An Efficient Software Implementation of Fugue

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Overview

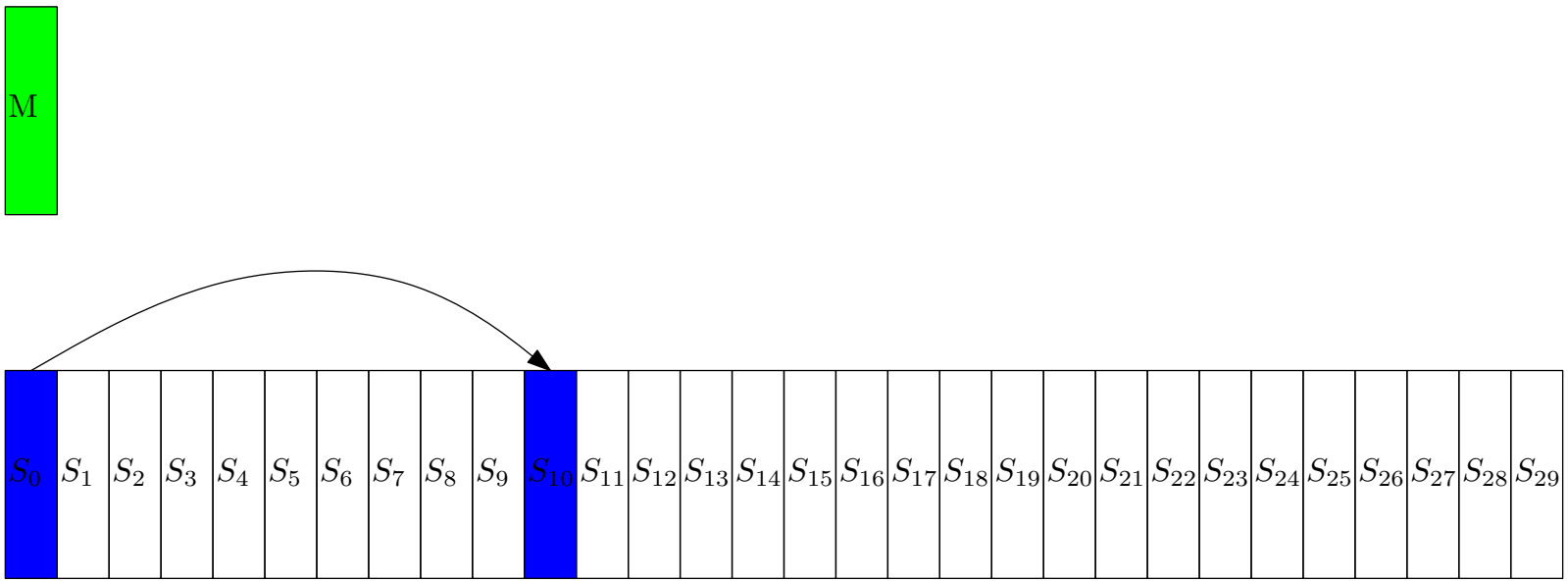
Outline

Fugue-256

S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}

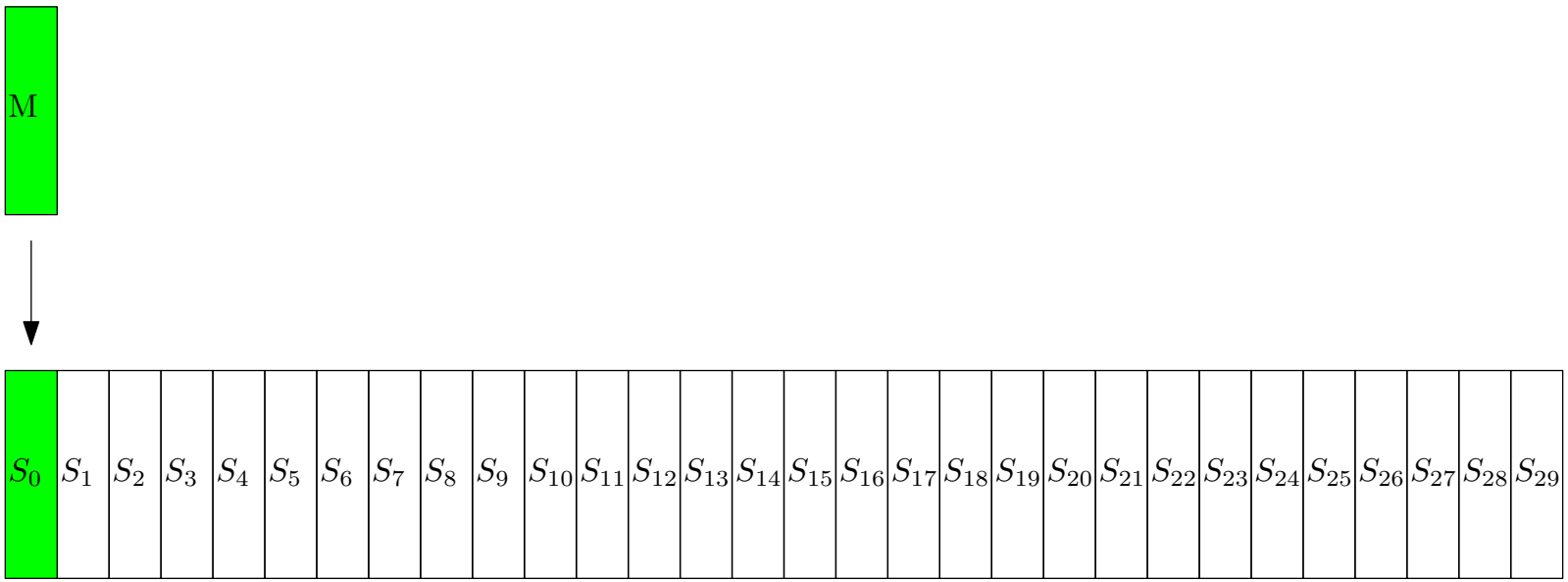
- $S_{10} += S_0$
- $S_0 = M$
- $S_8 += S_0$
- $S_1 += S_{24}$

TIX



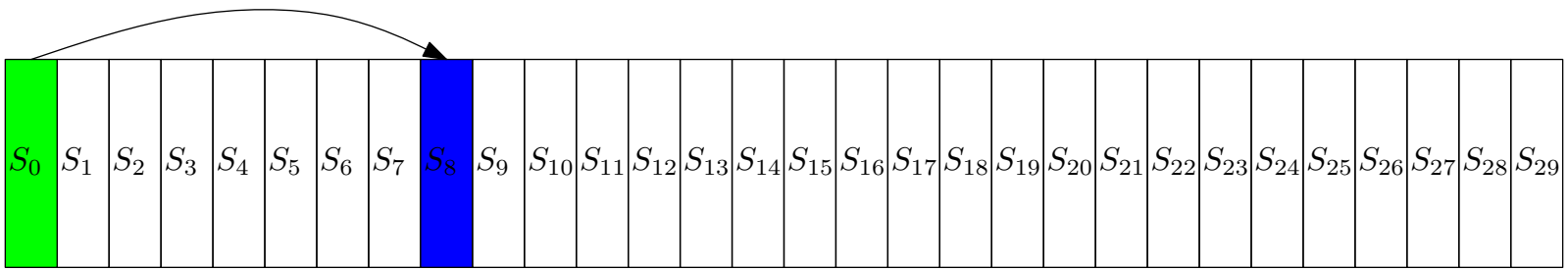
- $S_{10} += S_0$
- $S_0 = M$
- $S_8 += S_0$
- $S_1 += S_{24}$

TIX



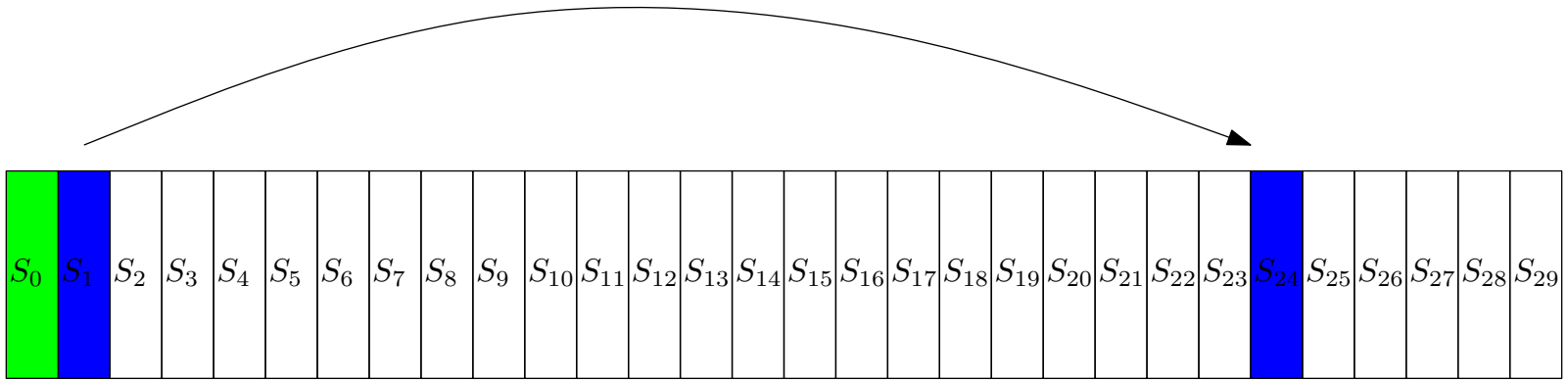
- $S_{10} += S_0$
- $S_0 = M$
- $S_8 += S_0$
- $S_1 += S_{24}$

TIX



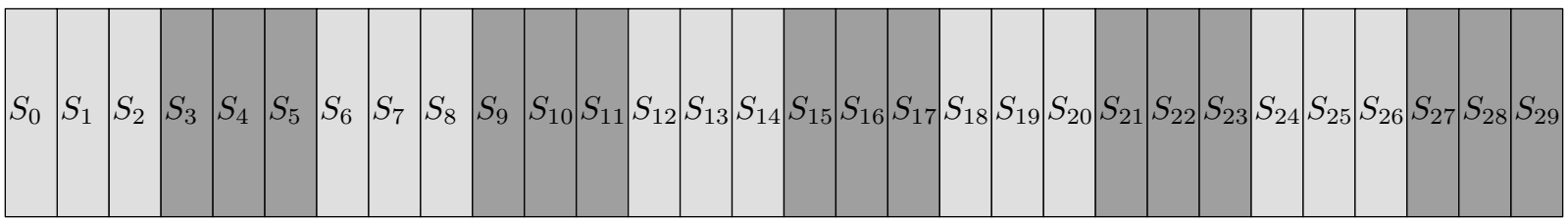
- $S_{10} += S_0$
- $S_0 = M$
- $S_8 += S_0$
- $S_1 += S_{24}$

TIX

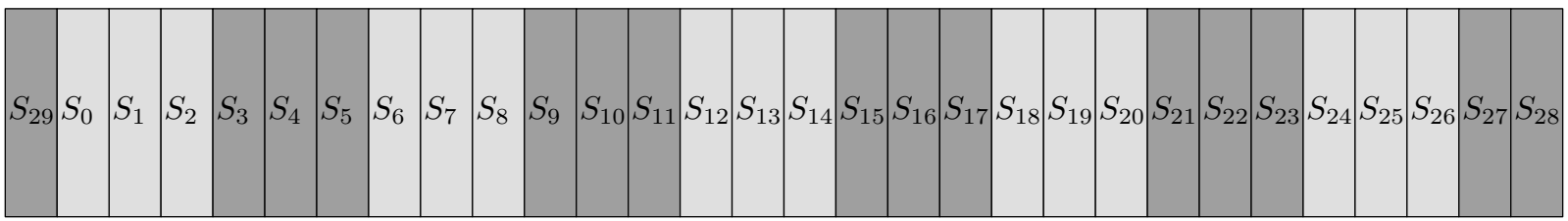


- $S_{10} += S_0$
- $S_0 = M$
- $S_8 += S_0$
- $S_1 += S_{24}$

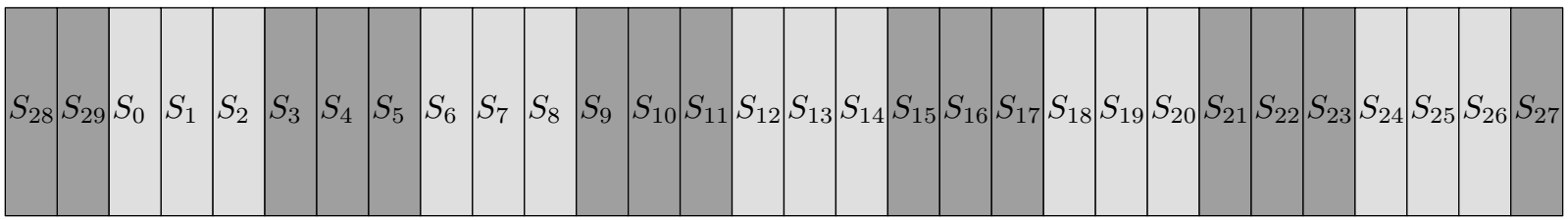
ROR3


S_0 S_1 S_2 S_3 S_4 S_5 S_6 S_7 S_8 S_9 S_{10} S_{11} S_{12} S_{13} S_{14} S_{15} S_{16} S_{17} S_{18} S_{19} S_{20} S_{21} S_{22} S_{23} S_{24} S_{25} S_{26} S_{27} S_{28} S_{29}

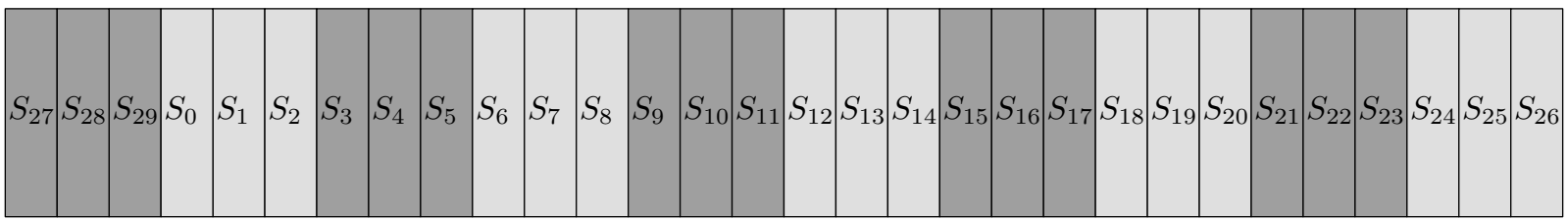
ROR3


S_{29} S_0 S_1 S_2 S_3 S_4 S_5 S_6 S_7 S_8 S_9 S_{10} S_{11} S_{12} S_{13} S_{14} S_{15} S_{16} S_{17} S_{18} S_{19} S_{20} S_{21} S_{22} S_{23} S_{24} S_{25} S_{26} S_{27} S_{28}

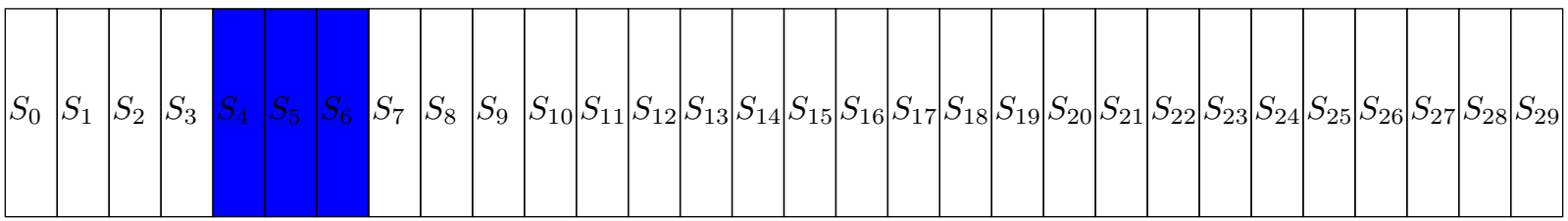
ROR3


S_{28} S_{29} S_0 S_1 S_2 S_3 S_4 S_5 S_6 S_7 S_8 S_9 S_{10} S_{11} S_{12} S_{13} S_{14} S_{15} S_{16} S_{17} S_{18} S_{19} S_{20} S_{21} S_{22} S_{23} S_{24} S_{25} S_{26} S_{27}

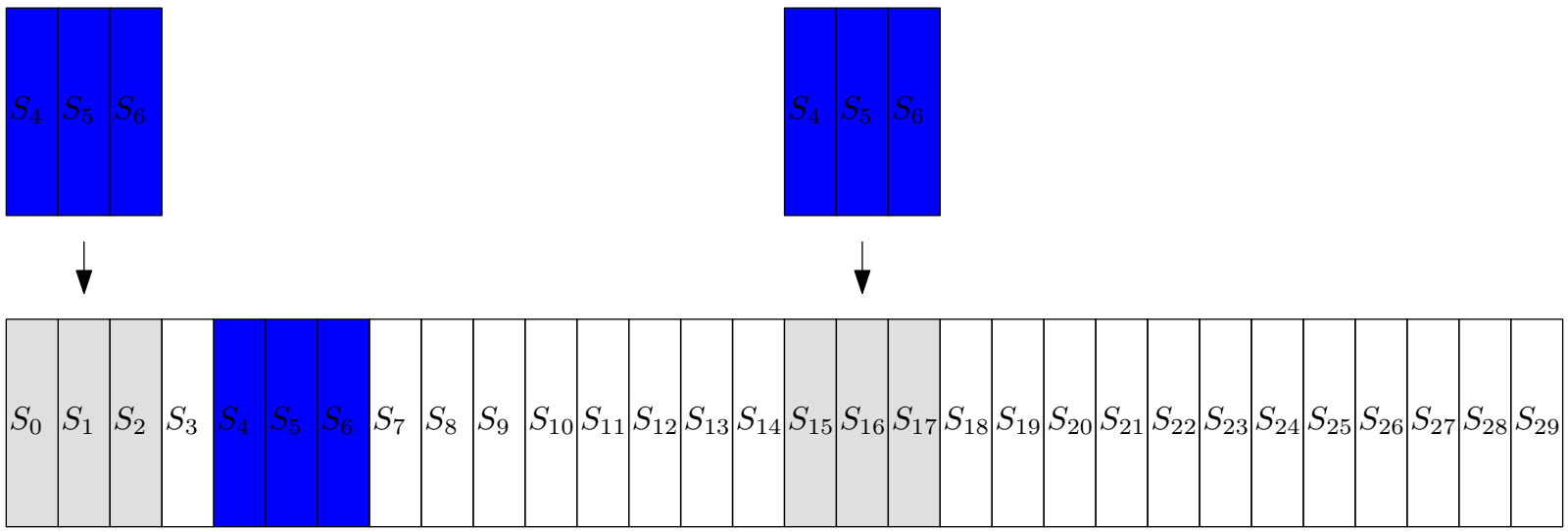
ROR3


S_{27} S_{28} S_{29} S_0 S_1 S_2 S_3 S_4 S_5 S_6 S_7 S_8 S_9 S_{10} S_{11} S_{12} S_{13} S_{14} S_{15} S_{16} S_{17} S_{18} S_{19} S_{20} S_{21} S_{22} S_{23} S_{24} S_{25} S_{26}

CMIX


S_0 S_1 S_2 S_3 S_4 S_5 S_6 S_7 S_8 S_9 S_{10} S_{11} S_{12} S_{13} S_{14} S_{15} S_{16} S_{17} S_{18} S_{19} S_{20} S_{21} S_{22} S_{23} S_{24} S_{25} S_{26} S_{27} S_{28} S_{29}

CMIX


S_4 S_5 S_6
S_4 S_5 S_6
S_0 S_1 S_2 S_3 S_4 S_5 S_6 S_7 S_8 S_9 S_{10} S_{11} S_{12} S_{13} S_{14} S_{15} S_{16} S_{17} S_{18} S_{19} S_{20} S_{21} S_{22} S_{23} S_{24} S_{25} S_{26} S_{27} S_{28} S_{29}

Substitution

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																										
x_2	x_6	x_{10}	x_{14}																										
x_3	x_7	x_{11}	x_{15}																										

- Each byte is replaced using AES s-box.

Substitution

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																										
x_2	x_6	x_{10}	x_{14}																										
x_3	x_7	x_{11}	x_{15}																										

- Each byte is replaced using AES s-box.

Super-MIX

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																										
x_2	x_6	x_{10}	x_{14}																										
x_3	x_7	x_{11}	x_{15}																										

$$
\begin{bmatrix}
1 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 4 & 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 7 & 1 & 1 & 4 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 4 & 7 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 7 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 1 & 0 & 4 \\
4 & 7 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 4 & 7 & 1 & 7 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 1 & 6 & 4 & 7 \\
7 & 1 & 6 & 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 7 & 4 & 7 & 1 & 6 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 4 & 7 & 1 \\
1 & 5 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 7 & 1 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 4 & 7 & 1 & 5 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix}
=
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix}
$$

Super-MIX

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																									
x_2	x_6	x_{10}	x_{14}																									
x_3	x_7	x_{11}	x_{15}																									

$$
\begin{bmatrix}
1 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 4 & 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 7 & 1 & 1 & 4 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 4 & 7 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 7 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 1 & 0 & 4 \\
4 & 7 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 4 & 7 & 1 & 7 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 1 & 6 & 4 & 7 \\
7 & 1 & 6 & 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 7 & 4 & 7 & 1 & 6 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 4 & 7 & 1 \\
1 & 5 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 7 & 1 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 4 & 7 & 1 & 5 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix}
=
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix}
$$

Super-MIX

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																									
x_2	x_6	x_{10}	x_{14}																									
x_3	x_7	x_{11}	x_{15}																									

$$
\begin{bmatrix}
1 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 4 & 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 7 & 1 & 1 & 4 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 4 & 7 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 7 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 1 & 0 & 4 \\
4 & 7 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 4 & 7 & 1 & 7 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 1 & 6 & 4 & 7 \\
7 & 1 & 6 & 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 7 & 4 & 7 & 1 & 6 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 4 & 7 & 1 \\
1 & 5 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 7 & 1 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 4 & 7 & 1 & 5 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix}
=
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix}
$$

Super-MIX

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																									
x_2	x_6	x_{10}	x_{14}																									
x_3	x_7	x_{11}	x_{15}																									

$$
\begin{bmatrix}
1 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 4 & 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 7 & 1 & 1 & 4 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 4 & 7 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 7 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 1 & 0 & 4 \\
4 & 7 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 4 & 7 & 1 & 7 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 1 & 6 & 4 & 7 \\
7 & 1 & 6 & 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 7 & 4 & 7 & 1 & 6 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 4 & 7 & 1 \\
1 & 5 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 7 & 1 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 4 & 7 & 1 & 5 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix}
=
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix}
$$

Super-MIX

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																										
x_2	x_6	x_{10}	x_{14}																										
x_3	x_7	x_{11}	x_{15}																										

$$
\begin{bmatrix}
1 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 4 & 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 7 & 1 & 1 & 4 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 4 & 7 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 7 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 1 & 0 & 4 \\
4 & 7 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 4 & 7 & 1 & 7 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 1 & 6 & 4 & 7 \\
7 & 1 & 6 & 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 7 & 4 & 7 & 1 & 6 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 4 & 7 & 1 \\
1 & 5 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 7 & 1 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 4 & 7 & 1 & 5 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix}
=
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix}
$$

Super-MIX

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																									
x_2	x_6	x_{10}	x_{14}																									
x_3	x_7	x_{11}	x_{15}																									

$$
\begin{bmatrix}
1 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 4 & 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 7 & 1 & 1 & 4 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 4 & 7 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 7 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 1 & 0 & 4 \\
4 & 7 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 4 & 7 & 1 & 7 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 1 & 6 & 4 & 7 \\
7 & 1 & 6 & 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 7 & 4 & 7 & 1 & 6 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 4 & 7 & 1 \\
1 & 5 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 7 & 1 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 4 & 7 & 1 & 5 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix}
=
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix}
$$

Super-MIX

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																										
x_2	x_6	x_{10}	x_{14}																										
x_3	x_7	x_{11}	x_{15}																										

$$
\begin{bmatrix}
1 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 4 & 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 7 & 1 & 1 & 4 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 4 & 7 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 7 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 1 & 0 & 4 \\
4 & 7 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 4 & 7 & 1 & 7 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 1 & 6 & 4 & 7 \\
7 & 1 & 6 & 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 7 & 4 & 7 & 1 & 6 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 4 & 7 & 1 \\
1 & 5 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 7 & 1 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 4 & 7 & 1 & 5 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix}
=
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix}
$$

Super-MIX

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																									
x_2	x_6	x_{10}	x_{14}																									
x_3	x_7	x_{11}	x_{15}																									

$$
\begin{bmatrix}
1 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 4 & 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 7 & 1 & 1 & 4 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 4 & 7 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 7 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 1 & 0 & 4 \\
4 & 7 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 4 & 7 & 1 & 7 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 1 & 6 & 4 & 7 \\
7 & 1 & 6 & 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 7 & 4 & 7 & 1 & 6 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 4 & 7 & 1 \\
1 & 5 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 7 & 1 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 4 & 7 & 1 & 5 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix}
=
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix}
$$

Super-MIX

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																										
x_2	x_6	x_{10}	x_{14}																										
x_3	x_7	x_{11}	x_{15}																										

$$
\begin{bmatrix}
1 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 4 & 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 7 & 1 & 1 & 4 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 4 & 7 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 7 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 1 & 0 & 4 \\
4 & 7 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 4 & 7 & 1 & 7 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 1 & 6 & 4 & 7 \\
7 & 1 & 6 & 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 7 & 4 & 7 & 1 & 6 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 4 & 7 & 1 \\
1 & 5 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 7 & 1 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 4 & 7 & 1 & 5 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix}
=
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix}
$$

Super-MIX

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																										
x_2	x_6	x_{10}	x_{14}																										
x_3	x_7	x_{11}	x_{15}																										

$$
\begin{bmatrix}
1 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 4 & 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 7 & 1 & 1 & 4 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 4 & 7 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 7 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 1 & 0 & 4 \\
4 & 7 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 4 & 7 & 1 & 7 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 1 & 6 & 4 & 7 \\
7 & 1 & 6 & 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 7 & 4 & 7 & 1 & 6 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 4 & 7 & 1 \\
1 & 5 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 7 & 1 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 4 & 7 & 1 & 5 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix}
=
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix}
$$

Super-MIX

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																										
x_2	x_6	x_{10}	x_{14}																										
x_3	x_7	x_{11}	x_{15}																										

$$
\begin{bmatrix}
1 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 4 & 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 7 & 1 & 1 & 4 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 4 & 7 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 7 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 1 & 0 & 4 \\
4 & 7 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 4 & 7 & 1 & 7 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 1 & 6 & 4 & 7 \\
7 & 1 & 6 & 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 7 & 4 & 7 & 1 & 6 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 4 & 7 & 1 \\
1 & 5 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 7 & 1 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 4 & 7 & 1 & 5 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix}
=
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix}
$$

Super-MIX

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																										
x_2	x_6	x_{10}	x_{14}																										
x_3	x_7	x_{11}	x_{15}																										

$$
\begin{bmatrix}
1 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 4 & 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 7 & 1 & 1 & 4 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 4 & 7 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 7 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 1 & 0 & 4 \\
4 & 7 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 4 & 7 & 1 & 7 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 1 & 6 & 4 & 7 \\
7 & 1 & 6 & 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 7 & 4 & 7 & 1 & 6 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 4 & 7 & 1 \\
1 & 5 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 7 & 1 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 4 & 7 & 1 & 5 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix}
=
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix}
$$

Super-MIX

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																										
x_2	x_6	x_{10}	x_{14}																										
x_3	x_7	x_{11}	x_{15}																										

$$
\begin{bmatrix}
1 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 4 & 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 7 & 1 & 1 & 4 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 4 & 7 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 7 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 1 & 0 & 4 \\
4 & 7 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 4 & 7 & 1 & 7 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 1 & 6 & 4 & 7 \\
7 & 1 & 6 & 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 7 & 4 & 7 & 1 & 6 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 4 & 7 & 1 \\
1 & 5 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 7 & 1 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 4 & 7 & 1 & 5 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix}
=
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix}
$$

Super-MIX

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																										
x_2	x_6	x_{10}	x_{14}																										
x_3	x_7	x_{11}	x_{15}																										

$$
\begin{bmatrix}
1 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 4 & 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 7 & 1 & 1 & 4 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 4 & 7 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 7 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 1 & 0 & 4 \\
4 & 7 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 4 & 7 & 1 & 7 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 1 & 6 & 4 & 7 \\
7 & 1 & 6 & 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 7 & 4 & 7 & 1 & 6 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 4 & 7 & 1 \\
1 & 5 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 7 & 1 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 4 & 7 & 1 & 5 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix}
=
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix}
$$

Super-MIX

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																										
x_2	x_6	x_{10}	x_{14}																										
x_3	x_7	x_{11}	x_{15}																										

$$
\begin{bmatrix}
1 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 4 & 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 7 & 1 & 1 & 4 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 4 & 7 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 7 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 1 & 0 & 4 \\
4 & 7 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 4 & 7 & 1 & 7 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 1 & 6 & 4 & 7 \\
7 & 1 & 6 & 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 7 & 4 & 7 & 1 & 6 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 4 & 7 & 1 \\
1 & 5 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 7 & 1 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 4 & 7 & 1 & 5 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix}
=
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix}
$$

Super-MIX

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																										
x_2	x_6	x_{10}	x_{14}																										
x_3	x_7	x_{11}	x_{15}																										

$$
\begin{bmatrix}
1 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 4 & 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 7 & 1 & 1 & 4 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 4 & 7 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 7 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 1 & 0 & 4 \\
4 & 7 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 4 & 7 & 1 & 7 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 1 & 6 & 4 & 7 \\
7 & 1 & 6 & 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 7 & 4 & 7 & 1 & 6 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 4 & 7 & 1 \\
1 & 5 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 7 & 1 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 4 & 7 & 1 & 5 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix}
=
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix}
$$

Super-MIX

x_0	x_4	x_8	x_{12}	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}
x_1	x_5	x_9	x_{13}																										
x_2	x_6	x_{10}	x_{14}																										
x_3	x_7	x_{11}	x_{15}																										

$$
\begin{bmatrix}
1 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 4 & 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 7 & 1 & 1 & 4 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 4 & 7 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 7 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 1 & 0 & 4 \\
4 & 7 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 4 & 7 & 1 & 7 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 1 & 6 & 4 & 7 \\
7 & 1 & 6 & 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 7 & 4 & 7 & 1 & 6 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 4 & 7 & 1 \\
1 & 5 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 7 & 1 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 4 & 7 & 1 & 5 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix}
=
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix}
$$

Outline

- 16 xmm registers, each 128-bits wide. (16-bytes, 8-words, 4-dwords, 2-quadwords)
- Binary operations, integer arithmetic, shifts, shuffles, etc.
- A necessary instruction: `pshufb` can be used as a 4×8 s-box.

Outline

SUBSTITUTE

```
movaps t1, xmmword ptr k_s0F          ;    1 : i
pandn t1, s0                          ;    1 = i¡¡4
psrld t1, 4                           ;    1 = i
pand s0, xmmword ptr k_s0F            ;    0 = k
movaps t2, xmmword ptr k_inv + 16     ;    2 : a/k
pshufb t2, s0                         ;    2 = a/k
pxor s0, t1                           ;    0 = j
movaps t3, xmmword ptr k_inv          ;    3 : 1/i
pshufb t3, t1                         ;    3 = 1/i
pxor t3, t2                           ;    3 = iak = 1/i + a/k
movaps t4, xmmword ptr k_inv          ;    4 : 1/j
pshufb t4, s0                         ;    4 = 1/j
pxor t4, t2                           ;    4 = jak = 1/j + a/k
movaps t2, xmmword ptr k_inv          ;    2 : 1/iak
pshufb t2, t3                         ;    2 = 1/iak
pxor t2, s0                           ;    2 = io
movaps t3, xmmword ptr k_inv          ;    3 : 1/jak
pshufb t3, t4                         ;    3 = 1/jak
xorps t3, t1                          ;    3 = jo
```

- Mike Hamburg, *Accelerating AES with Vector Permute Instructions*, CHES 2009.

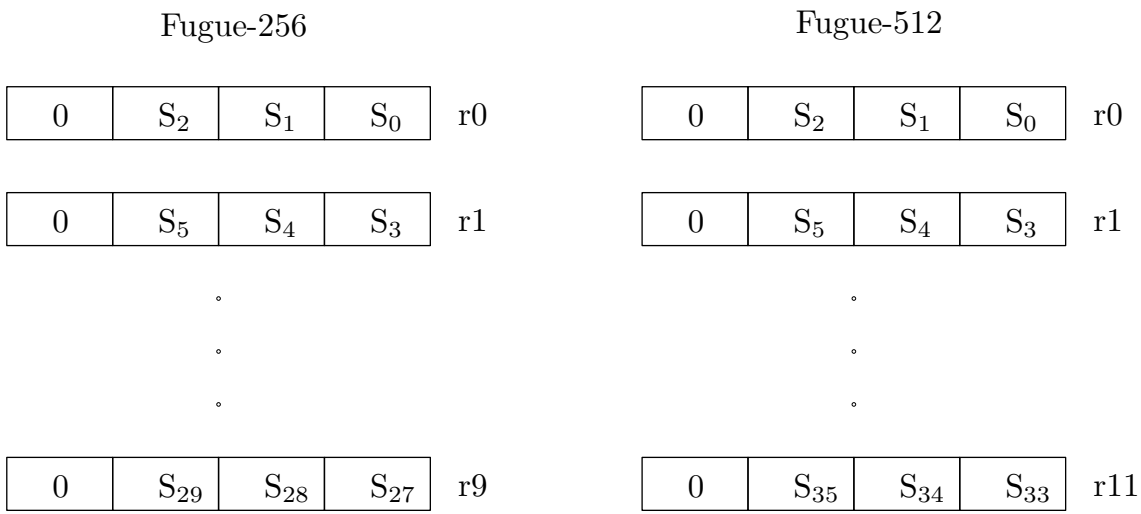
SUBSTITUTE

```
movaps t1, xmmword ptr k_s0F          ;    1 : i
pandn t1, s0                          ;    1 = i¡¡4
psrld t1, 4                           ;    1 = i
pand s0, xmmword ptr k_s0F            ;    0 = k
movaps t2, xmmword ptr k_inv + 16     ;    2 : a/k
pshufb t2, s0                         ;    2 = a/k
pxor s0, t1                           ;    0 = j
movaps t3, xmmword ptr k_inv          ;    3 : 1/i
pshufb t3, t1                         ;    3 = 1/i
pxor t3, t2                           ;    3 = iak = 1/i + a/k
movaps t4, xmmword ptr k_inv          ;    4 : 1/j
pshufb t4, s0                         ;    4 = 1/j
pxor t4, t2                           ;    4 = jak = 1/j + a/k
movaps t2, xmmword ptr k_inv          ;    2 : 1/iak
pshufb t2, t3                         ;    2 = 1/iak
pxor t2, s0                           ;    2 = io
movaps t3, xmmword ptr k_inv          ;    3 : 1/jak
pshufb t3, t4                         ;    3 = 1/jak
xorps t3, t1                          ;    3 = jo
```

- Mike Hamburg, *Accelerating AES with Vector Permute Instructions*, CHES 2009.
- A fast and constant time implementation of AES with SIMD instructions.

Outline

Representation of the State



TIX & CMIX

TIX256

```
pshufd t1, r0, 0f3h             ;    t1 = 0 0 S0 0
xorps r3, t1                    ;    S10 += S0
movss t1, dword ptr [msg]       ;    t1 = 0 0 0 I
TRANSFORM_INPUT t1, t2, t3
movss r0, t1                    ;    S0 = I
pslldq t1, 8                    ;    t1 = 0 S0 0 0
xorps r2, t1                    ;    S8 += S0
pshufd t1, r8, 0f3h             ;    t1 = 0 0 S24 0
xorps r0, t1                    ;    S1 += S24
```

CMIX

```
movaps t, r1                    ;    t = 0 S5 S4 S3
shufps t, r2, 0c9h              ;    t = 0 S6 S5 S4
xorps a, t                      ;    add to columns 0,1,2
xorps b, t                      ;    add to columns s/2, s/2+1, S/2+2
```

Super-MIX

Table: Super-Mix using 13 permutations

1×	0	1	2	3	7	1	2	2	11	12	1	6	15	0	5	10
	3	4	6	7	8	8	6	7	*	*	*	*	*	*	*	*
	4	5	9	11	12	13	13	11	*	*	*	*	*	*	*	*
	8	9	10	14	*	*	*	*	*	*	*	*	*	*	*	*
	12	13	14	15	*	*	*	*	*	*	*	*	*	*	*	*
4×	1	6	11	12	5	10	15	0	9	14	3	4	4	2	2	3
	*	*	*	*	*	*	*	*	*	*	*	*	8	9	7	7
	*	*	*	*	*	*	*	*	*	*	*	*	13	13	14	8
5×	*	*	*	*	*	*	*	*	*	*	*	*	12	1	6	11
6×	*	*	*	*	*	*	*	*	8	13	2	7	*	*	*	*
7×	2	7	8	13	6	11	12	1	4	1	0	3	14	3	4	9
	*	*	*	*	*	*	*	*	10	9	6	5	*	*	*	*
	*	*	*	*	*	*	*	*	12	15	14	11	*	*	*	*

Table: Super-Mix using 11 permutations

1x	0	1	2	3	7	1	2	2	11	12	1	6	15	0	5	10
	3	4	6	7	8	8	6	7	*	*	*	*	*	*	*	*
	4	5	9	11	12	13	13	11	*	*	*	*	*	*	*	*
	8	9	10	14	*	*	*	*	*	*	*	*	*	*	*	*
2x	**12**	**13**	**14**	**15**	*	*	*	*	**8**	**13**	**2**	**7**	**12**	**1**	**6**	**11**
4x	1	6	11	12	5	10	15	0	9	14	3	4	4	2	2	3
	12	**13**	**14**	**15**	*	*	*	*	**8**	**13**	**2**	**7**	8	9	7	7
	*	*	*	*	*	*	*	*	*	*	*	*	13	13	14	8
7x	2	7	8	13	6	11	12	1	4	1	0	3	14	3	4	9
	12	**13**	**14**	**15**	*	*	*	*	10	9	6	5	**12**	**1**	**6**	**11**
	*	*	*	*	*	*	*	*	12	15	14	11	*	*	*	*

Table: Super-Mix using 10 permutations

1×	0	1	2	7	8	1	2	2	11	12	1	6	15	0	5	10	
	4	5	6	3	7	8	13	11	10	15	14	5	12	9	10	14	
	3	4	9	11	12	13	6	7	*	*	*	*	8	13	14	15	+
	12	13	14	15	*	*	*	*	*	*	*	*	12	13	14	15	+
2×	*	*	*	*	8	13	2	7	10	15	14	5	12	1	6	11	
4×	1	6	11	12	5	10	15	0	9	14	3	4	4	2	2	3	
	13	13	14	8	8	13	2	7	10	15	14	5	8	9	7	7	
	13	13	14	8	8	13	2	7	8	13	2	7	13	13	14	8	+
7×	2	7	8	13	6	11	12	1	4	1	0	3	14	3	4	9	
	*	*	*	*	*	*	*	*	12	9	6	11	12	1	6	11	+

Optimizations

- Working in the transformed basis.
- Inline functions & loop unrolling.
- Combining subrounds.
- Microarchitectural optimizations.

Outline

Table: Benchmarks for Intel Core 2 Duo E8400 in cycles/byte

	Implementation	Fugue-224,-256	Fugue-384	Fugue-512
this paper	assembly	15.85	24.87	32.49
jutla-ssse3	intrinsics	18.41	N/A	N/A
jutla-sse4	intrinsics	17.45	N/A	N/A
opt-64	C	27.61	41.53	55.28

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opt-64	C	27.61	41.53	55.28

Table: Benchmarks for Intel Core i7-920 in cycles/byte

	Implementation	Fugue-224,-256	Fugue-384	Fugue-512
this paper	assembly	16.57	23.95	31.10
jutla-ssse3	intrinsics	20.97	N/A	N/A
jutla-sse4	intrinsics	38.50	N/A	N/A
opt-64	C	27.66	41.26	55.16

Outline

Summary

- A fast SIMD implementation of hash function Fugue.

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Future Work

- SIMD vs. AES-NI?

Summary

- A fast SIMD implementation of hash function Fugue.
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Future Work

- SIMD vs. AES-NI?
- Other candidates: ECHO, Grøstl and Shavite-3?