A SAT-based preimage analysis of reduced KECCAK hash functions

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The Second SHA-3 Candidate Conference
Santa Barbara, CA, August 23-24, 2010

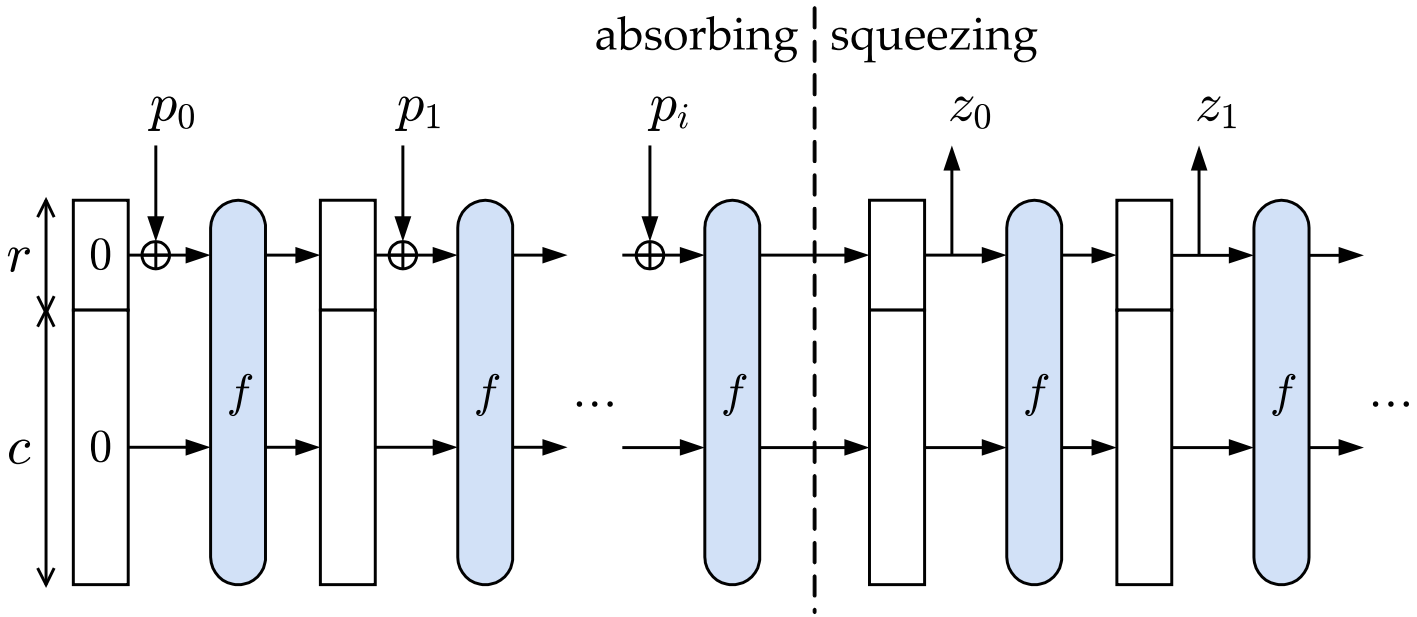
Overview

- Highlights of our results (relevant to the SHA-3 Contest)
- Keccak – some necessary basics
- SAT-based cryptanalysis – some necessary basics
- Our CryptLogVer toolkit
- Our experimental results on Keccak
- 2 other SHA-3 candidates
- Related work
- Conclusion and future directions

Highlights of our results (relevant to the SHA-3 Contest)

- Keccak is strong w.r.t. our analysis.
  - Our results suggest the strength of Keccak SHA-3 design.
  - We found a preimage only for very much reduced versions of Keccak.
  - 3-round Keccak-f[1600] with 40 unknown message bits.
- Our CNF size estimates for Grøstl and CubeHash suggest their very strong resistance against SAT-based cryptanalysis.

Keccak family sketch



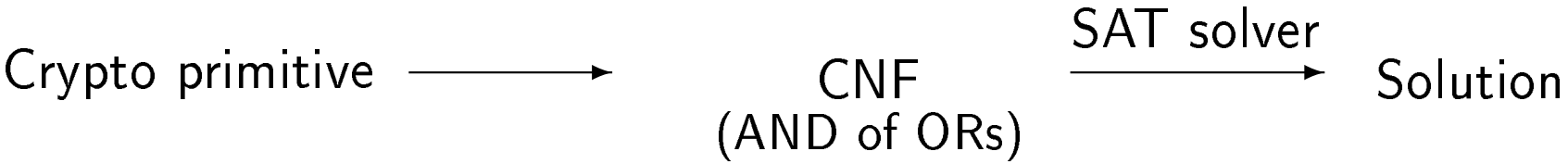
- Two main input parameters
  - r (called bitrate, default 1024 bits)
  - c (called capacity, default 576 bits)

  These parameters determine the state size, default 1600 bits.
- Keccak family variants operate also on smaller states:
  25- 50- 100- 200- 400- and 800-bit state

SAT-based cryptanalysis

Relatively recent method

Express your cryptographic primitive (e.g. a cipher, a hash function) in CNF and let a SAT-solver search for a solution.

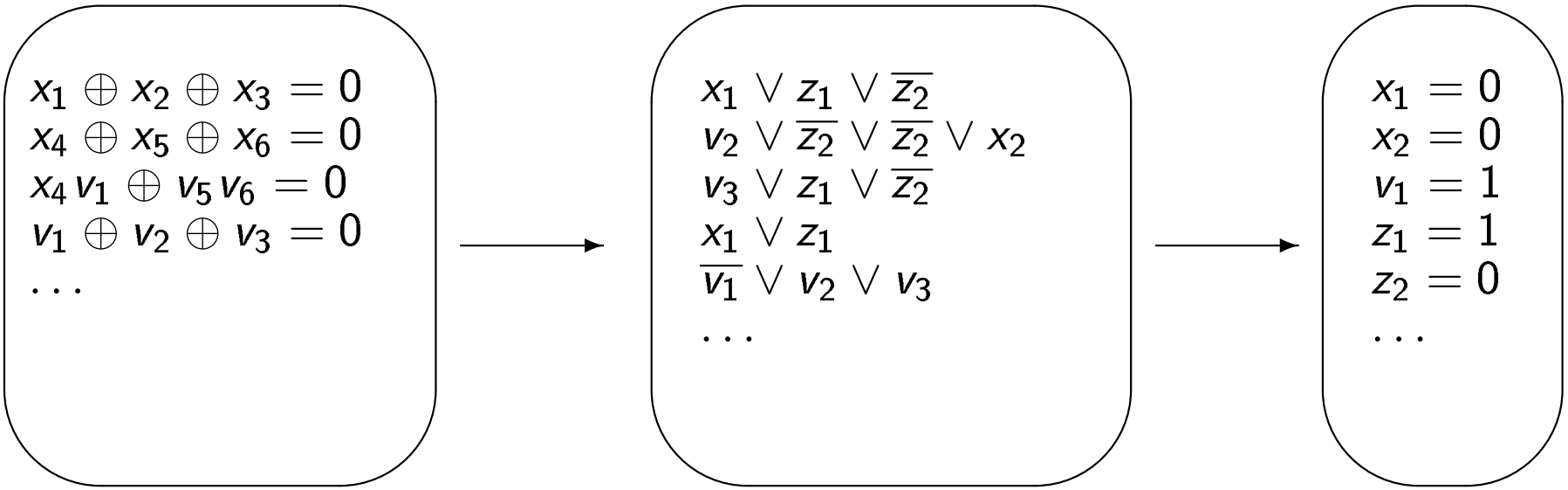


$$\varphi(x_1, \ldots, x_{1024}, \ldots, y_1, \ldots, y_{256})$$

Our CryptLogVer toolkit

3 applications:

1. Altera Quartus II Web Edition (free of charge)
2. Our CNF conversion:
   - From a system of boolean equations
   - To equisatisfiable CNF
   - In linear time, with many extra variables
3. SAT-solver PrecoSAT



Our experimental results on Keccak

Input parameters				Attack times [secs]	
Function	Number of rounds	Message size [bits]	Hash size [bits]	SAT-solver attack	Exhaustive search
Keccak[1024,576]	3	24	1024	2^{0}	2^{1}
Keccak[1024,576]	3	32	1024	$2^{3,3}$	2^{9}
Keccak[1024,576]	3	40	1024	$2^{10,8}$	2^{17}
Keccak[120,80]	3	24	80	$2^{2,5}$	$2^{-2,9}$
Keccak[120,80]	3	32	80	$2^{5,7}$	2^{5}
Keccak[120,80]	3	40	80	$2^{15,7}$	2^{13}
Keccak[24,26]	4	24	24	$2^{12,1}$	$2^{-13,5}$
Keccak[24,26]	5	24	24	$2^{12,8}$	$2^{-13,1}$

On Intel Xeon 2.5 GHz with 48 hours time limit.
Exhaustive search with speed-optimized implementation of Keccak Team.

2 other SHA-3 candidates

- Grøstl with 256-bit hash calls the AES S-box 1280 times.
  The AES S-box has been coded by our CryptLogVer toolkit as a formula with about 4800 clauses and 900 variables. A straigthforward calculation gives at least 1280 * 4800 = 6 mln 144 thousand clauses in total.
- Bernstein's CubeHash would have about
  1 mln 760 thousand clauses and 270 thousand variables.
- Full Keccak-f[1600] has
  775 thousand clauses and 181 thousand variables.
- Hence, no SAT-based attack looks feasible (with no extra financial effort). From this perspective: Keccak, Grøstl, and CubeHash seem to be very strong.
- Full SHA-1 has CNF with 181k clauses and 31k variables.

Related work

- The Keccak team used SAGE (computer algebra software) to solve the CICO problems with 12 unknown input bits, up to 8 rounds and with Keccak-f state widths from 40 to 200. As the number of unknown input bits grows, their method quickly becomes infeasible.
- Courtois and Bard, 2006: SAT-solvers can be a better option for cryptanalysis than computer algebra due to their much lower memory requirements.
- Rivest *et al.* tested their MD-6 with logical (SAT-based) analysis. They found collisions only for the first 11-rounds, as the best result.
- For full SHA-1, we obtained its CNF with 181k clauses and 31k variables. We found a short preimage for 27-round SHA-1 (out of its full 80 rounds).

Conclusion and future directions

- Our results suggest the strength of Keccak SHA-3 design.
  - We found a preimage only for very much reduced versions of Keccak.
  - E.g., 3-round Keccak-f[1600] with 40 unknown message bits.
- It might be interesting to try extrapolating our results for the full Keccak-f function (using a technique of Soos–Nohl–Castelluccia 2009)
- Grøstl and CubeHash seem to be also very strong against SAT-based cryptanalysis.
- SAT-based analysis of the other SHA-3 candidates might be interesting.

Thank you for your attention!

Questions, remarks, comments?