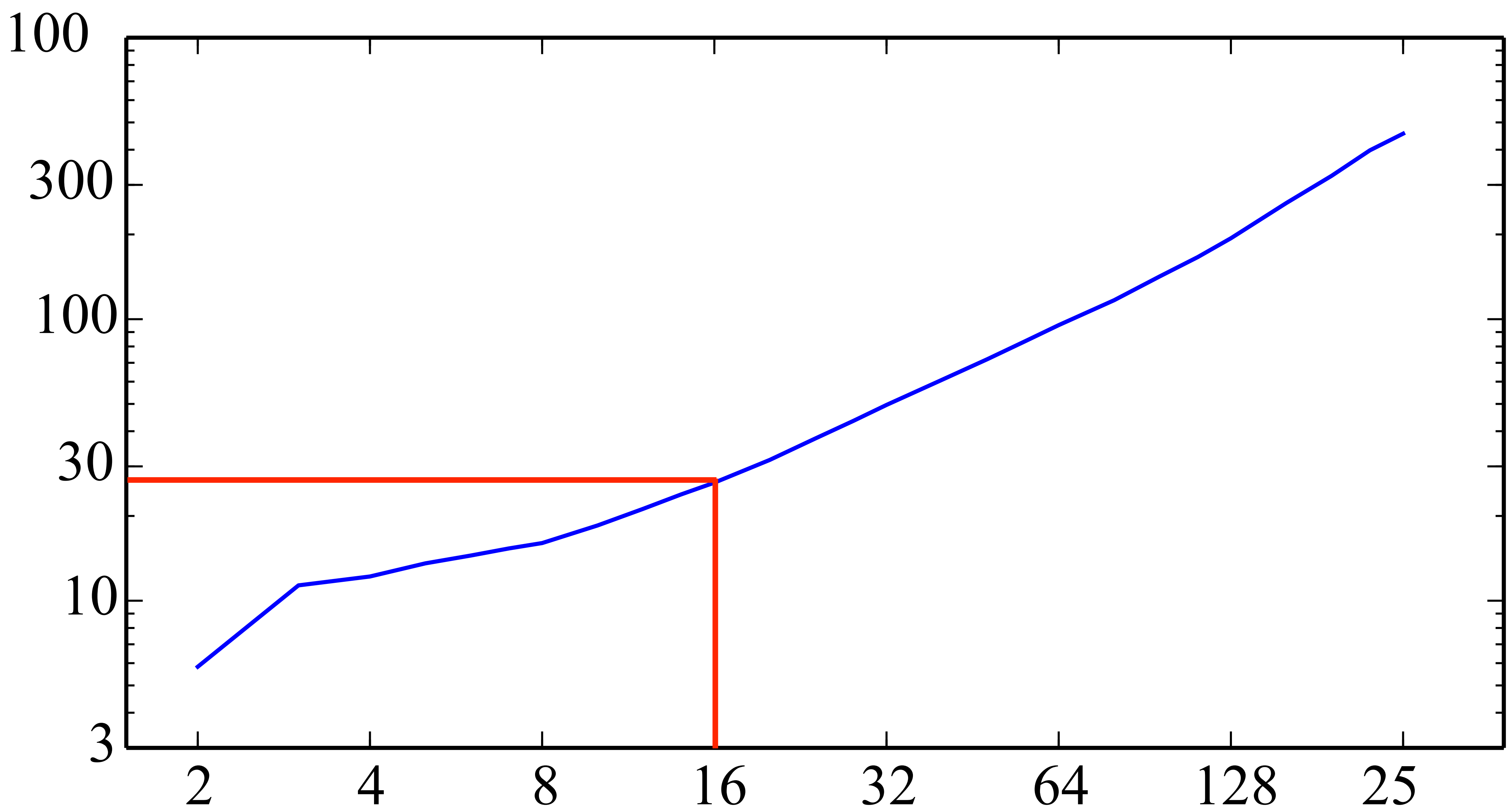

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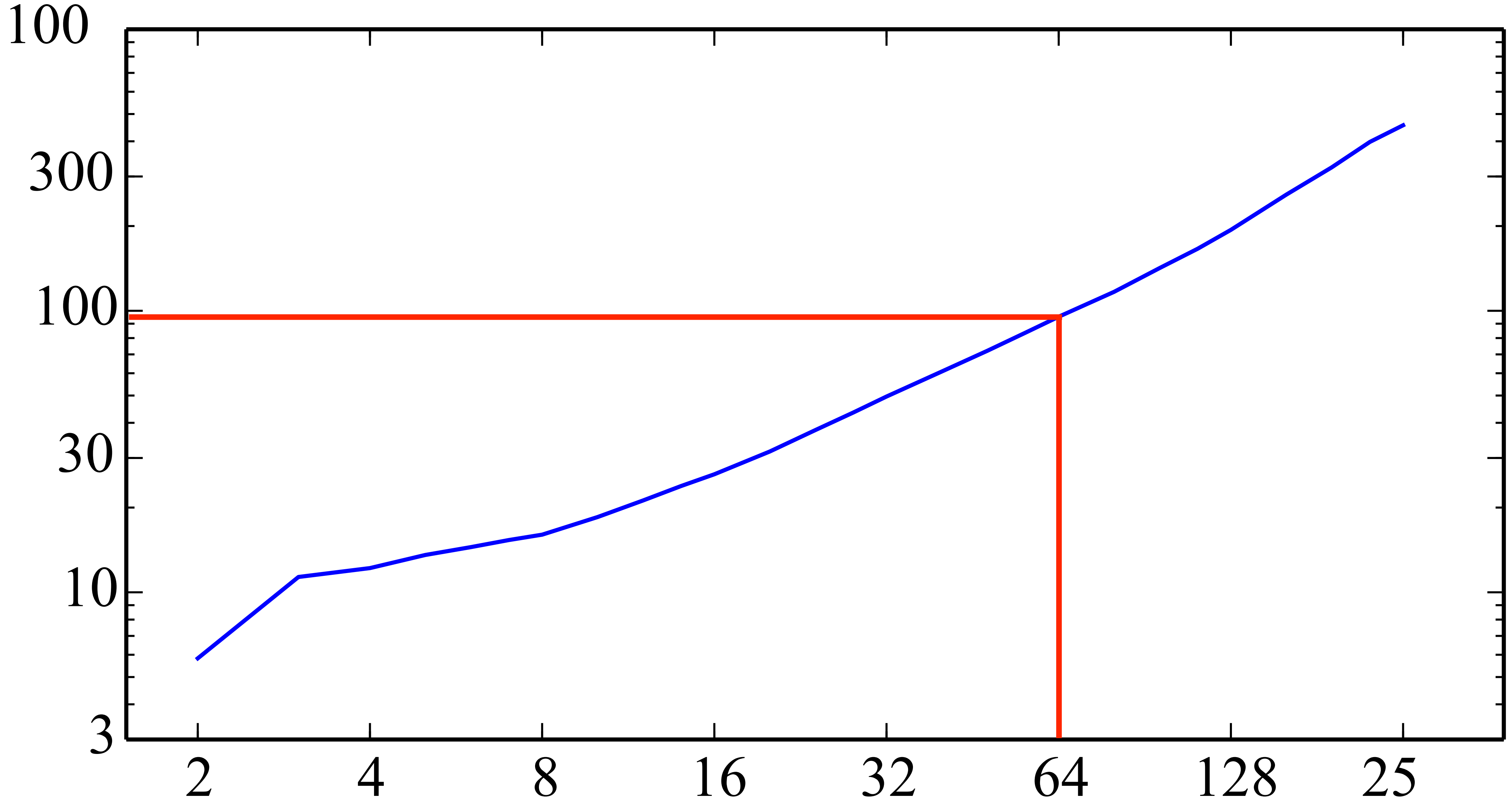

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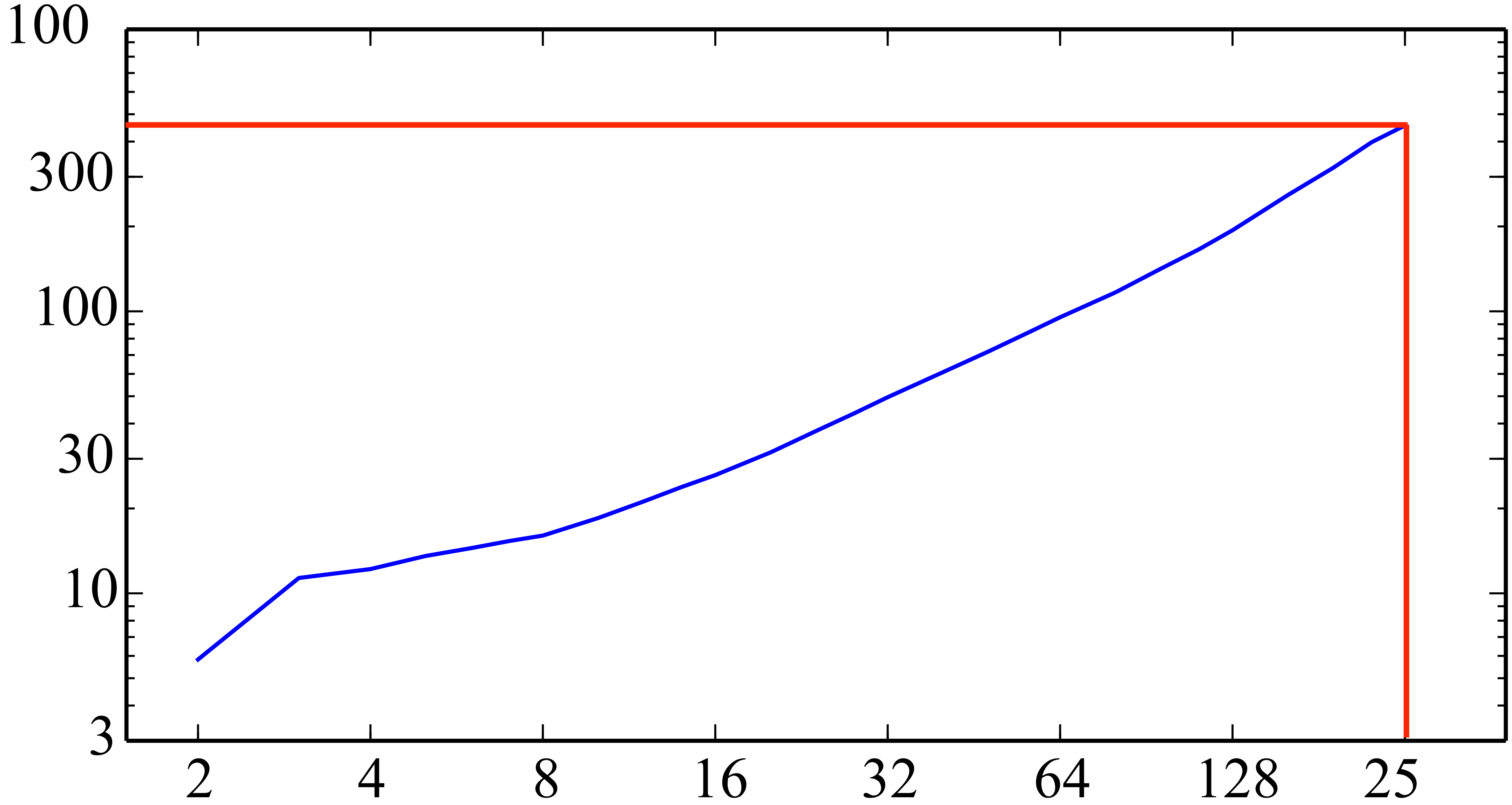
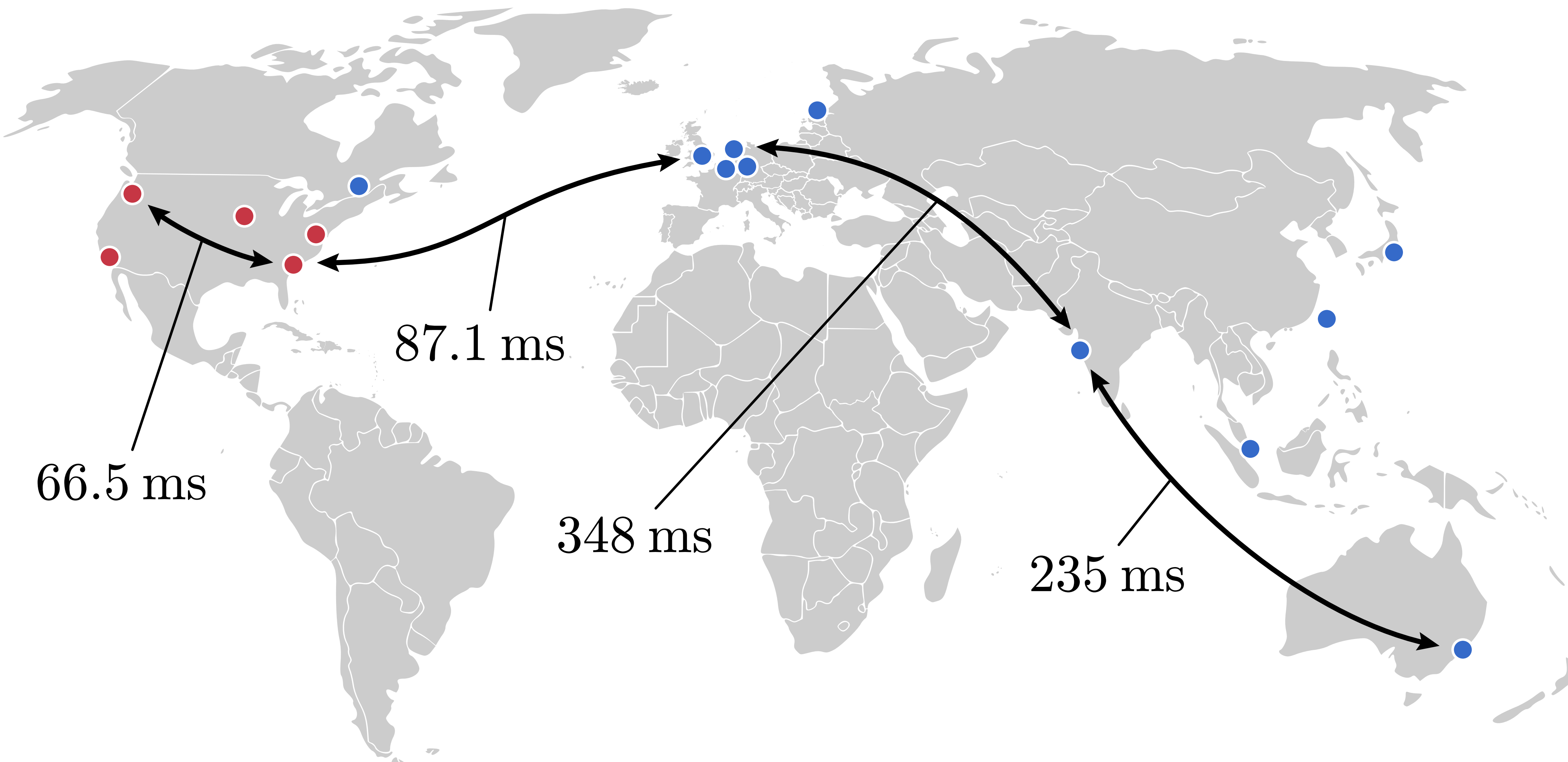


WAN Node Locations



WAN Benchmarks

All time values in milliseconds

Parties/Zones	Signing Rounds	Signing Time	Setup Time
5/1	9	13.6	67.9
5/5	9	288	328
16/1	10	26.3	181
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Comparison

All time figures in milliseconds

	Signing		Setup	
Protocol	$t = 2$	$t = 20$	$n = 2$	$n = 20$
This Work	**9.5**	**31.6**	**45.6**	**232**
GG18	77	509	–	–
LNR18	304	5194	~11000	~28000
BGG17	~650	~1500	–	–
GGN16	205	1136	–	–
Lindell17	36.8	–	2435	–
DKLs18	**3.8**	–	**43.4**	**177**

Note: Our figures are wall-clock times; includes network costs

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  - Well within LTE envelope (10Mbps) for responsivity

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  - eg. smartphones over LTE

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- Optimized computation **but communication well within practical range (65*t* KB/party)**
- **Wall-clock times**: Practical in realistic scenarios

Thank you!

https://gitlab.com/neucrypt/mpecdsa

Component-Wise Comparison

- LNR18 can be instantiated with “ECDSA assumptions”
- Heaviest components of signing (per party):
  - **LNR18**: $2t \times 2$P-MUL + $(83 + 78 \cdot t)$ Exponentiations
  - **This work**: $4t \times 2$P-MUL + $(5+t)$ Exponentiations

Check in Exponent

2. [sk/*k*] and [*1*/*k*] are consistent with *pk*

$$\sum_{i \in [n]} \left[\frac{\phi}{k}\right]_i \cdot \mathsf{pk} - \left[\frac{\phi}{k} \cdot \mathsf{sk}\right]_i \cdot G = 0$$

Check in Exponent

3. [sk/*k*] and [*1*/*k*] are consistent with *R*

$$\sum_{i \in [n]} \left[\frac{\phi}{k} \cdot \mathsf{sk} \right]_i \cdot R = \phi \cdot \mathsf{pk}$$