

Lattice-based distributed signing from the Fiat–Shamir with aborts paradigm

NIST MPTS Workshop

Based on “Two-round n -out-of- n and multi-signatures and trapdoor commitment from lattices”
(eprint 2020/1110)

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Background & Motivation

- Two approaches to lattice-based signatures among the NIST PQC standardization finalists:
 - Hash-and-sign [GPV08]: Falcon
 - **Fiat-Shamir with aborts** [Lyu09]: Dilithium
- FSwA-style signature has a structure similar to the DL-based counterparts.
 - Many existing works on round-efficient n -party Schnorr-style signatures.
 - Drijvers et al. [DEF⁺19] recently attacked & proposed **2-round** protocols.

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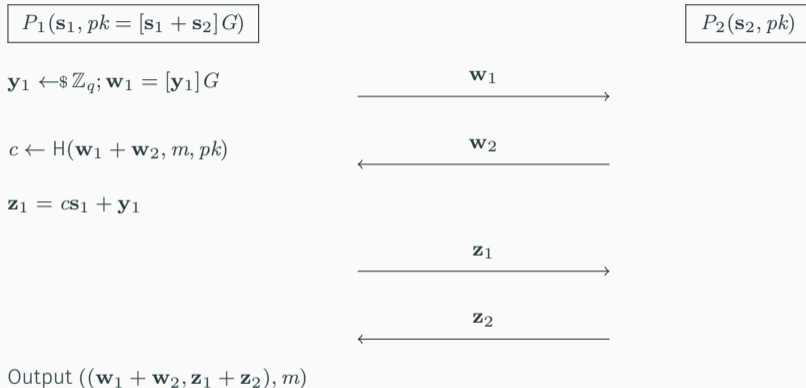
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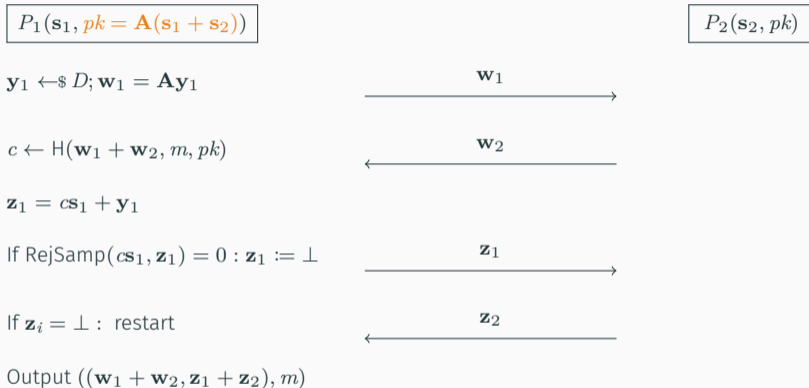
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Bare-bone 2-party signing: Schnorr vs Dilithium



- Round 1: Exchange “commitments” \mathbf{w}_i and locally derive a joint challenge c
- Round 2: Compute signature shares \mathbf{z}_i and exchange them

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Bare-bone 2-party signing: Schnorr vs Dilithium

$$P_1(s_1, pk = \mathbf{A}(s_1 + s_2))$$

$$y_1 \leftarrow_{\$} D; \mathbf{w}_1 = \mathbf{A}y_1$$

$$c \leftarrow H(\mathbf{w}_1 + \mathbf{w}_2, m, pk)$$

$$\mathbf{z}_1 = c\mathbf{s}_1 + y_1$$

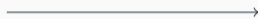
$$\text{If } \text{RejSamp}(c\mathbf{s}_1, \mathbf{z}_1) = 0 : \mathbf{z}_1 := \perp$$

$$\text{If } \mathbf{z}_i = \perp : \text{restart}$$

$$\text{Output } ((\mathbf{w}_1 + \mathbf{w}_2, \mathbf{z}_1 + \mathbf{z}_2), m)$$

$$P_2(s_2, pk)$$

\mathbf{w}_1



\mathbf{w}_2



\mathbf{z}_1

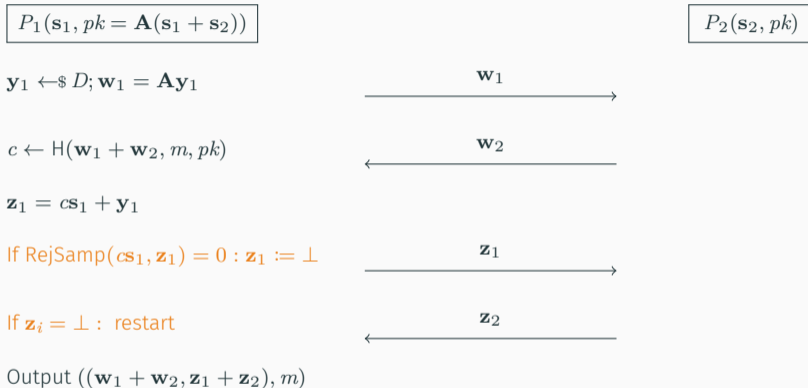


\mathbf{z}_2



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Recent observations in the DL-setting apply!

1. Variant of the **concurrent attack** against bare-bone 2-round protocols in DL
 - Idea: corrupt \tilde{P}_2 adaptively chooses \mathbf{w}_2 after seeing honest P_1 's \mathbf{w}_1
 - Vectorial variant of Wagner's k -list sum algorithm to find a valid forgery
2. Homomorphic commitment to the first message \mathbf{w}_i saves!
 - Per-message commitment key $ck = H(m, pk)$ is crucial to achieve secure 2-round protocol!

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Provably secure 2-round protocol: the final form

$$P_1(\mathbf{s}_1, pk = \mathbf{A}(\mathbf{s}_1 + \mathbf{s}_2))$$

$$ck \leftarrow H(m, pk)$$

$$\mathbf{y}_1 \leftarrow \$ D; \mathbf{w}_1 = \mathbf{A}\mathbf{y}_1$$

$$c \leftarrow H(com_1 + com_2, m, pk)$$

$$\mathbf{z}_1 = c\mathbf{s}_1 + \mathbf{y}_1$$

$$\text{If } \text{RejSamp}(c\mathbf{s}_1, \mathbf{z}_1) = 0 : (\mathbf{z}_1, \mathbf{w}_1, r_1) := (\perp, \perp)$$

If $\mathbf{z}_i = \perp$: restart

Output $((com_1 + com_2, \mathbf{z}_1 + \mathbf{z}_2, r_1 + r_2), m)$

$$P_2(\mathbf{s}_2, pk)$$

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Takeaways

- Progress in multi-party DL signing highly affects lattice-based counterparts!
- Several subtle differences:
 - Issue with “aborts”
 - Security proof is more involved
 - Need for many parallel repetitions in the n -party setting for large n
 - Poor quality of SIS solution in the security reduction for large n
 - Unclear if the same approach generalizes to t -out-of- n signing

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More details at <https://ia.cr/2020/1110>

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 Manu Drijvers, Kasma Edalatnejad, Bryan Ford, Eike Kiltz, Julian Loss, Gregory Neven, and Igors Stepanovs.

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Concurrent attack against bare-bone protocol

\mathcal{A} (malicious) has \mathbf{s}' ; P (honest) has \mathbf{s} ; joint public key is $\mathbf{t} = \mathbf{A}(\mathbf{s}' + \mathbf{s})$

1. \mathcal{A} starts k concurrent sessions on the same m ; receive $\mathbf{w}_1, \dots, \mathbf{w}_k$ from P
2. Let $\mathbf{w}^* = \mathbf{w}_1 + \dots + \mathbf{w}_k$; Find $m^*, \mathbf{w}'_1, \dots, \mathbf{w}'_k$ such that

$$\begin{aligned}c^* &= H(\mathbf{w}^*, m^*, \mathbf{t}) = H(\mathbf{w}_1 + \mathbf{w}'_1, m, \mathbf{t}) + \dots + H(\mathbf{w}_k + \mathbf{w}'_k, m, \mathbf{t}) \\ &= c_1 + \dots + c_k\end{aligned}$$

by solving a sparse, ternary variant of the generalized birthday problem for $(k+1)$ trees [Wag02]: GBP over $(C = \{c \in \mathbb{Z}^N : \|c\|_1 = \kappa \wedge \|c\|_\infty = 1\}, +)$

3. \mathcal{A} resumes the sessions by sending $\mathbf{w}'_1, \dots, \mathbf{w}'_k$; P returns $\mathbf{z}_1 = c_1\mathbf{s} + \mathbf{y}_1, \dots, \mathbf{z}_k = c_k\mathbf{s} + \mathbf{y}_k$.

4. Output a forgery $(\mathbf{w}^*, \mathbf{z}^*, m^*)$ where

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Why $(\mathbf{w}^*, \mathbf{z}^*, m^*)$ passes the verification:

- Thanks to the $(k+1)$ -list sum solver $c^* = H(\mathbf{w}^*, m^*, \mathbf{t}) = c_1 + \dots + c_k$
- The forgery \mathbf{z}^* satisfies

$$\begin{aligned}\mathbf{z}^* &= c^* \mathbf{s}' + \mathbf{z}_1 + \dots + \mathbf{z}_k \\ &= c^* \mathbf{s}' + (c_1 + \dots + c_k) \mathbf{s} + (\mathbf{y}_1 + \dots + \mathbf{y}_k) \\ &= c^* (\mathbf{s}' + \mathbf{s}) + (\mathbf{y}_1 + \dots + \mathbf{y}_k)\end{aligned}$$

- Hence we have

$$\begin{aligned}\mathbf{A} \mathbf{z}^* - c^* \mathbf{t} &= \mathbf{A} (\mathbf{y}_1 + \dots + \mathbf{y}_k) \\ &= \mathbf{w}^*\end{aligned}$$

- Verifier also checks $\|\mathbf{z}^*\|$ is small $\leadsto k$ should be sufficiently small.
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