

# **1<sup>st</sup> and 2<sup>nd</sup> Preimage Attacks on 7, 8 and 9 Rounds of Keccak-224,256,384,512**

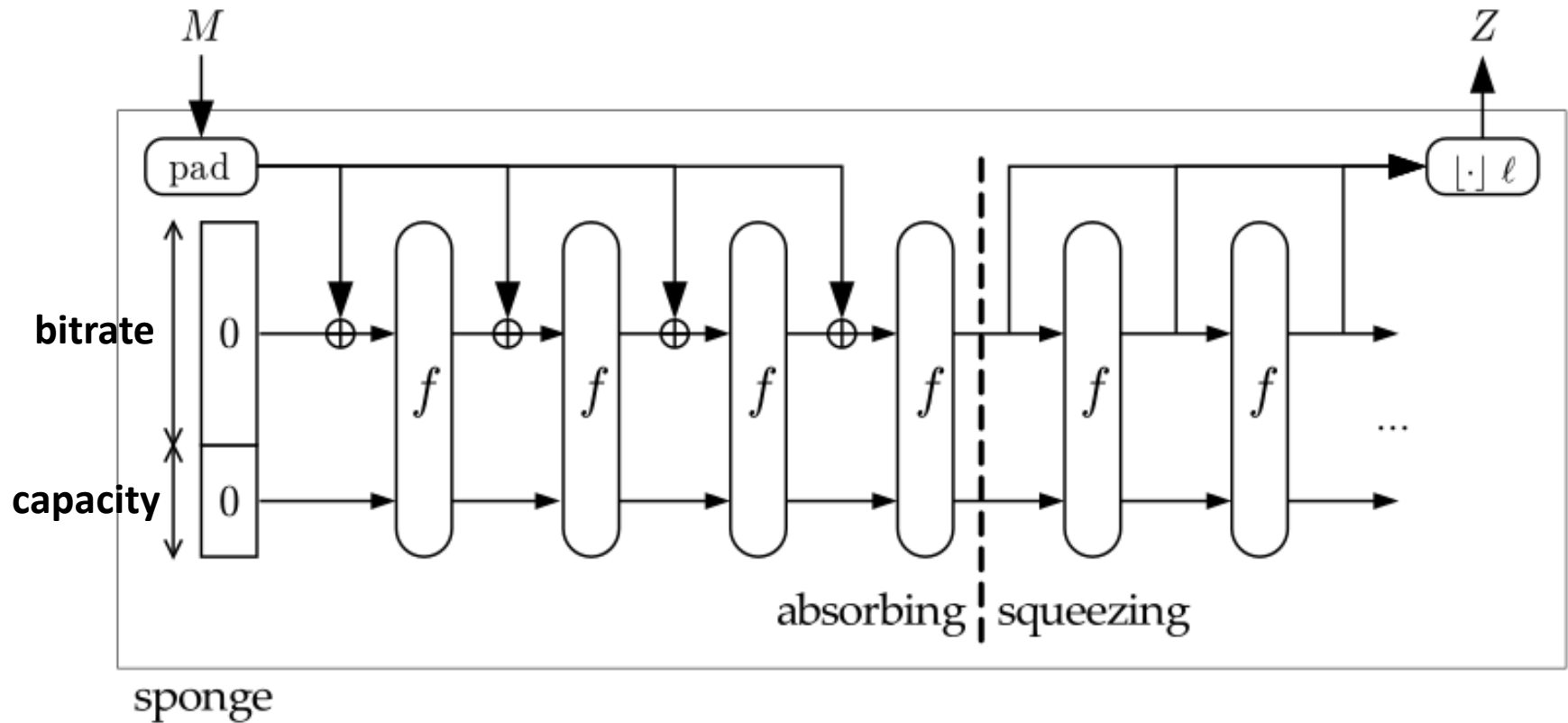
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Somitra Kumar Sanadhya<sup>1</sup>

<sup>1</sup>IIT-Delhi, India      <sup>2</sup>NSIT, India

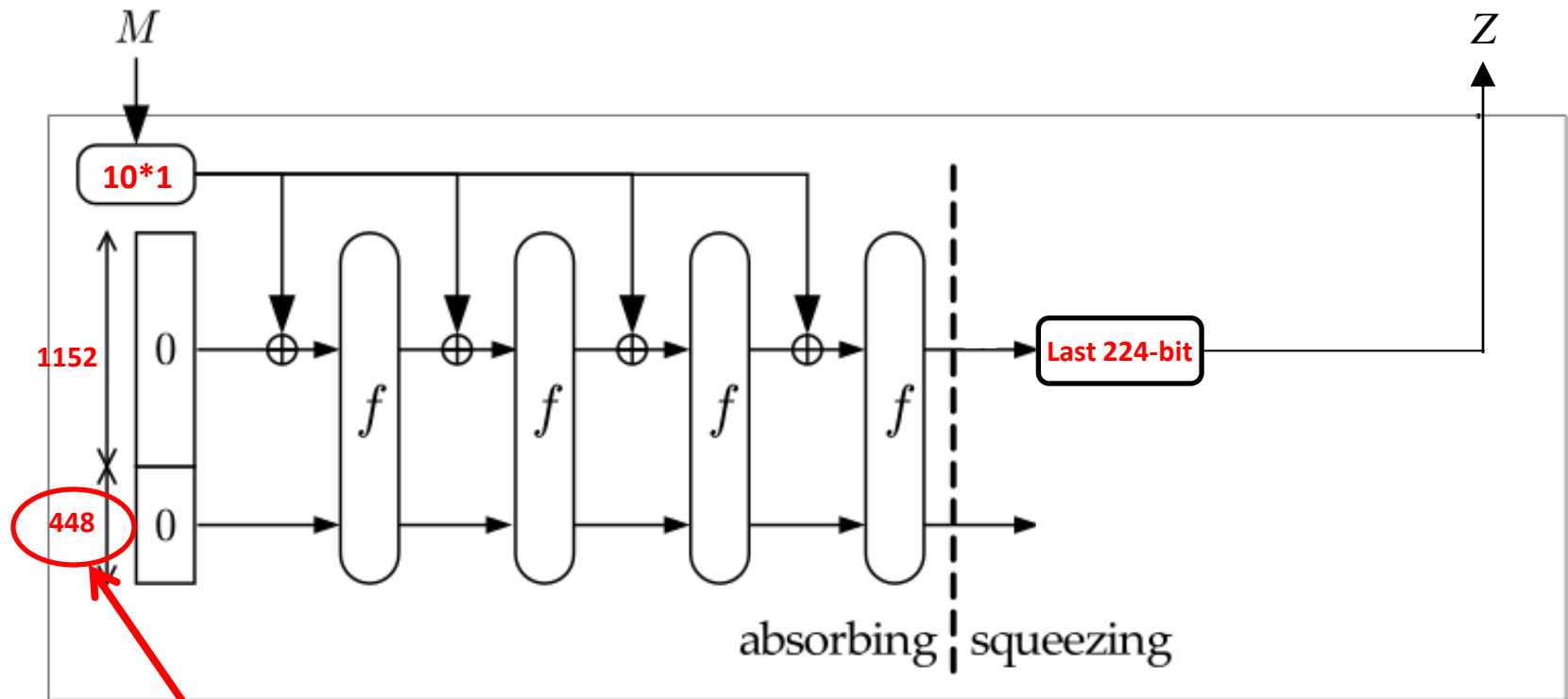
<sup>3</sup>Polish Academy of Sciences, Institute of Computer Science, Poland

Presented at 2014 SHA3 Workshop, Santa Barbara USA  
August 22 2014

# Sponge Construction

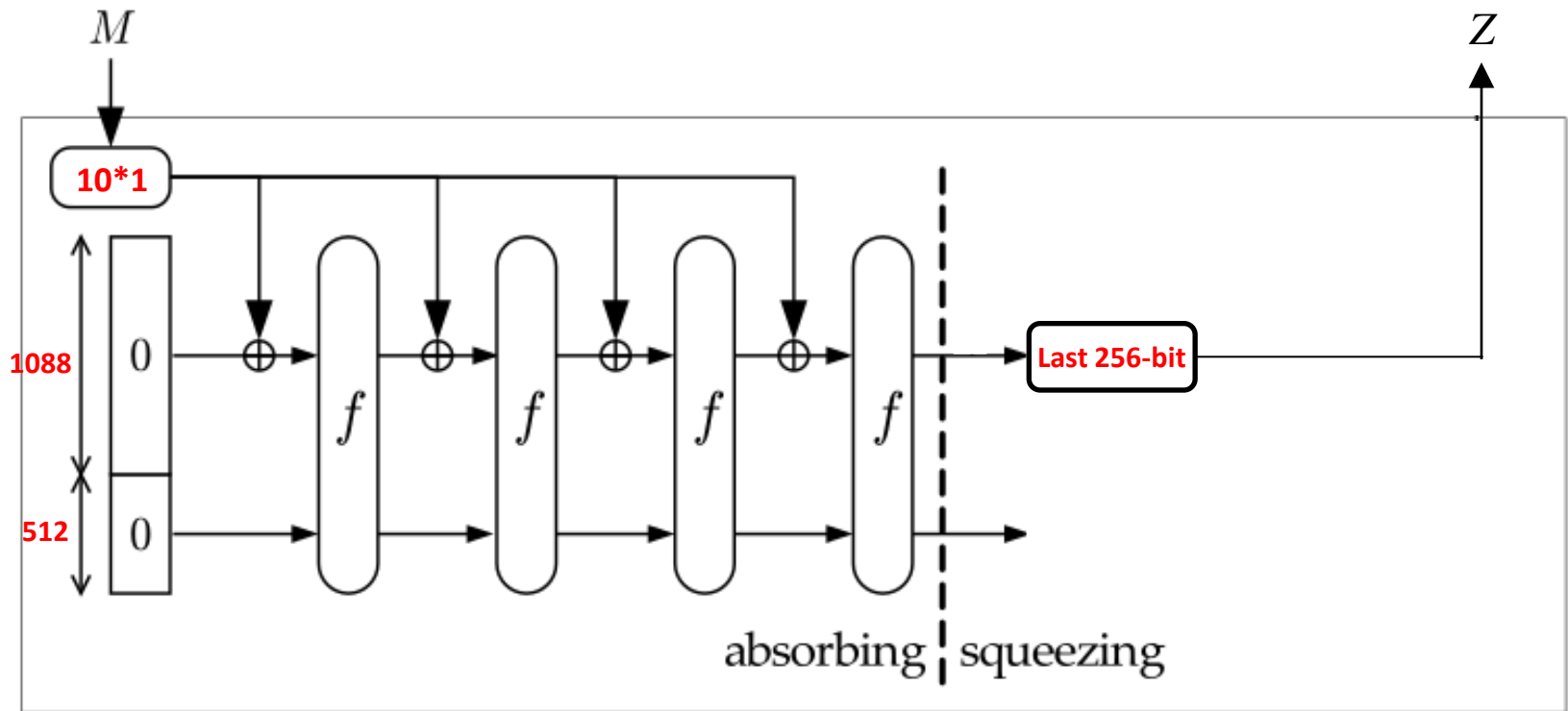


# Domain Extension of Keccak-224



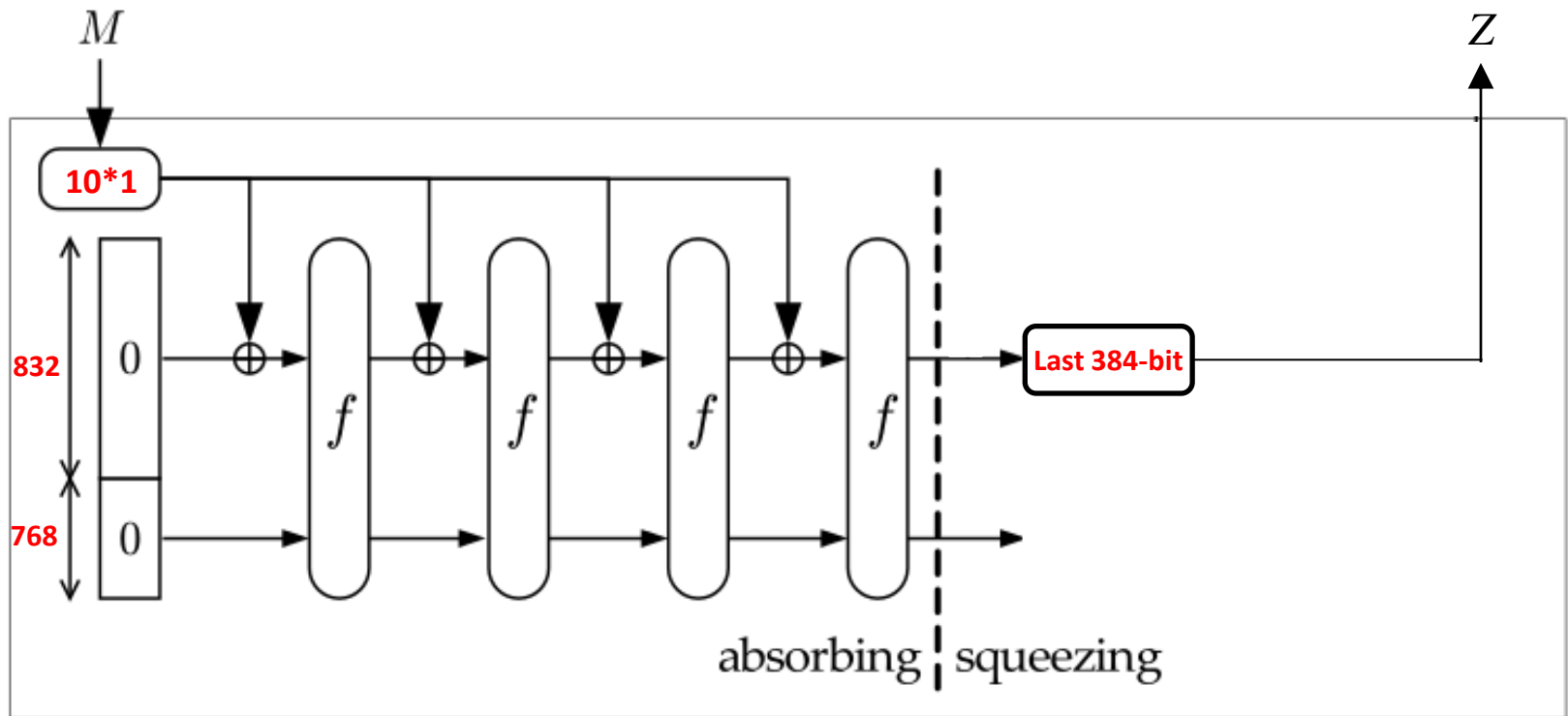
The size of capacity is double of the hash output size.

# Domain Extension of Keccak-256



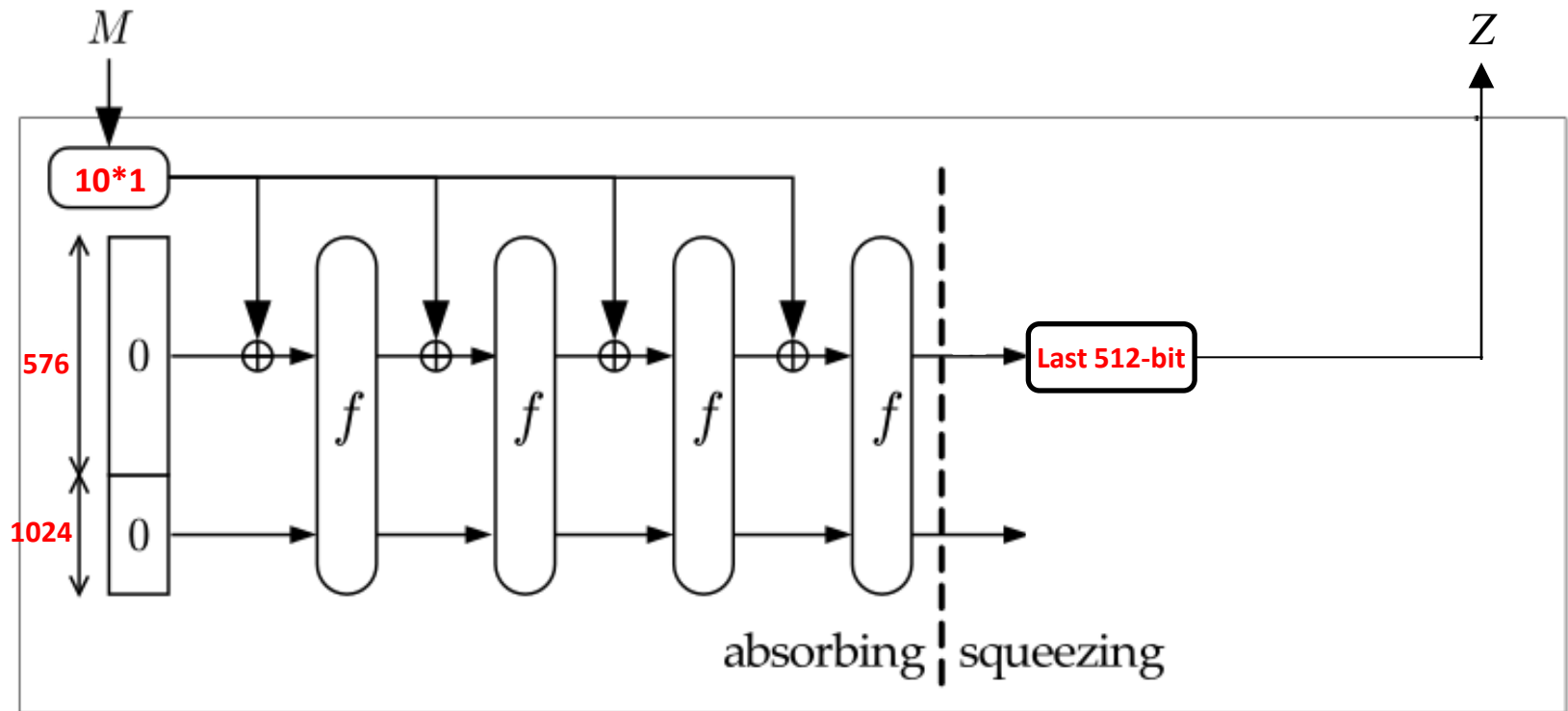
**The size of capacity is double of the hash output size.**

# Domain Extension of Keccak-384



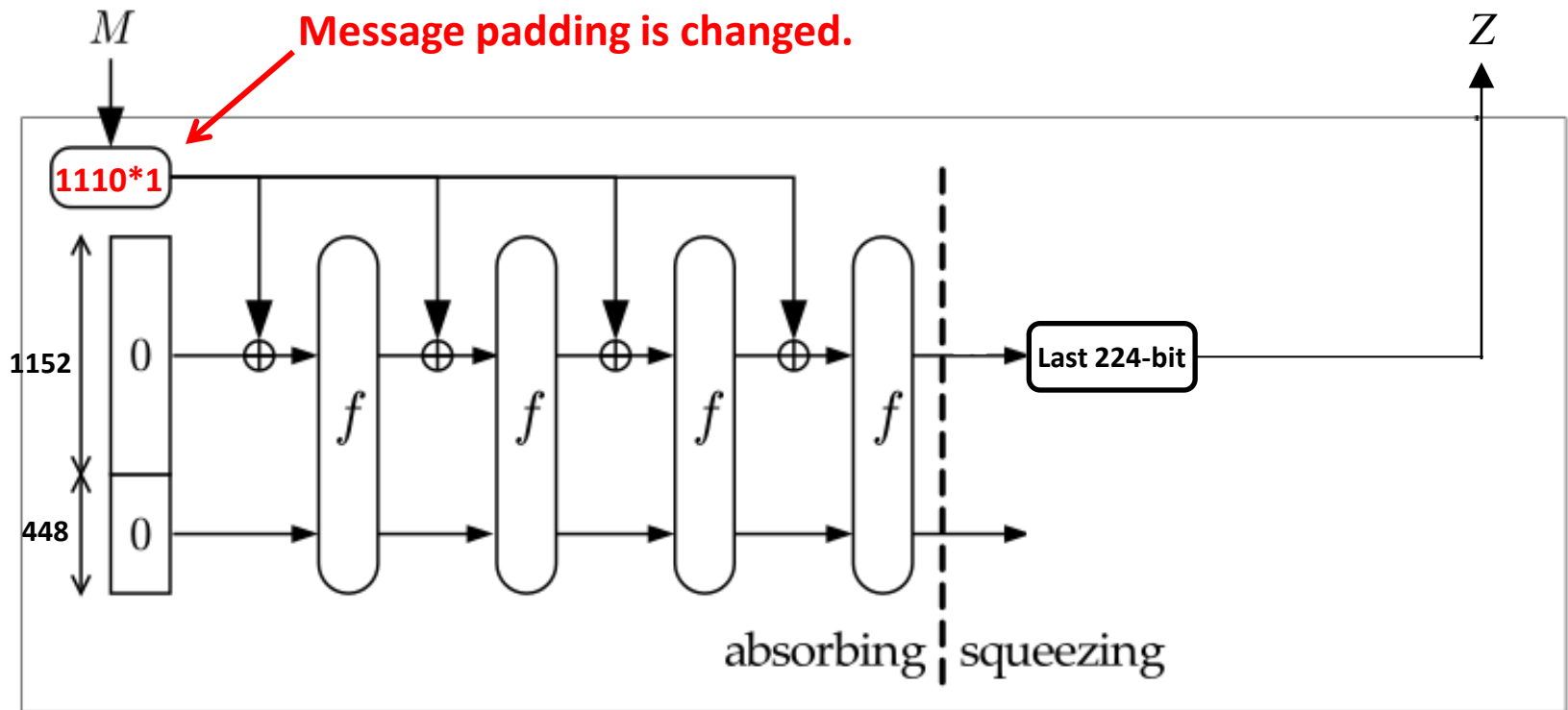
The size of capacity is double of the hash output size.

# Domain Extension of Keccak-512



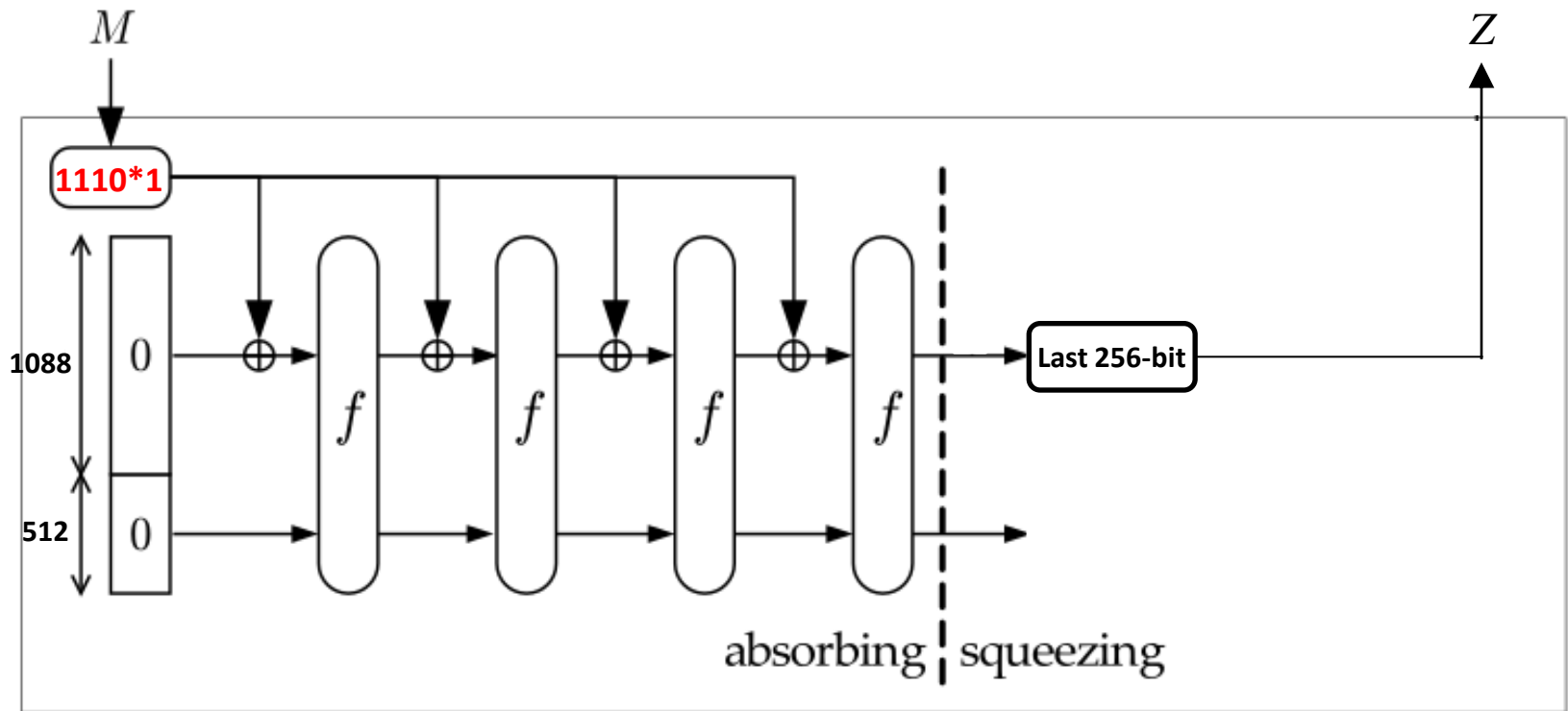
The size of capacity is double of the hash output size.

# Domain Extension of **SHA3-224**



The size of capacity is double of the hash output size.

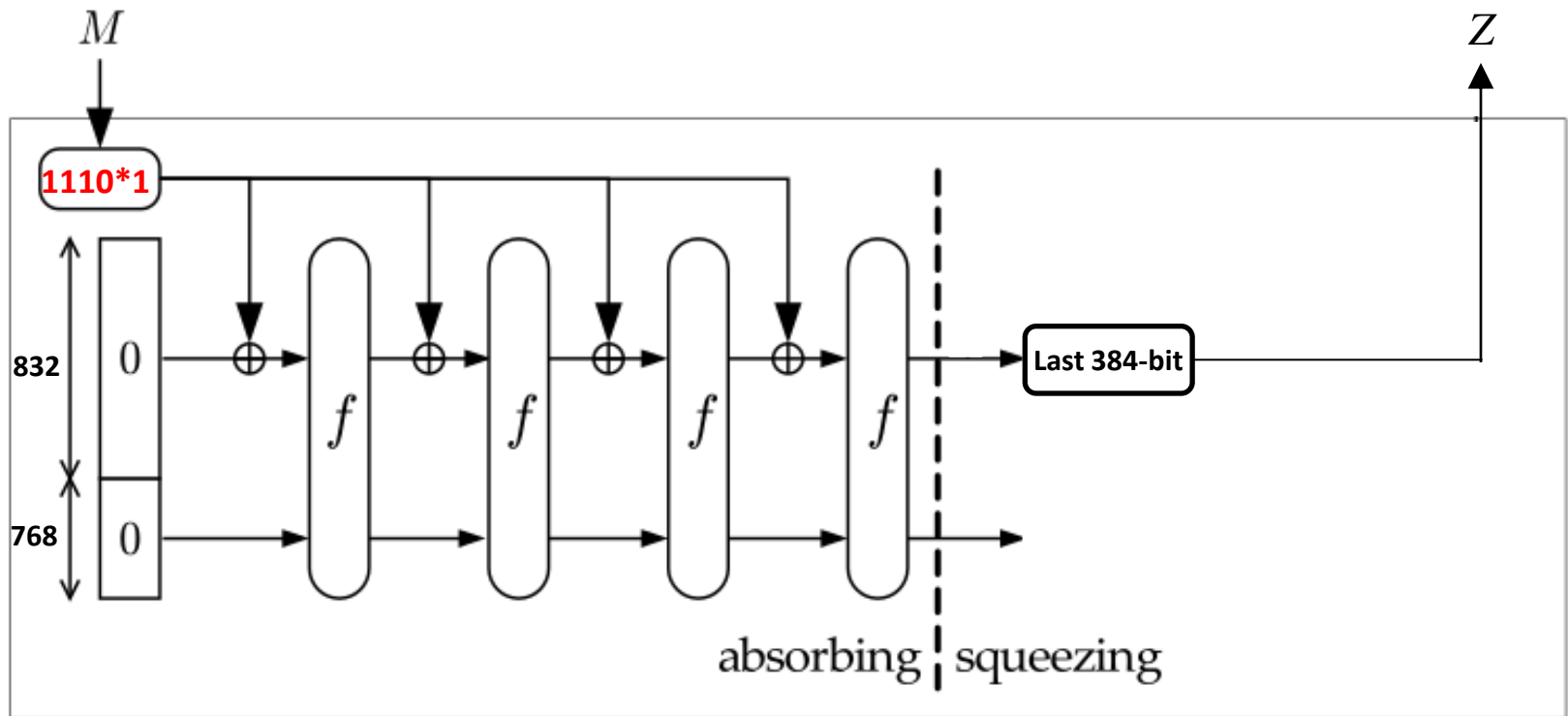
# Domain Extension of **SHA3-256**



**The size of capacity is double of the hash output size.**

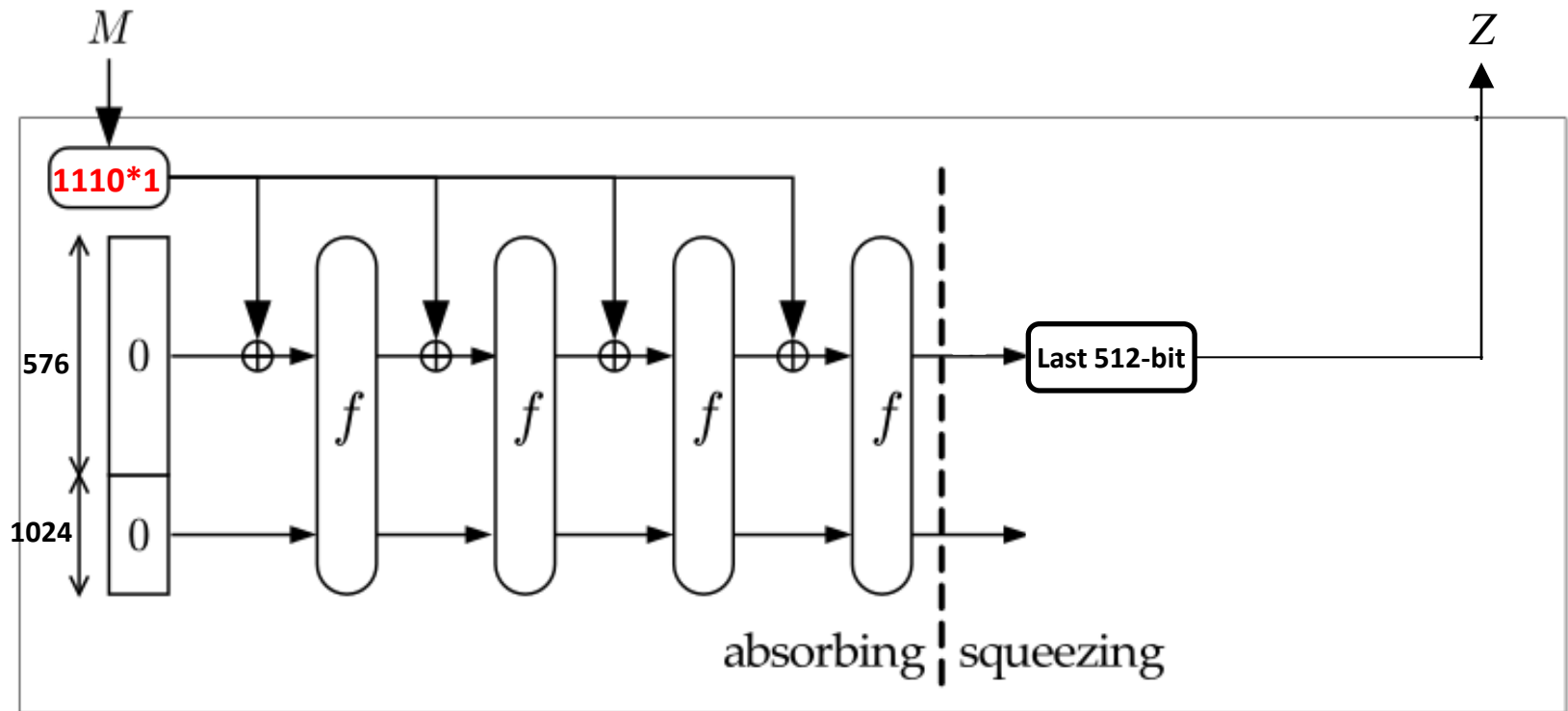


# Domain Extension of **SHA3-384**



The size of capacity is double of the hash output size.

# Domain Extension of **SHA3-512**



**The size of capacity is double of the hash output size.**

# 1600-bit Permutation $f$

- A 1600-bit state is described by  $a[x][y][z]$  for  $0 \leq x \leq 4, 0 \leq y \leq 4, 0 \leq z \leq 63$ .
- $f$  consists of 24 rounds. Each round is defined by  $\mathbf{R} = \iota \circ \chi \circ \pi \circ \rho \circ \theta$ .

$$\begin{array}{l}
 \text{degree 1} \\
 \text{(0.5 round)} \left\{ \begin{array}{l}
 \theta : a[x][y][z] \leftarrow a[x][y][z] \oplus \bigoplus_{y'=0}^4 a[x-1][y'][z] \oplus \bigoplus_{y'=0}^4 a[x+1][y'][z-1] \\
 \rho : a[x][y][z] \leftarrow a[x][y][z - (t+1)(t+2)/2], \\
 \text{with } t \text{ satisfying } 0 \leq t < 24 \text{ and } \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \text{ in } \mathbf{GF}(5)^{2 \times 2}, \\
 \text{or } t = -1 \text{ if } x = y = 0, \\
 \pi : a[x][y] \leftarrow a[x'][y'], \text{ with } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix},
 \end{array} \right. \\
 \text{degree 2} \\
 \text{(0.5 round)} \left\{ \begin{array}{l}
 \chi : a[x] \leftarrow a[x] \oplus (a[x+1] \oplus 1)a[x+2], \\
 \iota : a \leftarrow a \oplus \mathbf{RC}[i_r],
 \end{array} \right.
 \end{array}$$

# Number of Bit-operations of each Round

For  $0 \leq x \leq 4, 0 \leq y \leq 4, 0 \leq z \leq 63$ .

1600 bit-operations
1280 bit-operations
320 bit-operations

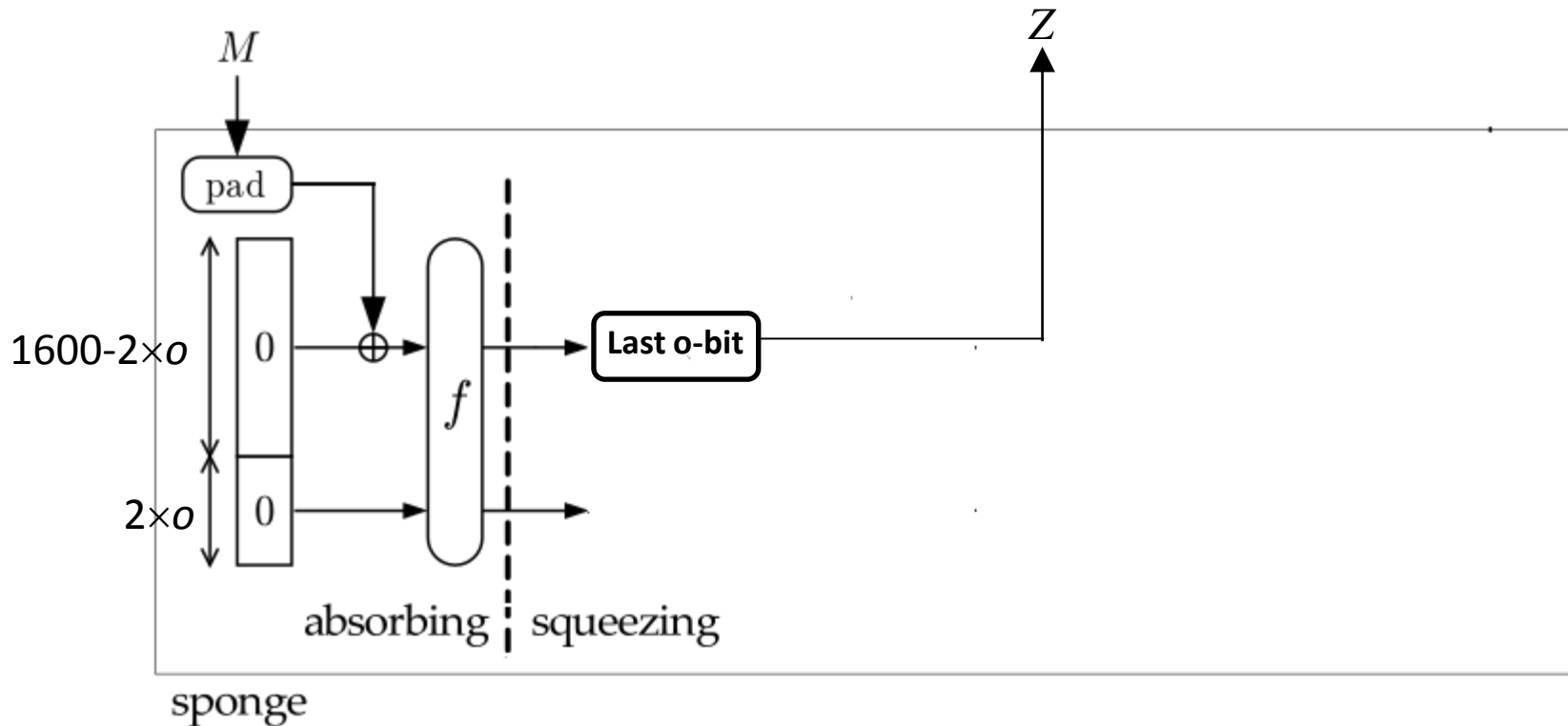
$$\begin{aligned}
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 \end{aligned}$$

64 bit-operations
4800 bit-operations

In total, at least **8064** (=1600+1280+320+4800+64) bit-operations are required to compute one round.

# General Preimage Attack Complexity for Keccak-n and SHA3-n based on r-round $f$

- So, given a  $o$ -bit hash value  $Z$ , we need  $r \times 8064 \times 2^o$  bit-operations to find its preimage with high probability.



# Polynomial Enumeration (used by Dinur and Shamir [FSE 2011] )

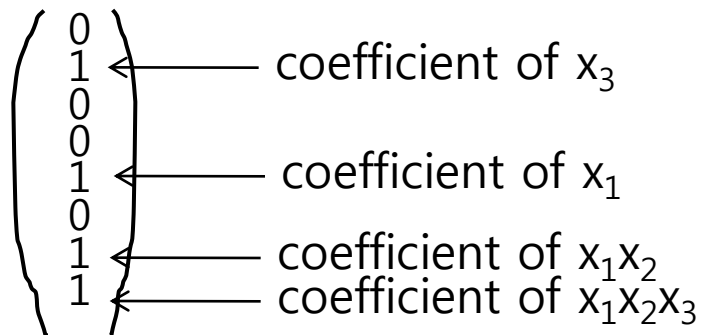
- Given a boolean function  $f_i$  ( $1 \leq i \leq b$ ) with  $n$ -bit input and degree  $d$ , where  $f_i$  is the  $i$ -th output bit of  $f$ ,
- polynomial enumeration algorithm is a way of constructing the truth table of  $f_i$  by the following two steps.
  - **Step 1:** Compute coefficients of  $f_i$ ,
    - **Time complexity:**  $\sum_{0 \leq j \leq d} (2^j \times_n C_j)$ .
  - **Step 2:** Construct the truth table of  $f_i$  using the fast Moebius transformation.
    - **Time complexity:**  $n \times 2^{n-1}$ .

# The Fast Moebius Transformation

- transforms the coefficient array of a boolean function to its truth table array.

For example,  $f(x_1, x_2, x_3) = x_1 \oplus x_1 x_2 x_3 \oplus x_1 x_2 \oplus x_3$

Coefficient Array



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Coefficient Array

0  
1  
0  
0  
0  
1  
0  
1

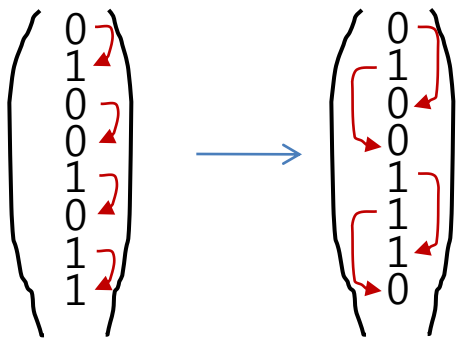


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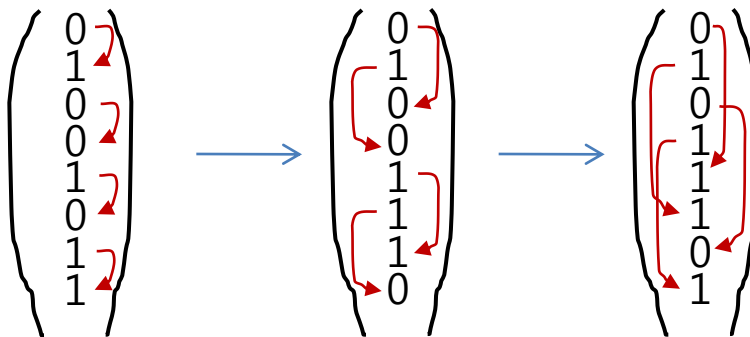


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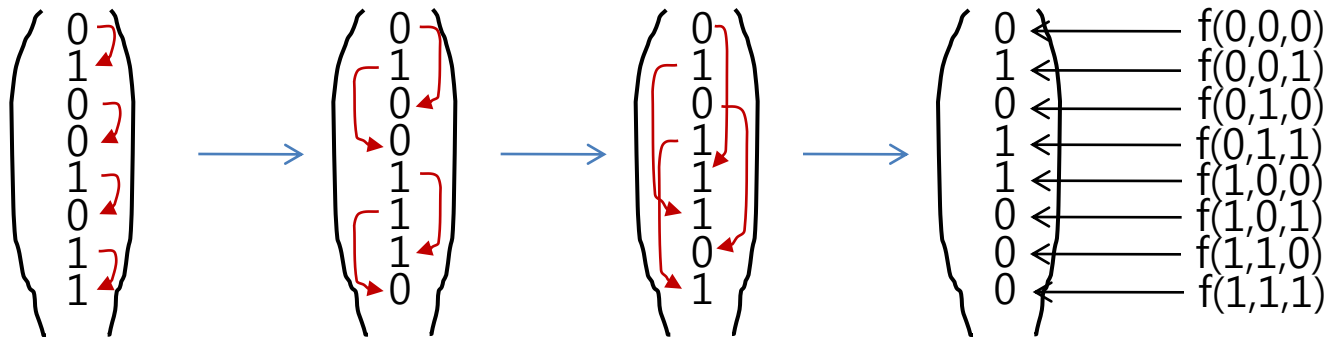
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For example,  $f(x_1, x_2, x_3) = x_1 \oplus x_1 x_2 x_3 \oplus x_1 x_2 \oplus x_3$

Coefficient Array

Truth Table Array




Complexity : for  $n$  variables,  $n \times 2^{n-1}$  1-bit XOR operations.

# Preimage Attack on H using Polynomial Enumeration (by Dinur and Shamir)


- Given a  $o$ -bit hash output  $Z$ ,
  - **Step 1:** By polynomial enumeration algorithm, efficiently find messages  $M$ 's which partially match over  $b$  bits of the given  $o$ -bit hash value.
  - **Step 2:** if there is  $M$  s.t.  $H(M)=Z$ , then return  $M$  else goes to Step 1.

# Improving Polynomial Enumeration

## (by Bernstein [NIST mailing list 2013] )

- Given a boolean function  $f_i$  ( $1 \leq i \leq b$ ) with  $n$ -bit input and degree  $d$ , where  $f_i$  is the  $i$ -th output bit of  $f$ .
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  - **Step 2:** Construct the truth table of  $f_i$  using the fast Moebius transformation.
    - **Time complexity:**  $n \times 2^{n-1}$ .

But this time complexity improvement requires big memory cost.

# Application to 6, 7, 8 rounds of Keccak-512 (by Bernstein)

- **6 rounds:**  $2^{176}$  bits of memory give a workload reduction by a factor 50 (~6 bits)
- **7 rounds:**  $2^{320}$  bits of memory give a workload reduction by a factor 37 (~5 bits)
- **8 rounds:**  $2^{508}$  bits of memory give a workload reduction by a factor 1.4 (half a bit)

# Our Results

- Bernstein only described the idea of improving Step 1 complexity. However, overall time and memory complexity of his attack is not clear.
- **Result 1:** Based on Bernstein's idea, we made **Algorithm 1** for generating the coefficient array of a boolean function with detailed time and memory complexity.
- **Result 2:** We provide a general preimage attack methodology on hash functions using Result 1 and meet-in-the-middle-matching technique.
- **Result 3:** Using Result 2, as an example, we further improve Bernstein's result upto 9 rounds of Keccak.



# Algorithm 1 for Generating the Coefficient Array of a Boolean Function (Result 1)

## Algorithm 1: Computing the Coefficient Static Array of a Boolean Function

**Input:** Boolean function  $f$  with  $n$ -bit input and having algebraic degree at most  $d$

**Result:** Coefficient static array  $C$  of size  $2^n$ , which is initialized with all zeros in the beginning

```
1 begin
2    $l=0$ ;
3   while  $l \leq d$  do
4     for  $A \in \alpha$  AND  $|A| = l$  do
5        $y=0$ ;
6        $i=0$ ;
7        $y=f(S_A)$ ;
8        $\text{Sum}_0[S_A] = y$ ;
9       while  $i < l$  do
10         $y = y \oplus \text{Sum}_i[S_{A,i+1}]$ ;
11         $i=i+1$ ;
12         $\text{Sum}_i[S_A] = y$ ;
13       $C[S_A] = y$ , where  $C_A$  is also same as  $C[S_A]$ ;
14     $l=l+1$ ;
```

**Time Complexity:**  $5 \times (\sum_{l=0}^d l \times \binom{n}{l}) + T \times \sum_{l=0}^d \binom{n}{l}$

**Memory Complexity:**  $(2d + 1) \times 2^n + 2^n$

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```

$\alpha = \{A : |A| \leq d \text{ and } A \subset \{1, 2, \dots, n\}\}$

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```

The time complexity of  $f$  is  $T$ .  
(in terms of number of bit-operations)

Step 7

**Time Complexity:**  $5 \times \left( \sum_{l=0}^d l \times \binom{n}{l} \right) + T \times \sum_{l=0}^d \binom{n}{l}$

**Memory Complexity:**  $(2d + 1) \times 2^n + 2^n$

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14     $l=l+1$ ;

```

2 bit-operations (1 XOR, 1-bit memory access of static array Sum)

2 bit-operations are needed on average

1 bit -operation (1-bit update of static array Sum)

Step 10,11,12

**Time Complexity:**  $5 \times \left( \sum_{l=0}^d l \times \binom{n}{l} \right) + T \times \sum_{l=0}^d \binom{n}{l}$

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```

**Current Sum Arrays:** Each Sum array (which is static) has  $2^n$  elements of size 1-bit. We need at most  $d+1$  current Sum arrays.

**Previous Sum Arrays :** Each Sum array (which is static) has  $2^n$  elements of size 1-bit. We need at most  $d$  previous Sum arrays.

**Coefficient Array** (which is static) has  $2^n$  elements of size 1-bit.

**Time Complexity:**  $5 \times \left( \sum_{l=0}^d l \times \binom{n}{l} \right) + T \times \sum_{l=0}^d \binom{n}{l}$

**Memory Complexity:**  $(2d + 1) \times 2^n + 2^n$

# Our General Preimage Attack on $H=H_2 \circ H_1$

## (Result 2)

⑤ Repeat  $2^{o-n}$  times

②

Polynomial  
Enumeration  
(Algorithm 1  
and the fast  
Moebius  
Transformation)

Message M with n variables

$H_1$

(with Time Comp.  $T'$ )

③ q-bit matching ( $q \geq b$ )

$H_2$

(with Time Comp.  $T''$ )

b bits

Given: o-bit hash value h

④ Matching with  
remaining o-b bits

①

Table Look-up  
(a large  
memory may  
be required)

# Complexity of Our General Preimage Attack

## (Result 2)

**Time Complexity:**

$$\begin{aligned}
 & b \times 2^q \times T'' + 2^{q-b} \times (q-b) \times q + \\
 & 2^{o-n} \times \left[ (T' \times \sum_{j=0}^d \binom{n}{j}) + ((2w+3) \times q \times \sum_{j=0}^d j \times \binom{n}{j}) + (q \times n \times 2^{n-1}) \right] + \\
 & 2^{o-n} \times [(T \times 2^{n-b}) + (\max\{(q-b), 1\} \times 2^n \times q)],
 \end{aligned}$$

① Generating lookup Table for  $H_2$   
 ② Algorithm 1 (here,  $w=1$ )  
 ② the fast Moebius Transformation  
 ③ Matching over  $q$ -bit  
 ④ Matching over remaining  $o-q$  bits (where  $T=T'+T''$ )  
 ⑤

**Memory Complexity:**

$$q \times 2^{q-b} + (2d + q + 1) \times 2^n.$$

Lookup Table for  $H_2$   
 $q$  Coefficient arrays and  $2d+1$  Sum arrays of size  $2^n$  for Polynomial Enumeration

# Application to Keccak (Result 3)

⑤ Repeat  $2^{o-n}$  times

②

Polynomial  
Enumeration  
(Algorithm 1  
and the fast  
Moebius  
Transformation)

①

Inverting  
(no memory  
required)

Message M

$H_1$

First  $r-0.5$  rounds  
(degree:  $2^{r-1}$ )

③ q-bit matching ( $q=b=5$  or  $10$ )

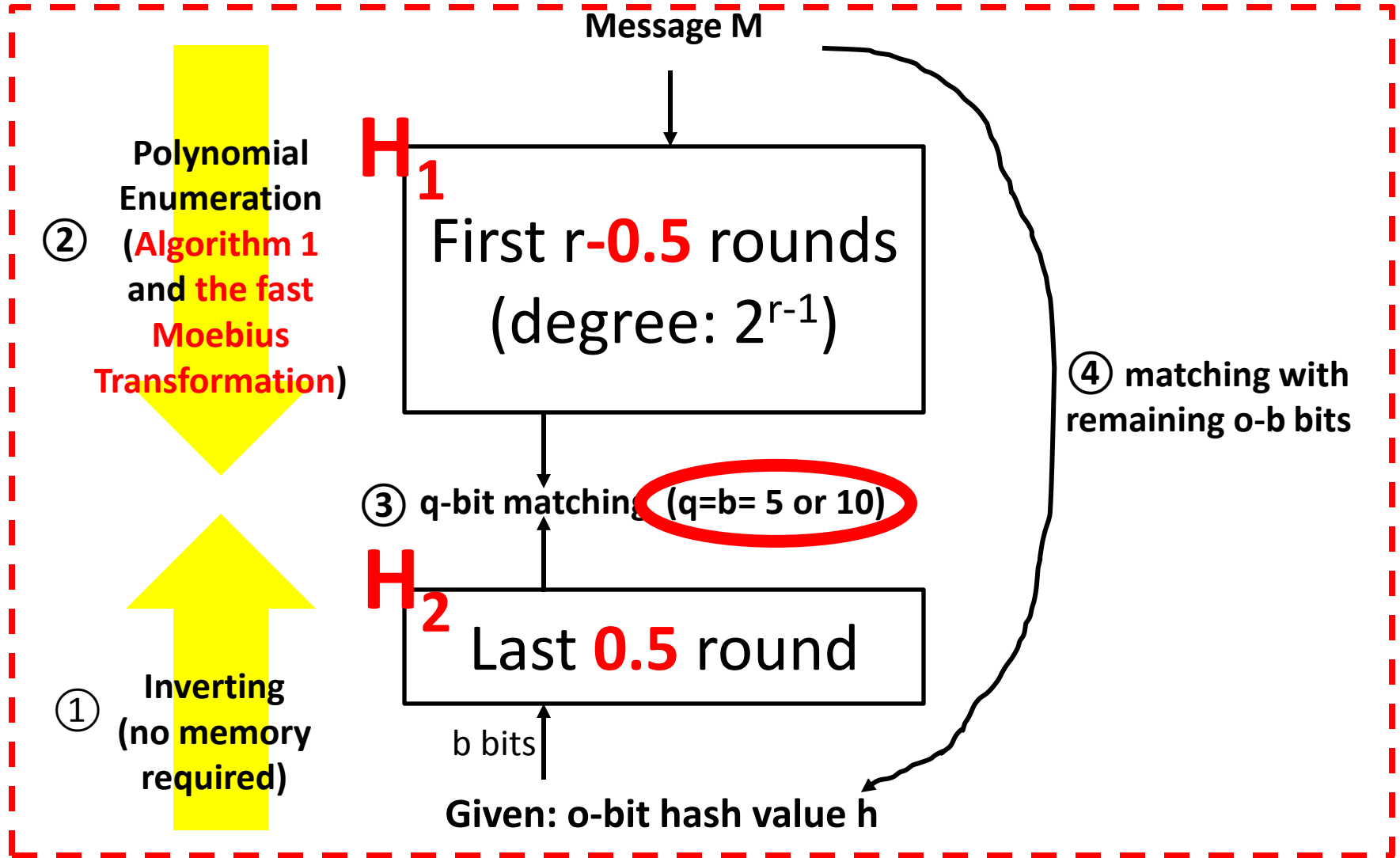
$H_2$

Last  $0.5$  round

b bits

Given: o-bit hash value h

④ matching with  
remaining o-b bits





# 1<sup>st</sup> and 2<sup>nd</sup> Preimage Attacks on 6, 7, 8, 9 rounds of Keccak (Result 3)

Version	Reference	No. of Rounds	Type of attack	Time Complexity	Memory Complexity	Improvement Factor
Keccak-256	[18]	2	Preimage	$2^{33}$		$2^{223}$
Keccak-512	[17]	3	Preimage	$2^{506}$		64
Keccak-512	<b>Bernstein's results</b>		Preimage	$2^{506}$		64
Keccak-512	[12, 8]	6	2nd Preimage	$2^{506}$	$2^{176}$	50
	[12, 8, 14]	7	"	$2^{507}$	$2^{320}$	37
	[12, 8, 14]	8	"	$2^{511.4}$	$2^{508}$	1.44
	This work, § 7	6	Preimage/ 2nd Preimage	$2^{509.19}$	$2^{98.91}$	7.01
	This work, § 7	7	"	$2^{509.39}$	$2^{172.52}$	6.13
Keccak-224 Keccak-256 Keccak-384 Keccak-512	<b>Our results</b>		8	$2^{509.73}$	$2^{315.29}$	4.81
	This work, § 8	7	"	$2^{218.11}$	$2^{180.12}$	58.66
	This work, § 8	8	"	$2^{255.64}$	$2^{254.03}$	1.29
	This work, § 8	8	"	$2^{378.74}$	$2^{324.06}$	38.36
	This work, § 8	6	"	$2^{505.58}$	$2^{104.23}$	85.70
	This work, § 8	7	"	$2^{506.11}$	$2^{180.12}$	59.34
	This work, § 8	8	"	$2^{506.74}$	$2^{324.07}$	38.36
	This work, § 8	9	"	$2^{511.70}$	$2^{510.02}$	1.23

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Keccak-512	[17]	3	Preimage	$2^{506}$		64
Keccak-512	Bernstein's results		Preimage	$2^{506}$		64
Keccak-512	[12, 8]	6	2nd Preimage	$2^{506}$	$2^{176}$	50
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	This work, § 7	6	Preimage/ 2nd Preimage	$2^{509.19}$	$2^{98.91}$	7.01
	This work, § 7	7	"	$2^{509.39}$	$2^{172.52}$	6.13
	Our results	8	"	$2^{509.73}$	$2^{315.29}$	4.81
Keccak-224	This work, § 8	7	"	$2^{218.11}$	50 → 85.70	58.66
Keccak-256	This work, § 8	8	"	$2^{255.64}$	$2^{254.03}$	1.29
Keccak-384	This work, § 8	8	"	$2^{378.74}$	$2^{324.06}$	38.36
Keccak-512	This work, § 8	6	"	$2^{505.58}$	$2^{104.23}$	85.70
	This work, § 8	7	"	$2^{506.11}$	$2^{180.12}$	59.34
	This work, § 8	8	"	$2^{506.74}$	$2^{324.07}$	38.36
	This work, § 8	9	"	$2^{511.70}$	$2^{510.02}$	1.23

# 1<sup>st</sup> and 2<sup>nd</sup> Preimage Attacks on 6, 7, 8, 9 rounds of Keccak (Result 3)

Version	Reference	No. of Rounds	Type of attack	Time Complexity	Memory Complexity	Improvement Factor
Keccak-256	[18]	2	Preimage	$2^{33}$		$2^{223}$
Keccak-512	[17]	3	Preimage	$2^{506}$		64
Keccak-512	<b>Bernstein's results</b>		Preimage	$2^{506}$		64
Keccak-512	[12, 8]	6	2nd Preimage	$2^{506}$	$2^{176}$	50
	[12, 8, 14]	7	"	$2^{507}$	$2^{320}$	37
	[12, 8, 14]	8	"	$2^{511.4}$	$2^{508}$	1.44
	This work, § 7	6	Preimage/ 2nd Preimage	$2^{509.19}$	$2^{98.91}$	7.01
	This work, § 7	7	"	$2^{509.39}$	$2^{172.52}$	6.13
	<b>Our results</b>		8	$2^{509.73}$	$2^{315.29}$	4.81
Keccak-224	This work, § 8	7	"	$2^{218.11}$	$2^{180.12}$	58.66
Keccak-256	This work, § 8	8	"	$2^{255.64}$		1.29
Keccak-384	This work, § 8	8	"	$2^{378.74}$	$2^{324.88}$	38.36
Keccak-512	This work, § 8	6	"	$2^{505.58}$	$2^{104.23}$	85.70
	This work, § 8	7	"	$2^{506.11}$	$2^{180.12}$	59.34
	This work, § 8	8	"	$2^{506.74}$	$2^{324.07}$	38.36
	This work, § 8	9	"	$2^{511.70}$	$2^{510.02}$	1.23

37 → 59.34

# 1<sup>st</sup> and 2<sup>nd</sup> Preimage Attacks on 6, 7, 8, 9 rounds of Keccak (Result 3)

Version	Reference	No. of Rounds	Type of attack	Time Complexity	Memory Complexity	Improvement Factor
Keccak-256	[18]	2	Preimage	$2^{33}$		$2^{223}$
Keccak-512	[17]	3	Preimage	$2^{506}$		64
Keccak-512	<b>Bernstein's results</b>		Preimage	$2^{506}$		64
Keccak-512	[12, 8]	6	2nd Preimage	$2^{506}$	$2^{176}$	50
	[12, 8, 14]	7	"	$2^{507}$	$2^{320}$	37
	[12, 8, 14]	8	"	$2^{511.4}$	$2^{508}$	1.44
	This work, § 7	6	Preimage/ 2nd Preimage	$2^{509.19}$	$2^{98.91}$	7.01
	This work, § 7	7	"	$2^{509.39}$	$2^{172.52}$	6.13
	<b>Our results</b>		"	$2^{509.73}$	$2^{315.29}$	4.81
Keccak-224	This work, § 8	7	"	$2^{218.11}$	$2^{180.12}$	58.66
Keccak-256	This work, § 8	8	"	$2^{255.64}$	$2^{254.03}$	1.29
Keccak-384	This work, § 8	8	"	$2^{378.74}$		38.36
Keccak-512	This work, § 8	6	"	$2^{505.58}$	$2^{104.23}$	85.70
	This work, § 8	7	"	$2^{506.11}$	$2^{180.12}$	59.34
	This work, § 8	8	"	$2^{506.74}$	$2^{324.07}$	38.36
	This work, § 8	9	"	$2^{511.70}$	$2^{510.02}$	1.23

1.44 → 38.36

# 1<sup>st</sup> and 2<sup>nd</sup> Preimage Attacks on 6, 7, 8, 9 rounds of Keccak (Result 3)

Version	Reference	No. of Rounds	Type of attack	Time Complexity	Memory Complexity	Improvement Factor
Keccak-256	[18]	2	Preimage	$2^{33}$		$2^{223}$
Keccak-512	[17]	3	Preimage	$2^{506}$		64
Keccak-512	<b>Bernstein's results</b>		Preimage	$2^{506}$		64
Keccak-512	[12, 8]	6	2nd Preimage	$2^{506}$	$2^{176}$	50
	[12, 8, 14]	7	"	$2^{507}$	$2^{320}$	37
	[12, 8, 14]	8	"	$2^{511.4}$	$2^{508}$	1.44
	This work, § 7	6	Preimage/ 2nd Preimage	$2^{509.19}$	$2^{98.91}$	7.01
	This work, § 7	7	"	$2^{509.39}$	$2^{172.52}$	6.13
	<b>Our results</b>		8	$2^{509.73}$	$2^{315.29}$	4.81
Keccak-224	This work, § 8	7	"	$2^{218.11}$	$2^{180.12}$	58.66
Keccak-256	This work, § 8	8	"	$2^{255.64}$	$2^{254.03}$	1.29
Keccak-384	This work, § 8	8	"	$2^{378.74}$	$2^{324.06}$	38.36
Keccak-512	This work, § 8	6	"	$2^{505.58}$	$2^{104.23}$	85.70
	This work, § 8	7	"	$2^{506.11}$		59.34
	This work, § 8	8	"	$2^{506.74}$	$2^{324.07}$	38.36
	This work, § 8	9	"	$2^{511.70}$	$2^{510.02}$	1.23

**New : 1.23**

# Work in Progress

- **Message Modification:** Good selection of position of message lanes will not double the degree by bypassing chi step ( $\chi$ ) of the round function of Keccak.
- Very careful memory and time complexity analysis required (at the complexities close to exhaustive search)
- Our preliminary analysis shows
  - 1<sup>st</sup> and 2<sup>nd</sup> preimage attacks on **9 rounds** of Keccak-256 with improvement factor 1.14
  - 1<sup>st</sup> and 2<sup>nd</sup> preimage attacks on **10 rounds** of Keccak-512 with improvement factor 1.05

# Conclusion

- None of the attacks threatens the security of Keccak as the attack complexities are already close to brute force by the time we cross 9 rounds of Keccak.
- In fact, this work shows the limits of polynomial enumeration method-based preimage attacks against Keccak.
- Our Attack on reduced rounds of Keccak can be applied to reduced rounds of SHA3 with the same complexity and same number of rounds.