Subspace Distinguisher for 5/8 Rounds of the ECHO-256 Hash Function

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Outline

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Analysis of SHA-3 Candidates

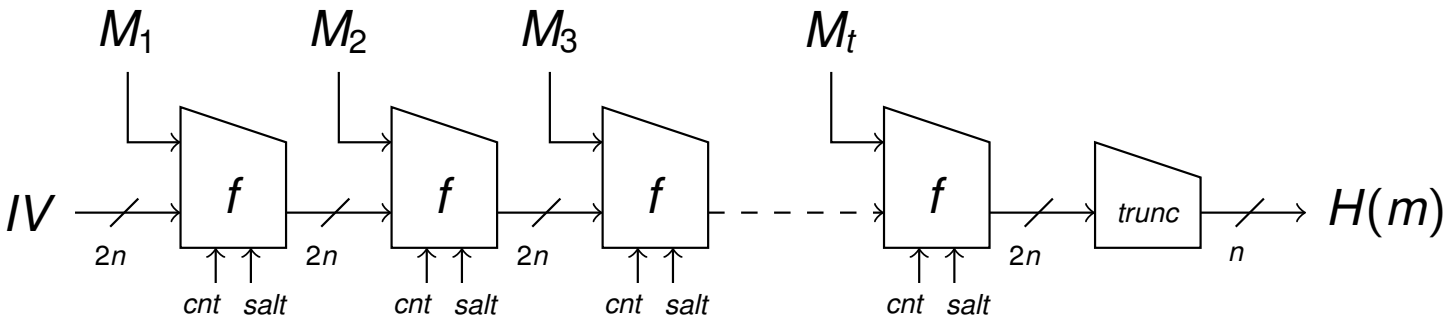
- Mostly attacks on building blocks
  - most of the time distinguishers
  - often high complexities, weak properties
  - sometimes better generic attacks exist
  - unknown how properties extend to hash function
  - can serve as lower bound for attacks on hash function
- Few attacks on hash function
  - can help in detecting more interesting properties
  - easier to compare attacks on hash functions
  - better evaluation of security margins
  - more work for attacker

How to Evaluate Attacks on SHA-3 Candidates?

- Number of attacked rounds (security parameter)
- Complexity of the attacks
- Which part has been attacked
- Which property has been found
- How much freedom is left in the attacks?
- Number of (non-incremental) independent results
- Effort spent on each hash function?
- Is the hash function easy to analyze?

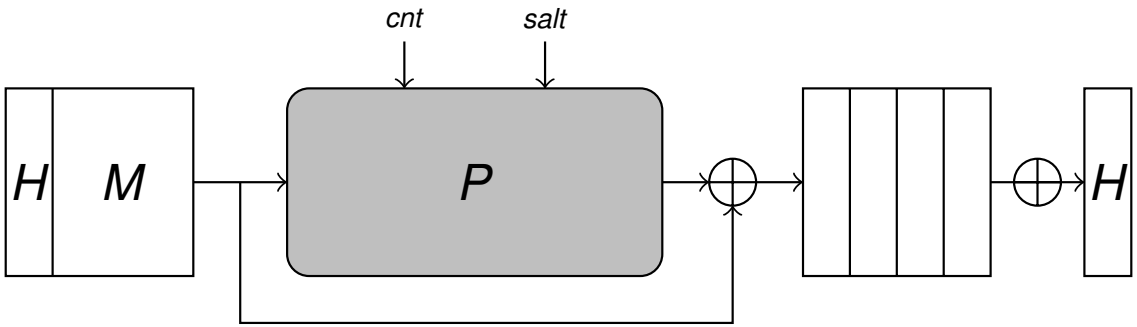
Outline

Description of ECHO [BBG+08]



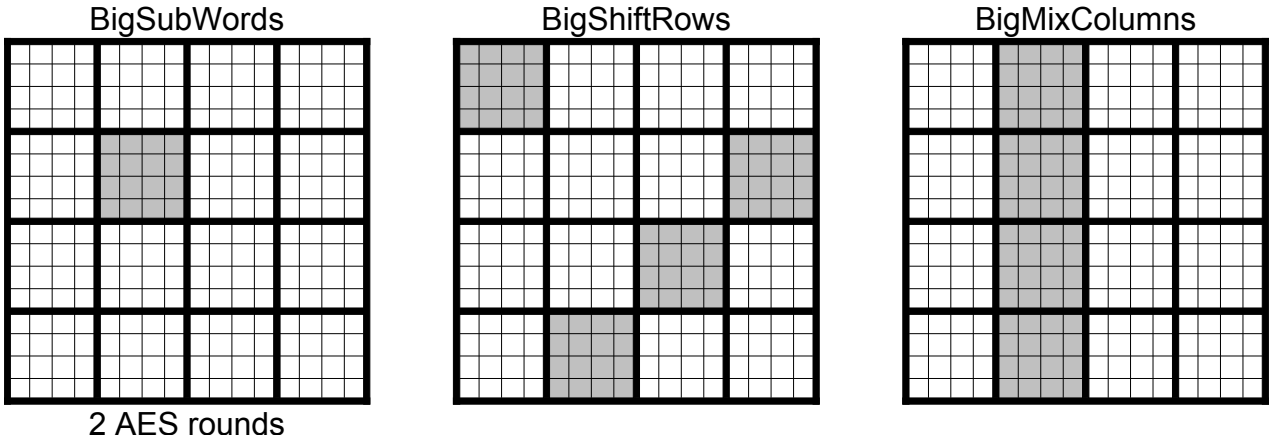
- Hash function submitted to the NIST SHA-3 competition
  - Round 2 candidate
  - AES based design
- Iterated hash function
  - Haifa design principle
  - double-pipe construction
  - output transformation: simple truncation
  - security based on non-randomness of compression function

The Compression Function of ECHO-256



- ECHO Permutation P
  - large 2048-bit permutation
  - tweak inputs (counter, salt)
  - AES based round transformations
- Finalization:
  - feed-forward of (H, M)
  - compression of 1024-bit columns

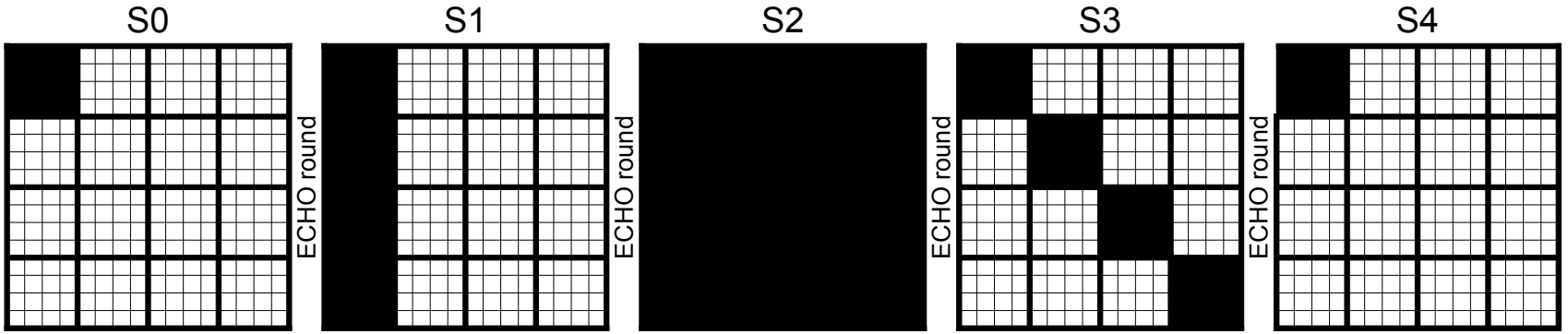
Round Transformations of ECHO



- ECHO state:
  - 4×4 AES states
  - AES inside AES
  - 8 rounds (ECHO-256) or 10 rounds (ECHO-512)
- Round transformations:
  - 128-bit BigSubWords (two standard AES rounds)
  - BigShiftRows shifts AES states
  - BigMixColumns mixes AES states (16 parallel MixColumns)

Outline

Simple 4-Round Truncated Differential Path



- Minimal 4-round AES path at word level [MPRS09]
  - number of active words: $1 \rightarrow 4 \rightarrow 16 \rightarrow 4 \rightarrow 1$
  - full active SubBytes layer
  - 800 active S-boxes

$\Rightarrow$ just enough freedom for Rebound Attack with one inbound phase

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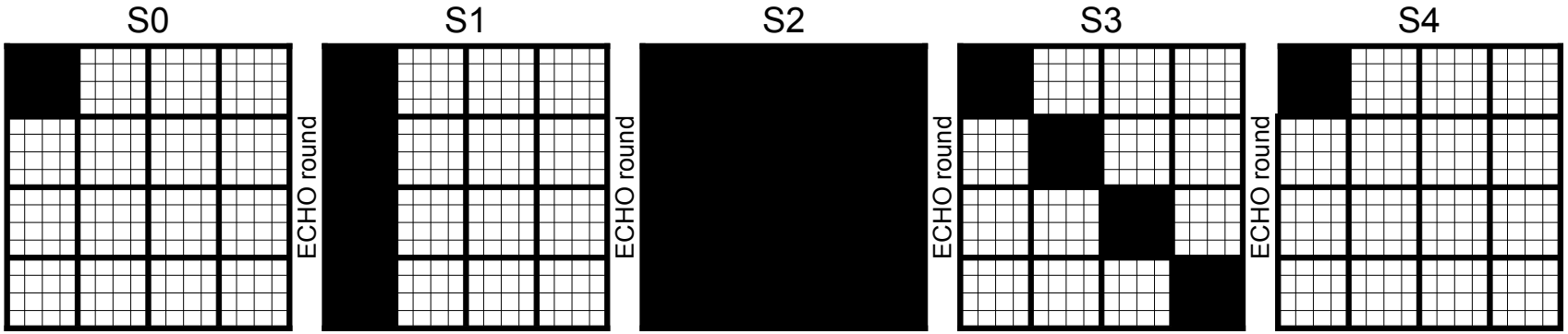

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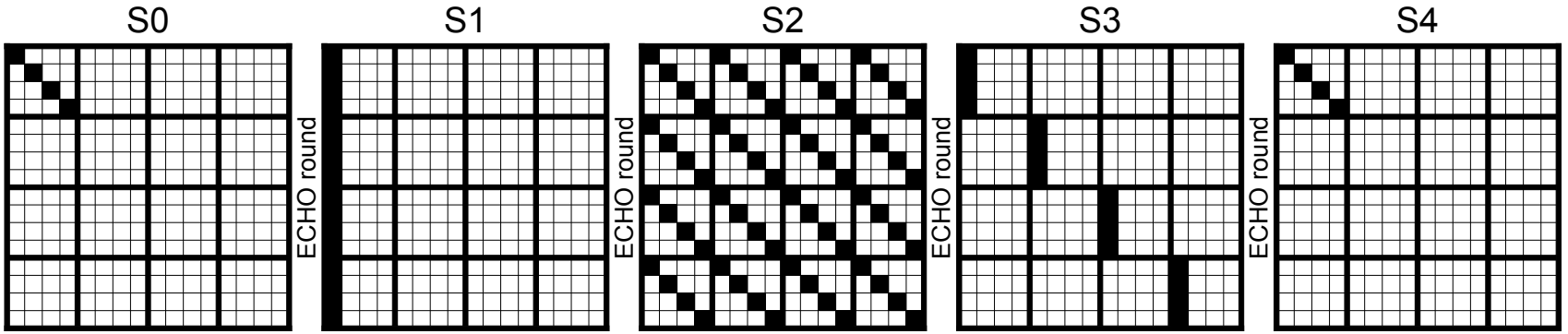
- Nevertheless: distinguishers for full ECHO-256 permutation
  - due to many improvements in the last year [GP10, Pey10a, Pey10b, SLW+10]

Sparse 4-Round Truncated Differential Path



- Sparse AES states in full active ECHO states
  - same number of active words: $1 \rightarrow 4 \rightarrow 16 \rightarrow 4 \rightarrow 1$
  - only 245 active S-boxes (lower bound: 200 [BBG+08])

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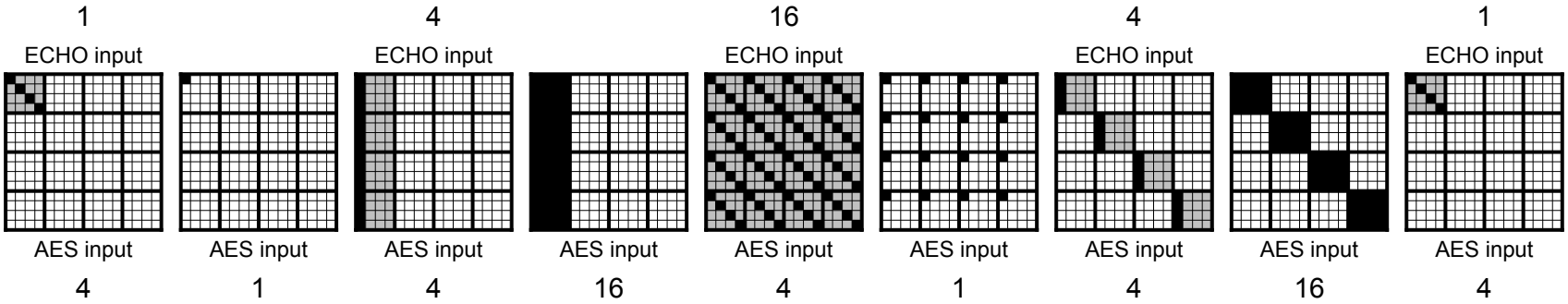

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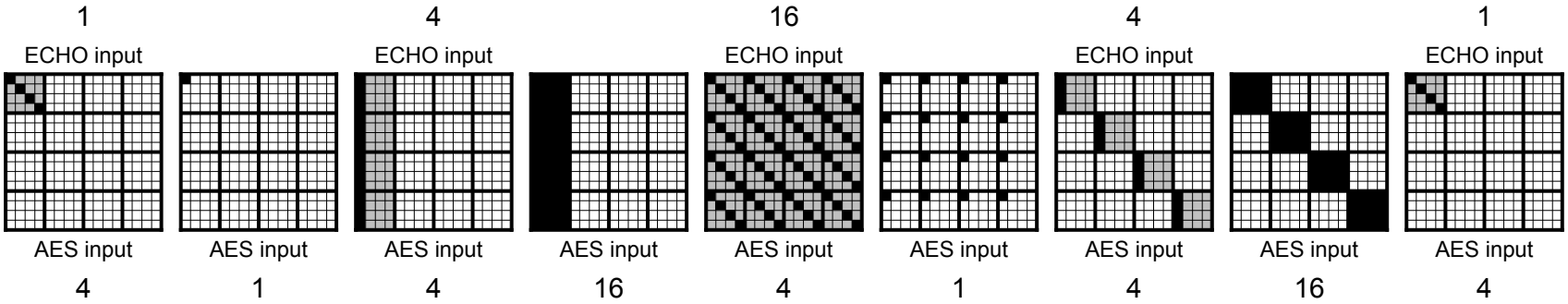

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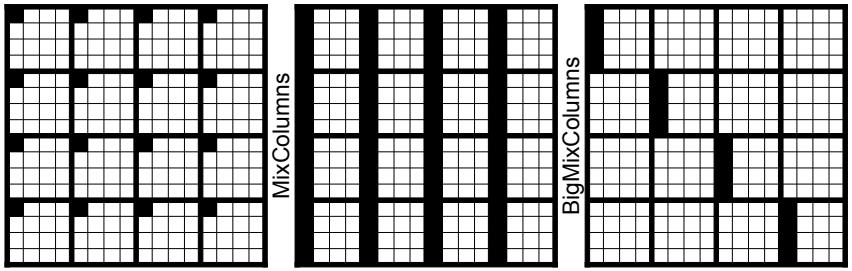
- Rebound Attack with multiple inbound phases
  - Whirlpool, LANE [LMR+09, MNPN+09]
  - less complexity
  - longer paths

MixColumns and BigMixColumns



- Properties of MC and BMC:
  - both have branch number 5
  - applied to 1x16 column of state
- Combined SuperMixColumns transformation (also [SLW+10]):
  - has branch number 8
  - ideal case: branch number 17

$\Rightarrow$ Sparse truncated differential paths

Computing the Available Degrees of Freedom

- For a given (truncated) differential path
  - determine probability of this path P_{dp}
  - total number of possible input pairs N_{in}
  - estimate the number of "surviving" output pairs:

$$N_{out} = N_{in} \cdot P_{dp}$$

  - degrees of freedom: $log_2(N_{out})$

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- $log_2(N_{out}) << 0$:
  - attack does not work with high probability
  - cannot be repeated with new input pair
  - restart with new (probably less efficient) path
- $log_2(N_{out}) >> 0$:
  - many right pairs exist (we need to find just one)
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This evaluation is independent of an actual Attack!

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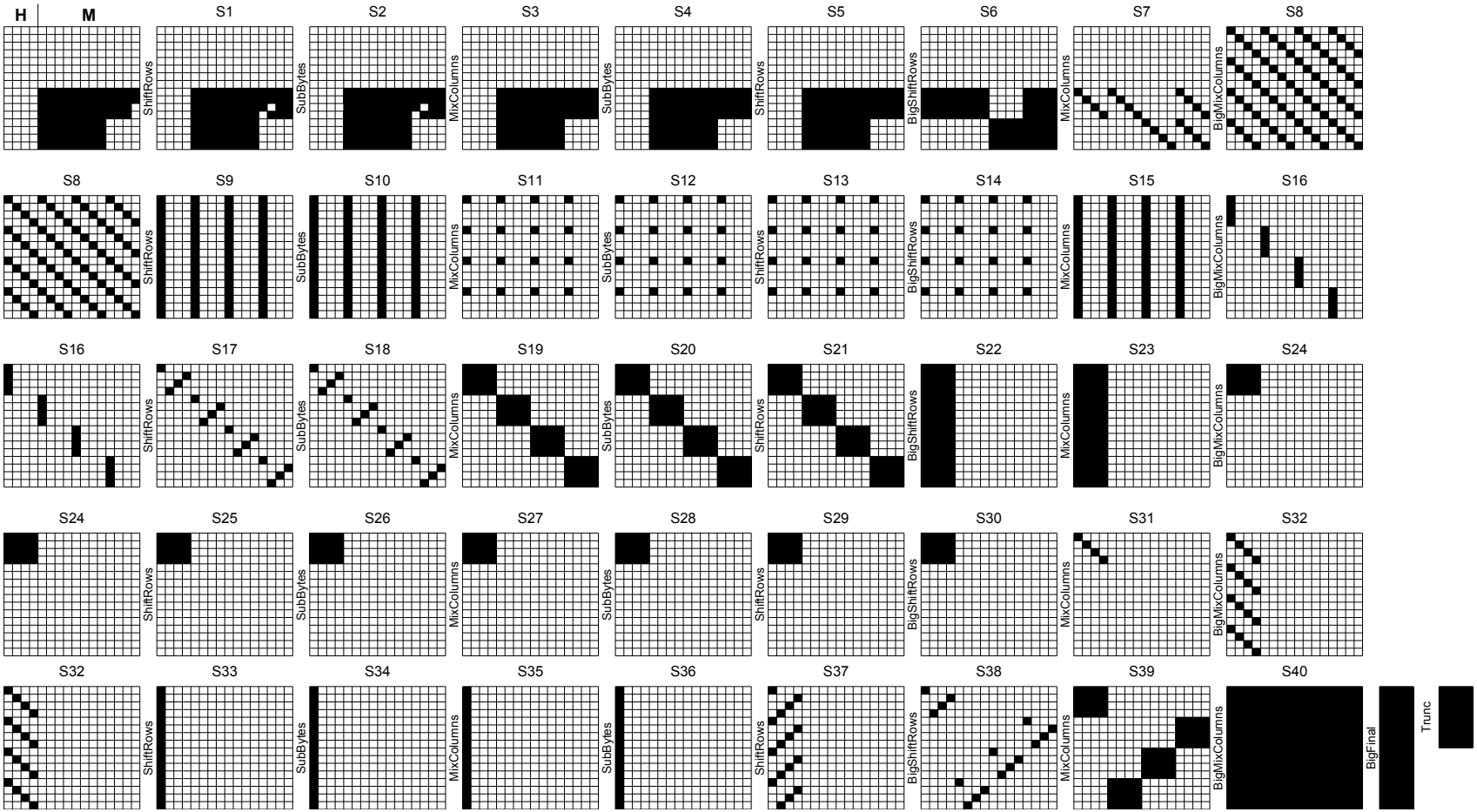
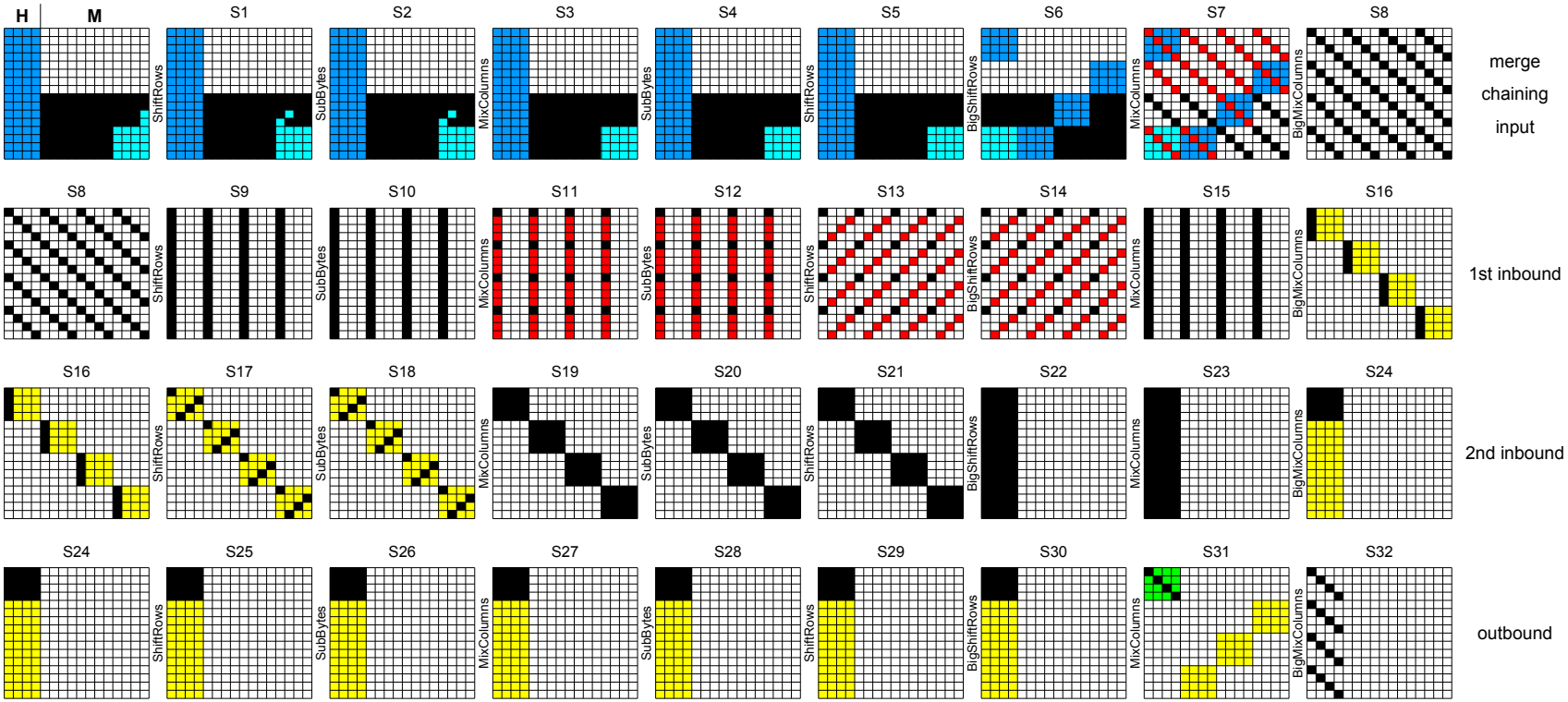
Rebound Attack with Multiple Inbound Phases



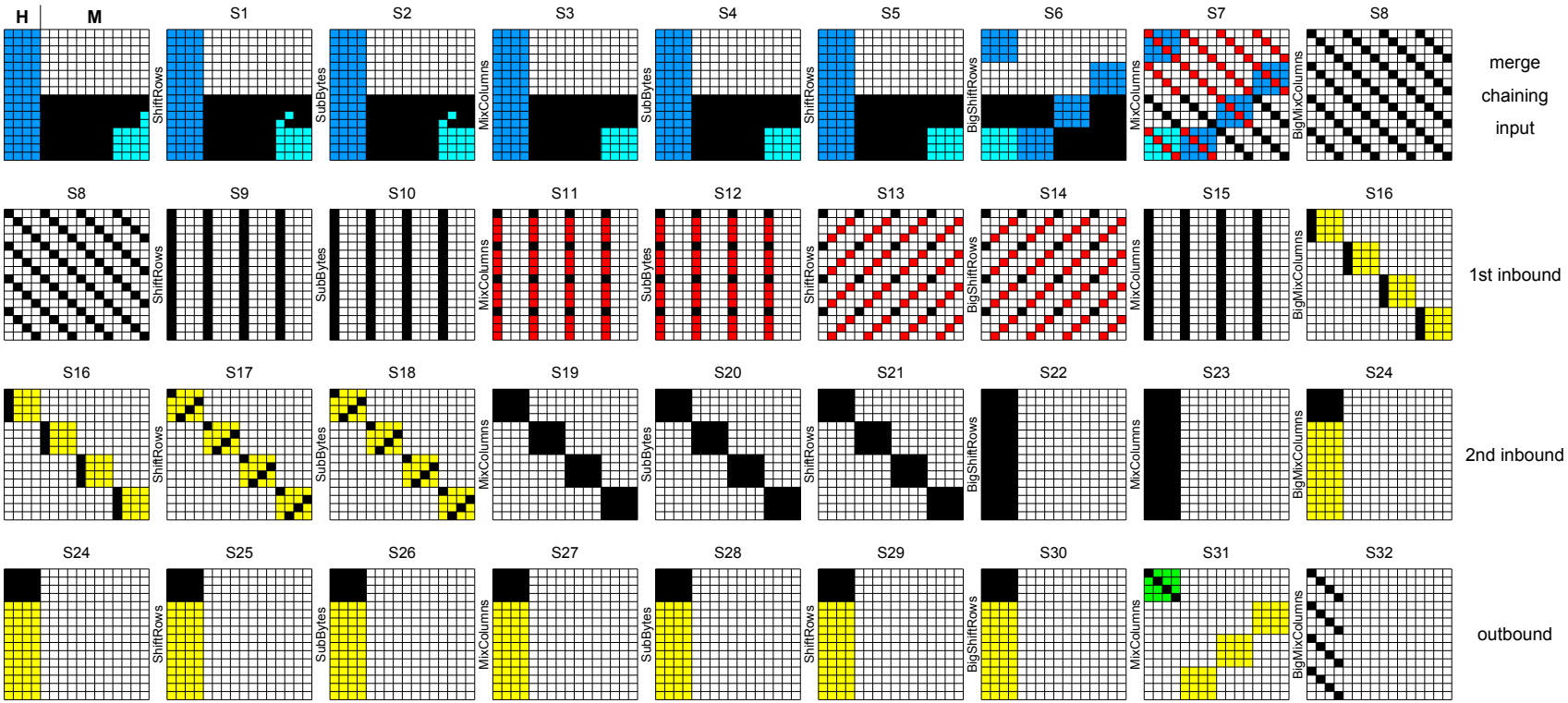
- enough freedom for more than one inbound phase
- merge independent solutions using birthday effect

Outline of the Rebound Attack



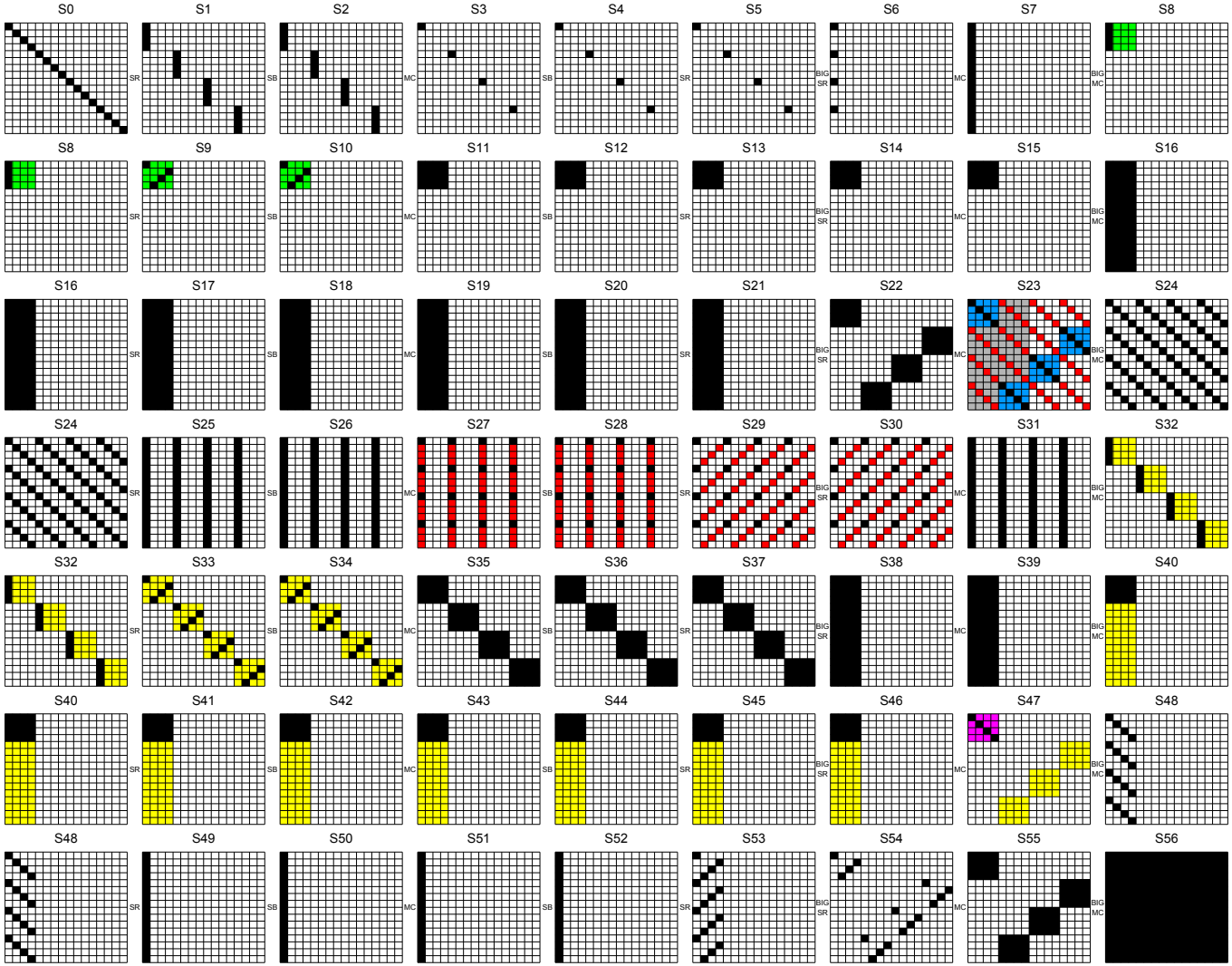
- 1st inbound phase (■, complexity 2^{32}, average 1)
- merge chaining input (■, complexity 2^{32})
- 2nd inbound phase (■, complexity 2^{64}, average 1)
- outbound phase (■, complexity 2^{96})
- merge inbound phases (□, complexity 2^{64})

Rough Estimation of Degrees of Freedom



- 1st inbound phase $\sim 1/4$ of the state (■)
- chaining input $\sim 1/4$ of the state (■)
- 2nd inbound phase $\sim 1/4$ of the state (■)
- available freedom $\sim 1/4$ of the state
  also freedom in differences (■)

Compression Function Dist. for 7/8 Rounds



$\sim$800 degrees of freedom ($\sim 2^{800}$ pairs for this path exist)

Outline

Results and Related Analysis

Function	Target	Rounds	Time	Memory	Type	Ref.
ECHO-256 (8 Rounds)	hash function	5	2^{96}	2^{64}	distinguisher	this work
		4.5	2^{96}	2^{64}	near-collision	this work
		4	2^{64}	2^{64}	collision	this work
	compression function	7	2^{152}	2^{64}	distinguisher[1]	this work
		7	2^{107}	2^{64}	distinguisher*	this work
		6.5	2^{152}	2^{64}	free-start near-collision	this work
		6.5	2^{96}	2^{64}	free-start near-collision*	this work
		4.5	2^{96}	2^{32}	distinguisher	[Pey10b]
		3	2^{64}	2^{32}	free-start collision	[Pey10b]
		3	2^{96}	2^{32}	semi-free-start collision*	[Pey10b]
ECHO-512 (10 Rounds)	compression function	7	2^{162}	2^{64}	distinguisher	this work
		7	2^{106}	2^{64}	distinguisher*	this work
		6.5	2^{152}	2^{64}	free-start near-collision	this work
		6.5	2^{96}	2^{64}	free-start near-collision*	this work
		6.5	2^{96}	2^{32}	distinguisher	[Pey10b]
		3	2^{96}	2^{32}	(semi-)free-start collision*	[Pey10b]

* chosen salt

[1] limited birthday distinguisher

(all attacks also without chosen salt)

Conclusions

- Easy analysis of AES-based hash functions
  - construct good paths by hand
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  - get good results in short amount of time

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- Sparse truncated differential paths in ECHO:
  - only 1/4 of the state is active
  - sparse also at input and output (“survives” final folding)
  - still leaving large ($>$ 800) degrees of freedom

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  - still leaving large (> 800) degrees of freedom

  $\Rightarrow$ Rebound Attack with multiple inbound phases
- How to protect against this type of attack?
  - avoid sparse (truncated) differential paths
  - strictly follow wide-trail design strategy: fixed-key AES, Grøstl, ... (best attacks on Grøstl: 0-3 degrees of freedom)

  $\Rightarrow$ not enough freedom for multiple inbound phases

Thank you for your Attention!

Questions?

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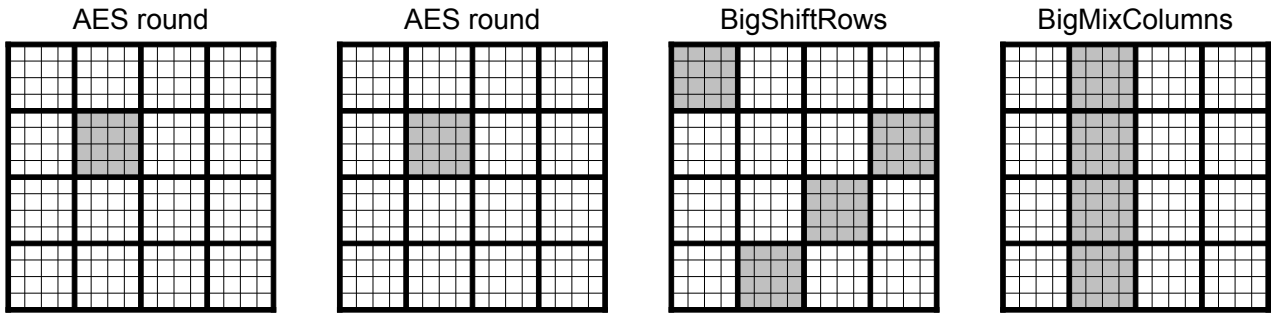
Internal AES Round Transformations



- 10 AES-like round transformations:
  - SubBytes
  - ShiftRows
  - MixColumns
  - AddRoundKey (counter)
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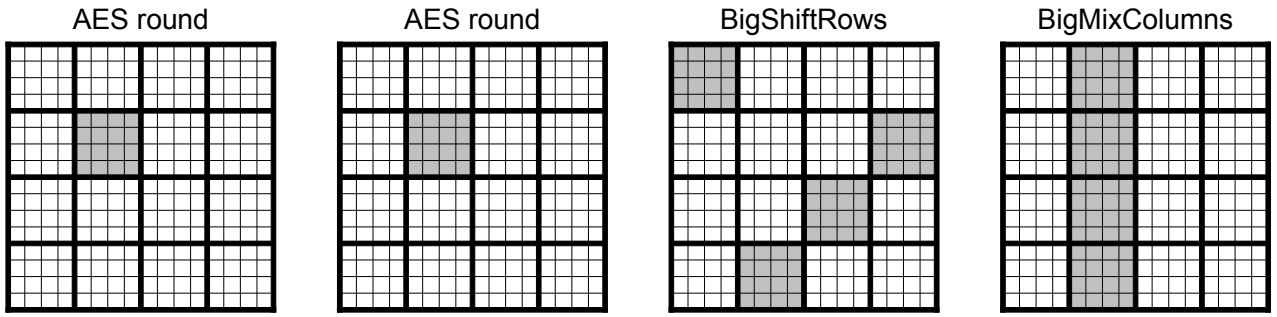

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⇐ non-linear

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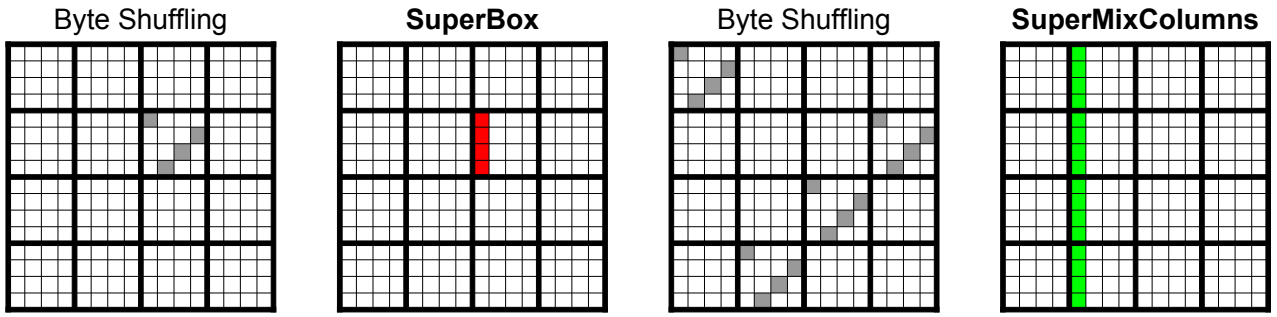

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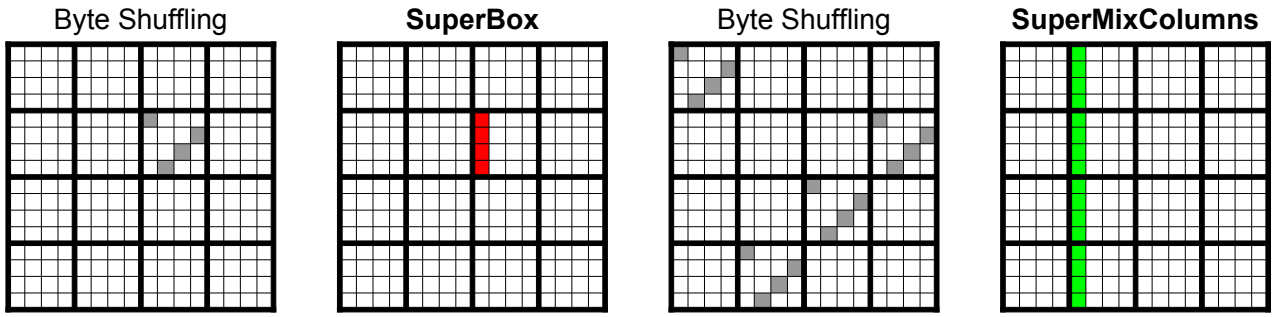

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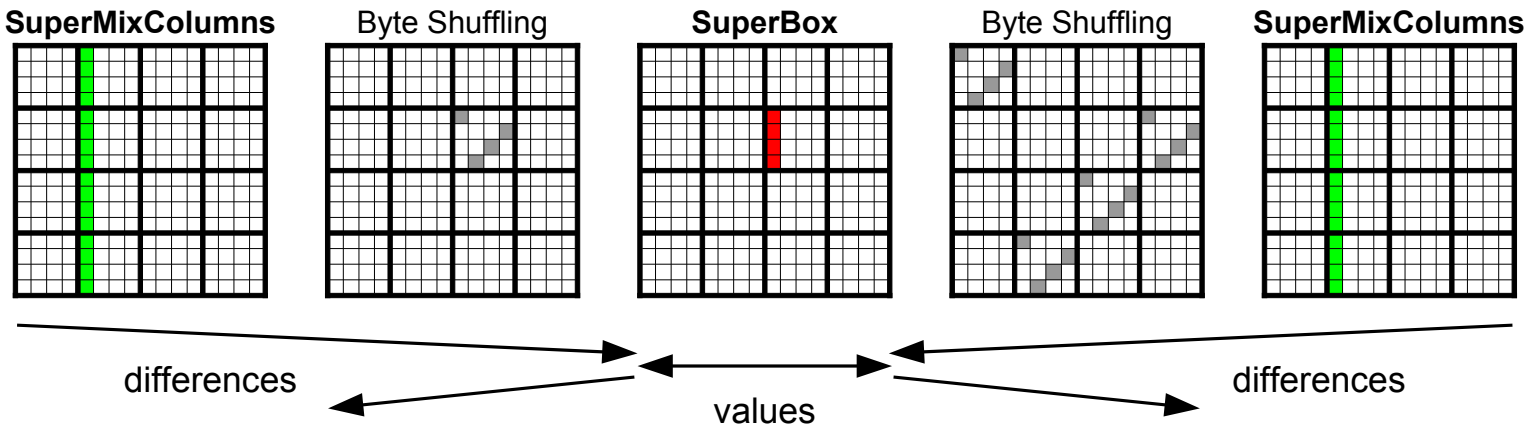
Rebound Attack on ECHO



- Inbound phase:
  - choosing valid differences for SuperMixColumns
  - matching resulting differences in SuperBoxes
- SuperBox match:
  - to get input values of SuperBoxes (complexity 1)
  - using differential distribution table (DDT) of size 2^{64}
- Rebound Attack with multiple inbound phases:
  - enough freedom for more than one inbound phase
  - merge independent solutions using birthday effect