

Advanced Topics in Combinatorial Methods for Testing

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Solutions to the oracle problem



How to automate checking correctness of output



- **Creating test data is the easy part!**
- How do we check that the code worked correctly on the test input?
 - **Crash testing** server or other code to ensure it does not crash for any test input (like ‘fuzz testing’)
 - Easy but limited value
 - **Built-in self test with embedded assertions** – incorporate assertions in code to check critical states at different points in the code, or print out important values during execution
 - **Full scale model-checking** using mathematical model of system and model checker to generate expected results for each input
 - expensive but tractable

Crash Testing

- Like “fuzz testing” - send packets or other input to application, watch for crashes
- Unlike fuzz testing, input is non-random; cover all t-way combinations
- May be more efficient - random input generation requires several times as many tests to cover the t-way combinations in a covering array

Limited utility, but can detect high-risk problems such as:

- buffer overflows
- server crashes

Built-in Self Test through Embedded Assertions

Simple example:

```
assert( x != 0);    // ensure divisor is not zero
```

Or pre and post-conditions:

```
/requires amount >= 0;
```

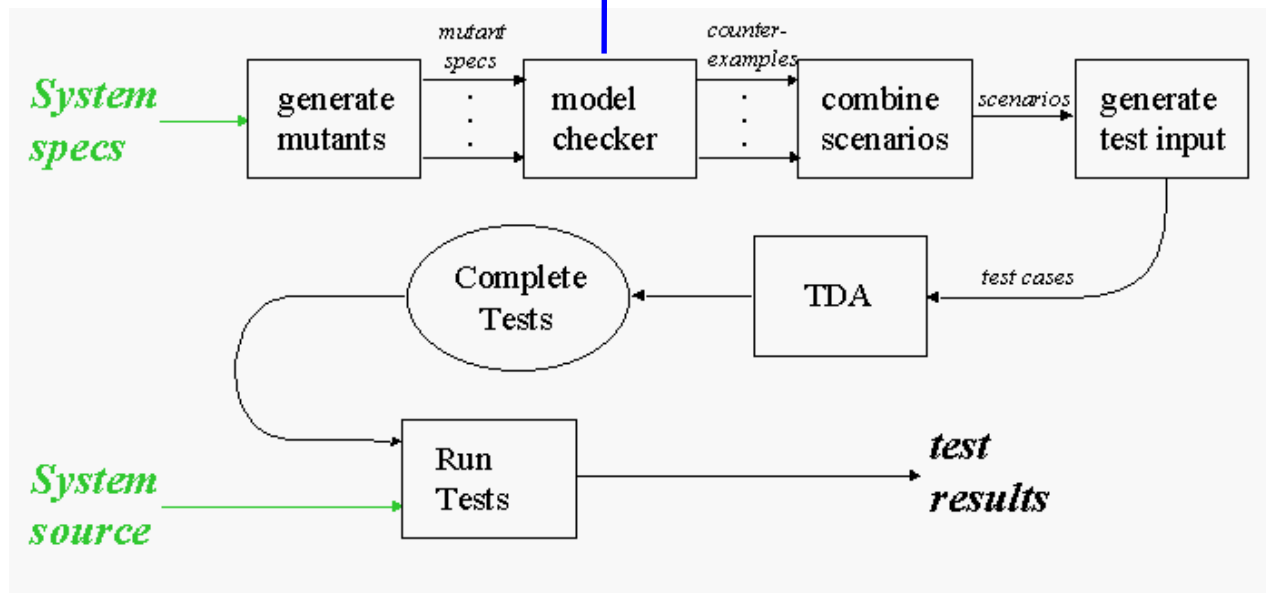
```
/ensures balance == \old(balance) - amount &&  
\result == balance;
```

Built-in Self Test

Assertions check properties of expected result:
ensures balance == \old(balance) - amount
&& \result == balance;

- Reasonable assurance that code works correctly across the range of expected inputs
- May identify problems with handling unanticipated inputs
- Example: Smart card testing
 - Used Java Modeling Language (JML) assertions
 - Detected 80% to 90% of flaws

Using model checking to produce tests



- Model-checker test production: if assertion is not true, then a counterexample is generated.

- This can be converted to a test case.

Model checking example

```
-- specification for a portion of tcas - altitude separation.
-- The corresponding C code is originally from Siemens Corp.
Research
-- Vadim Okun 02/2002
MODULE main
VAR
  Cur_Vertical_Sep : { 299, 300, 601 };
  High_Confidence : boolean;
  ...
init(alt_sep) := START_;
  next(alt_sep) := case
    enabled & (intent_not_known | !tcas_equipped) : case
      need_upward_RA & need_downward_RA : UNRESOLVED;
      need_upward_RA : UPWARD_RA;
      need_downward_RA : DOWNWARD_RA;
      1 : UNRESOLVED;
    esac;
  1 : UNRESOLVED;
esac;
  ...
SPEC AG ((enabled & (intent_not_known | !tcas_equipped) &
!need_downward_RA & need_upward_RA) -> AX (alt_sep = UPWARD_RA))
-- "FOR ALL executions,
-- IF enabled & (intent_not_known ....
-- THEN in the next state alt_sep = UPWARD_RA"
```


Computation Tree Logic

- The usual logic operators, plus temporal:
 - A φ - All: φ holds on all paths starting from the current state.
 - E φ - Exists: φ holds on some paths starting from the current state.
 - G φ - Globally: φ has to hold on the entire subsequent path.
 - F φ - Finally: φ eventually has to hold
 - X φ - Next: φ has to hold at the next state
- [others not listed]
- execution paths
- states on the execution paths
- ```
SPEC AG ((enabled & (intent_not_known |
!tcas_equipped) & !need_downward_RA & need_upward_RA)
-> AX (alt_sep = UPWARD_RA))
```
- "FOR ALL executions,  
IF enabled & (intent\_not\_known ....  
THEN in the next state alt\_sep = UPWARD\_RA"

# What is the most effective way to integrate combinatorial testing with model checking?

- Given  $AG(P \rightarrow AX(R))$   
“for all paths, in every state,  
if P then in the next state, R holds”
- For k-way variable combinations,  $v1 \ \& \ v2 \ \& \ \dots \ \& \ vk$
- $v_i$  abbreviates “var1 = val1”
- Now combine this constraint with assertion to produce counterexamples. Some possibilities:

1.  $AG(v1 \ \& \ v2 \ \& \ \dots \ \& \ vk \ \& \ P \rightarrow AX \ !(R))$

2.  $AG(v1 \ \& \ v2 \ \& \ \dots \ \& \ vk \rightarrow AX \ !(1))$

3.  $AG(v1 \ \& \ v2 \ \& \ \dots \ \& \ vk \rightarrow AX \ !(R))$

# What happens with these assertions?

1.  $AG(v1 \ \& \ v2 \ \& \ \dots \ \& \ vk \ \& \ P \ \rightarrow \ AX \ !(R))$

P may have a negation of one of the  $v_i$ , so we get

$0 \ \rightarrow \ AX \ !(R)$

always true, so no counterexample, no test.

This is too restrictive!

1.  $AG(v1 \ \& \ v2 \ \& \ \dots \ \& \ vk \ \rightarrow \ AX \ !(1))$

The model checker makes non-deterministic choices for variables not in  $v1..vk$ , so all R values may not be covered by a counterexample.

This is too loose!

2.  $AG(v1 \ \& \ v2 \ \& \ \dots \ \& \ vk \ \rightarrow \ AX \ !(R))$

Forces production of a counterexample for each R.

This is just right!

# More testing Examples

Name: Julia T. Teacher: Mrs H.

12/12

## First Grade Spelling Test

★ 1. soft

★ 2. lost

★ 3. goat

★ 4. toast

★ 5. load

# Buffer Overflows

- Empirical data from the National Vulnerability Database
  - Investigated > 3,000 denial-of-service vulnerabilities reported in the NIST NVD for period of 10/06 – 3/07
  - Vulnerabilities triggered by:
    - Single variable – 94.7%  
example: *Heap-based buffer overflow in the SFTP protocol handler for Panic Transmit ... allows remote attackers to execute arbitrary code via a long ftps:// URL.*
    - 2-way interaction – 4.9%  
example: *single character search string in conjunction with a single character replacement string, which causes an "off by one overflow"*
    - 3-way interaction – 0.4%  
example: *Directory traversal vulnerability when register\_globals is enabled and magic\_quotes is disabled and .. (dot dot) in the page parameter*

# Example: Finding Buffer Overflows

```
1. if (strcmp(conn[sid].dat->in_RequestMethod, "POST")==0) {
2. if (conn[sid].dat->in_ContentLength<MAX_POSTSIZE) {
3.
4. conn[sid].PostData=calloc(conn[sid].dat->in_ContentLength+1024,
5. sizeof(char));
6.
7. pPostData=conn[sid].PostData;
8. do {
9. rc=recv(conn[sid].socket, pPostData, 1024, 0);
10.
11. pPostData+=rc;
12. x+=rc;
13. } while ((rc==1024) || (x<conn[sid].dat->in_ContentLength));
14. conn[sid].PostData[conn[sid].dat->in_ContentLength]='\0';
15. }
```

## Interaction: request-method="POST", content-length = -1000, data= a string > 24 bytes

```
1. if (strcmp(conn[sid].dat->in_RequestMethod, "POST")==0) {
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11. }
```

## Interaction: request-method="POST", content-length = -1000, data= a string > 24 bytes

```

1. if (strcmp(conn[sid].dat->in_RequestMethod, "POST")==0) { true branch
2. if (conn[sid].dat->in_ContentLength<MAX_POSTSIZE) {

3. conn[sid].PostData=calloc(conn[sid].dat->in_ContentLength+1024,
sizeof(char));

4. pPostData=conn[sid].PostData;
5. do {
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2. if (conn[sid].dat->in_ContentLength<MAX_POSTSIZE) true branch

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sizeof(char));
 Allocate -1000 + 1024 bytes = 24 bytes

4. pPostData=conn[sid].PostData;
5. do {
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7. pPostData+=rc;
8. x+=rc;
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11. }

```

**Boom!**



# Example: Modeling & Simulation

- “Simured” network simulator
  - Kernel of ~ 5,000 lines of C++ (not including GUI)
- Objective: detect configurations that can produce deadlock:
  - Prevent connectivity loss when changing network
  - Attacks that could lock up network
- Compare effectiveness of random vs. combinatorial inputs
- Deadlock combinations discovered
- Crashes in >6% of tests w/ valid values (Win32 version only)

# Simulation Input Parameters

|    | Parameter  | Values      |
|----|------------|-------------|
| 1  | DIMENSIONS | 1,2,4,6,8   |
| 2  | NODOSDIM   | 2,4,6       |
| 3  | NUMVIRT    | 1,2,3,8     |
| 4  | NUMVIRTINJ | 1,2,3,8     |
| 5  | NUMVIRTEJE | 1,2,3,8     |
| 6  | LONBUFFER  | 1,2,4,6     |
| 7  | NUMDIR     | 1,2         |
| 8  | FORWARDING | 0,1         |
| 9  | PHYSICAL   | true, false |
| 10 | ROUTING    | 0,1,2,3     |
| 11 | DELFIFO    | 1,2,4,6     |
| 12 | DELCROSS   | 1,2,4,6     |
| 13 | DELCHANNEL | 1,2,4,6     |
| 14 | DELSWITCH  | 1,2,4,6     |

$5 \times 3 \times 4 \times 4 \times 4 \times 4 \times 2 \times 2 \times 2 \times 4 \times 4 \times 4 \times 4 \times 4$   
= 31,457,280 configurations

Are any of them dangerous?

If so, how many?

Which ones?

# Network Deadlock Detection

## Deadlocks Detected: combinatorial

| t | Tests | 500<br>pkts | 1000<br>pkts | 2000<br>pkts | 4000<br>pkts | 8000<br>pkts |
|---|-------|-------------|--------------|--------------|--------------|--------------|
| 2 | 28    | 0           | 0            | 0            | 0            | 0            |
| 3 | 161   | 2           | 3            | 2            | 3            | 3            |
| 4 | 752   | 14          | 14           | 14           | 14           | 14           |

## Average Deadlocks Detected: random

| t | Tests | 500<br>pkts | 1000<br>pkts | 2000<br>pkts | 4000<br>pkts | 8000<br>pkts |
|---|-------|-------------|--------------|--------------|--------------|--------------|
| 2 | 28    | 0.63        | 0.25         | 0.75         | 0.50         | 0.75         |
| 3 | 161   | 3           | 3            | 3            | 3            | 3            |
| 4 | 752   | 10.13       | 11.75        | 10.38        | 13           | 13.25        |

# Network Deadlock Detection

Detected 14 configurations that can cause deadlock:

$$14 / 31,457,280 = 4.4 \times 10^{-7}$$

Combinatorial testing found more deadlocks than random, including some that might never have been found with random testing

Why do this testing? Risks:

- accidental deadlock configuration: low
- deadlock config discovered by attacker: **much higher**  
(because they are looking for it)

# Coverage Measurement





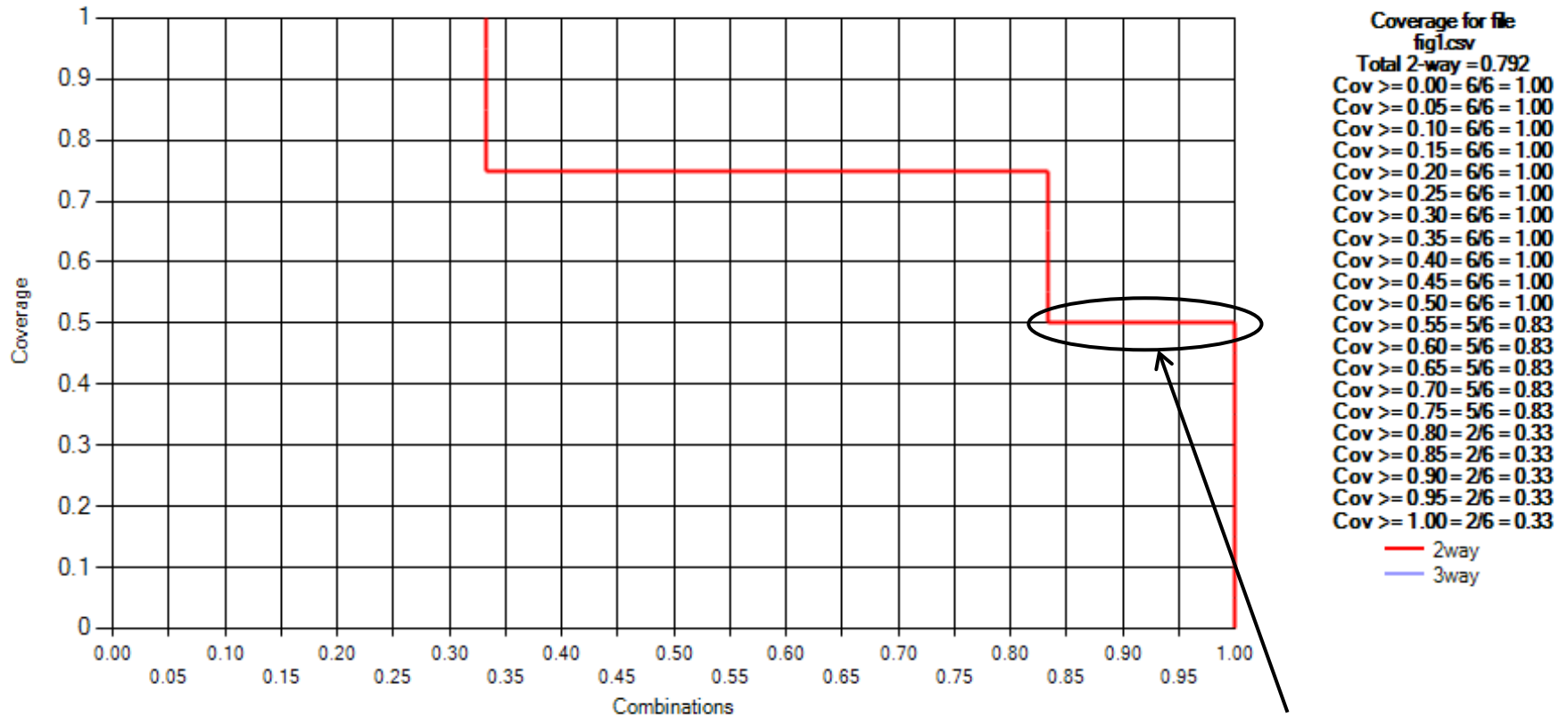
# Combinatorial Coverage Measurement

| Tests | Variables |   |   |   |
|-------|-----------|---|---|---|
|       | a         | b | c | d |
| 1     | 0         | 0 | 0 | 0 |
| 2     | 0         | 1 | 1 | 0 |
| 3     | 1         | 0 | 0 | 1 |
| 4     | 0         | 1 | 1 | 1 |
| 5     | 0         | 1 | 0 | 1 |
| 6     | 1         | 0 | 1 | 1 |
| 7     | 1         | 0 | 1 | 0 |
| 8     | 0         | 1 | 0 | 0 |

| Variable pairs | Variable-value combinations covered | Coverage |
|----------------|-------------------------------------|----------|
| <i>ab</i>      | 00, 01, 10                          | .75      |
| <i>ac</i>      | 00, 01, 10                          | .75      |
| <i>ad</i>      | 00, 01, 11                          | .75      |
| <i>bc</i>      | 00, 11                              | .50      |
| <i>bd</i>      | 00, 01, 10, 11                      | 1.0      |
| <i>cd</i>      | 00, 01, 10, 11                      | 1.0      |

100% coverage of 33% of combinations  
75% coverage of half of combinations  
50% coverage of 16% of combinations

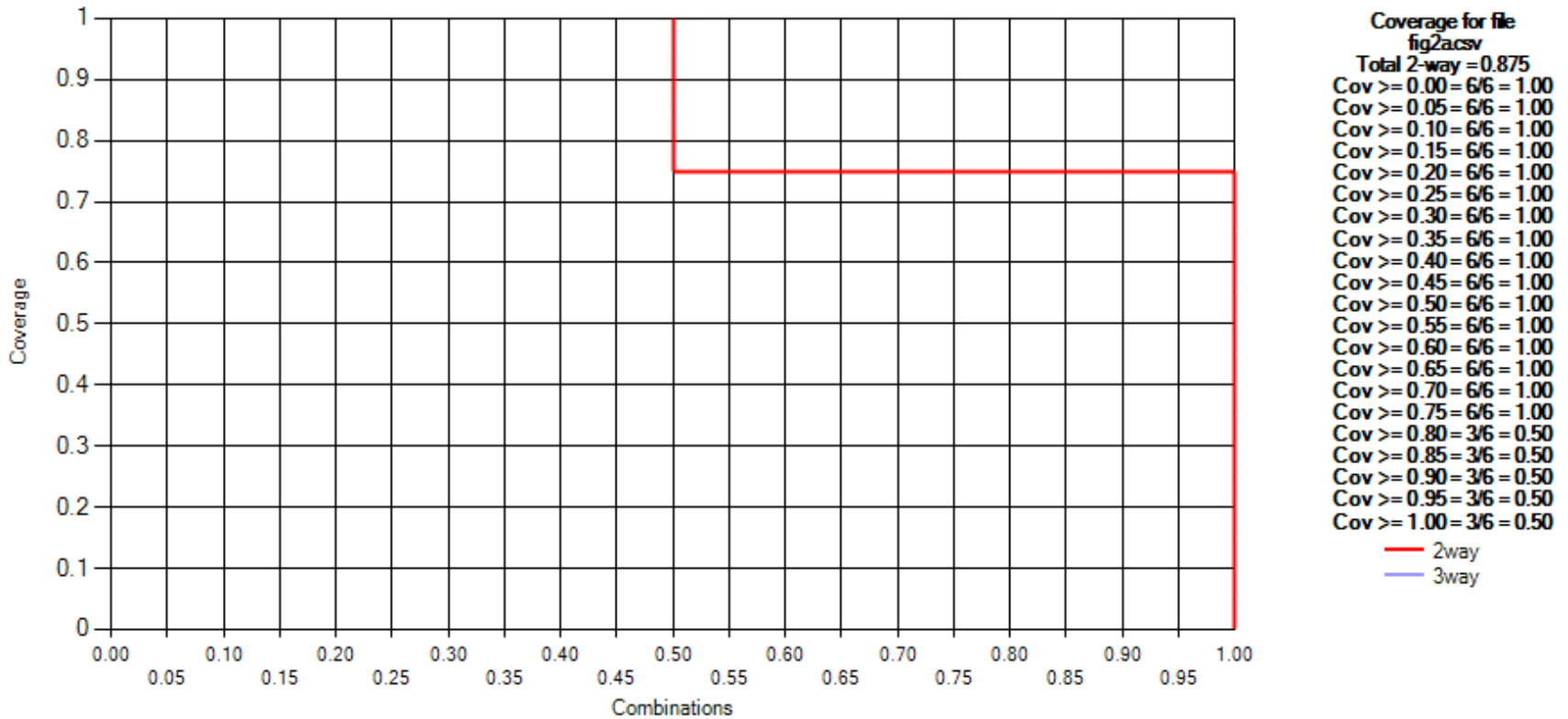
# Graphing Coverage Measurement



100% coverage of 33% of combinations  
75% coverage of half of combinations  
50% coverage of 16% of combinations

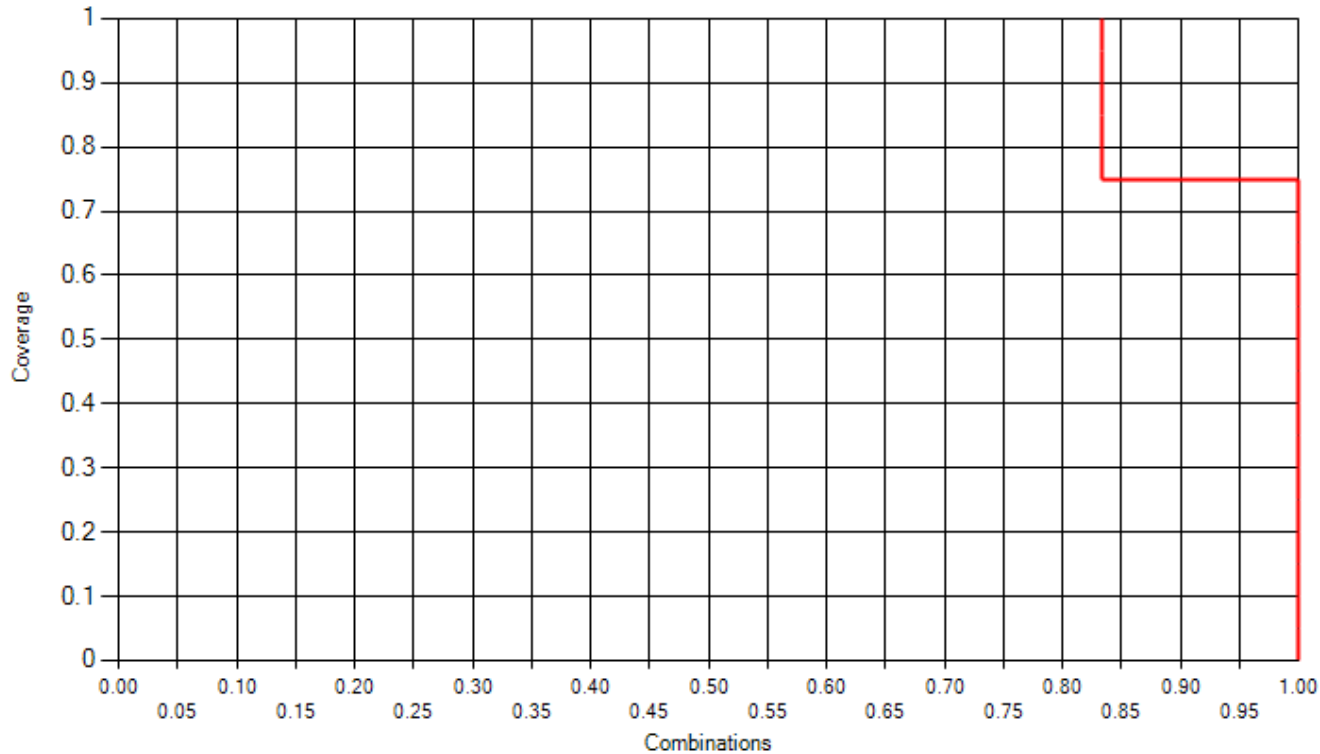
Bottom line:  
All combinations  
covered to at least 50%

# Adding a test



Coverage after adding test [1,1,0,1]

# Adding another test

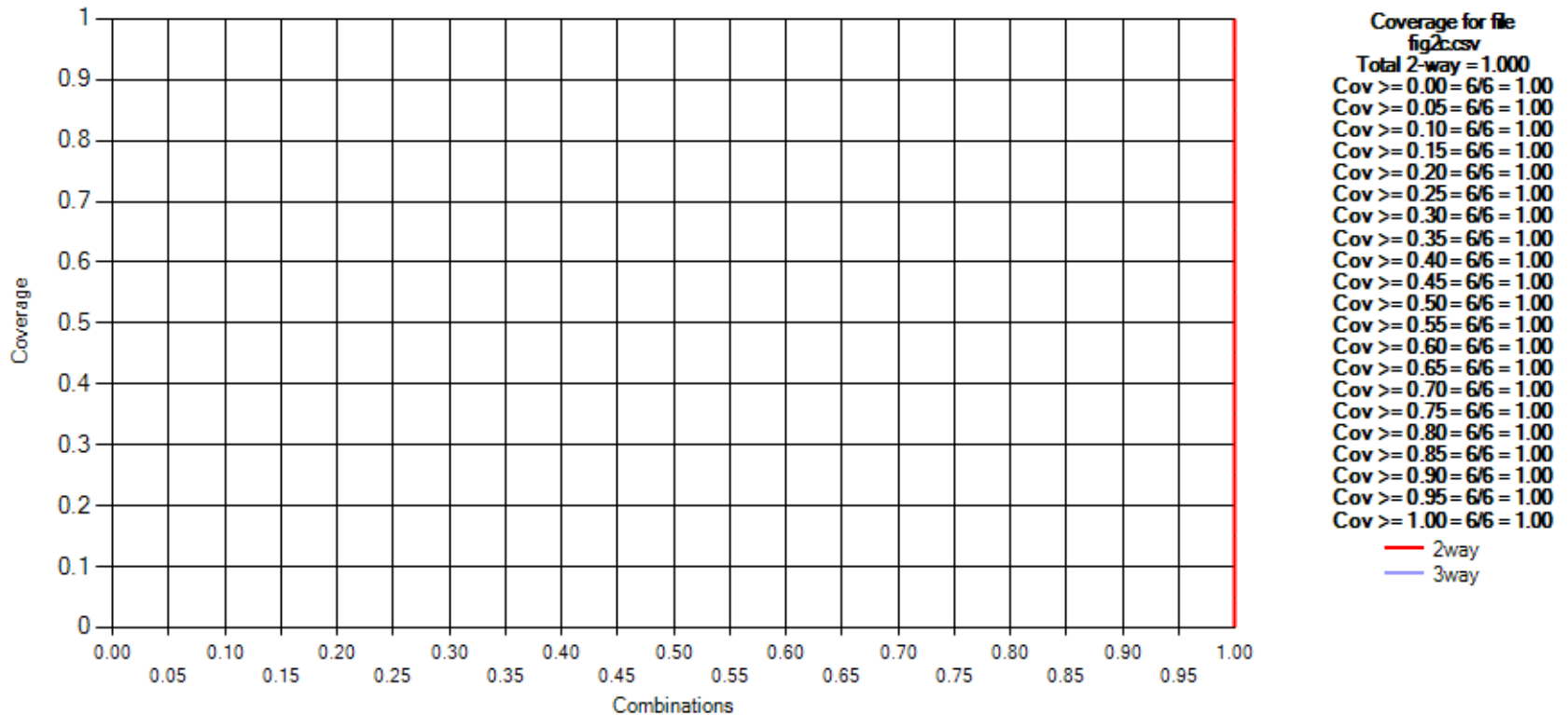


Coverage for file  
fig2b.csv  
Total 2-way = 0.958  
Cov >= 0.00 = 6/6 = 1.00  
Cov >= 0.05 = 6/6 = 1.00  
Cov >= 0.10 = 6/6 = 1.00  
Cov >= 0.15 = 6/6 = 1.00  
Cov >= 0.20 = 6/6 = 1.00  
Cov >= 0.25 = 6/6 = 1.00  
Cov >= 0.30 = 6/6 = 1.00  
Cov >= 0.35 = 6/6 = 1.00  
Cov >= 0.40 = 6/6 = 1.00  
Cov >= 0.45 = 6/6 = 1.00  
Cov >= 0.50 = 6/6 = 1.00  
Cov >= 0.55 = 6/6 = 1.00  
Cov >= 0.60 = 6/6 = 1.00  
Cov >= 0.65 = 6/6 = 1.00  
Cov >= 0.70 = 6/6 = 1.00  
Cov >= 0.75 = 6/6 = 1.00  
Cov >= 0.80 = 5/6 = 0.83  
Cov >= 0.85 = 5/6 = 0.83  
Cov >= 0.90 = 5/6 = 0.83  
Cov >= 0.95 = 5/6 = 0.83  
Cov >= 1.00 = 5/6 = 0.83

— 2way  
— 3way

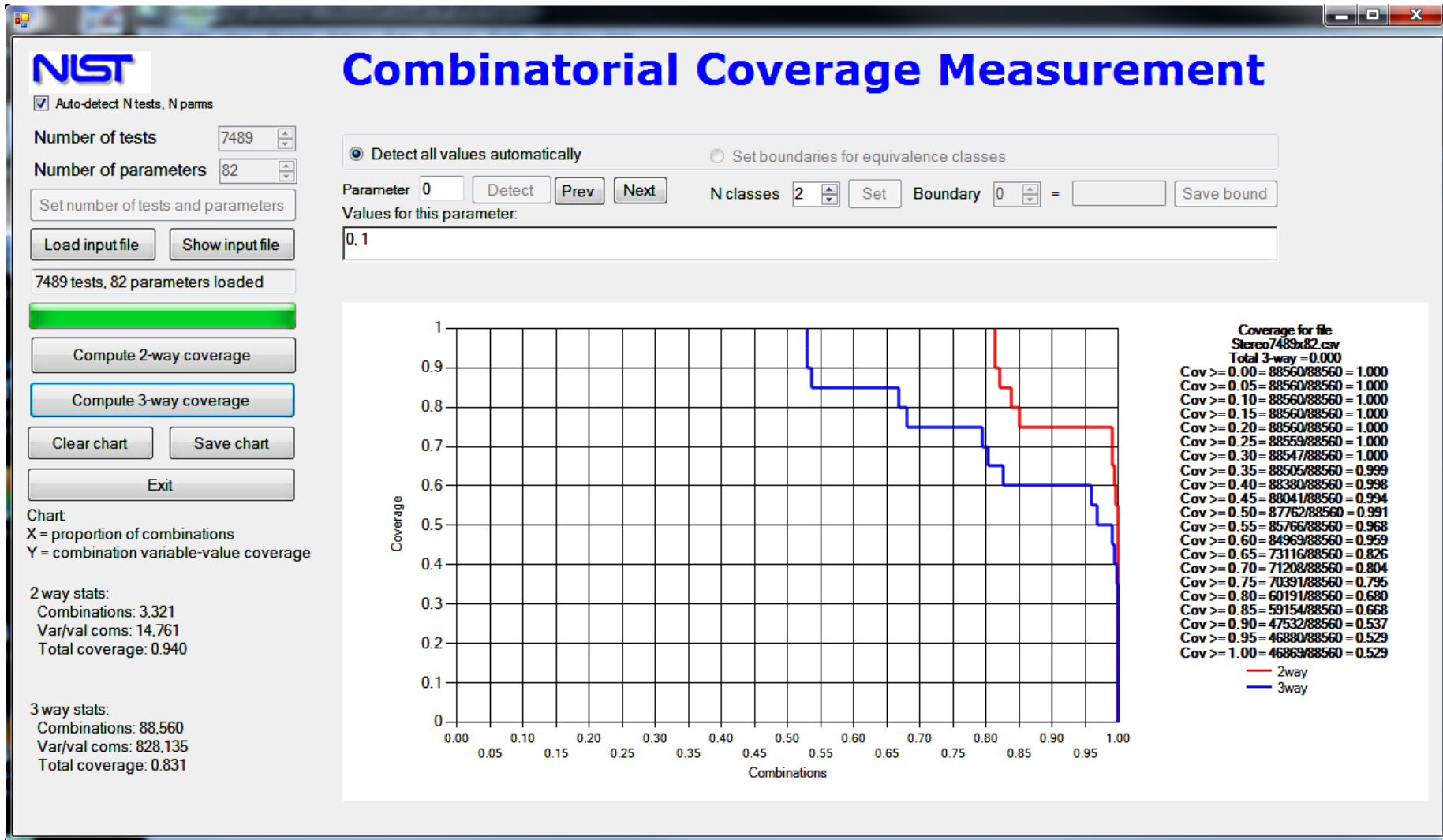
Coverage after adding test [1,0,1,1]

# Additional test completes coverage

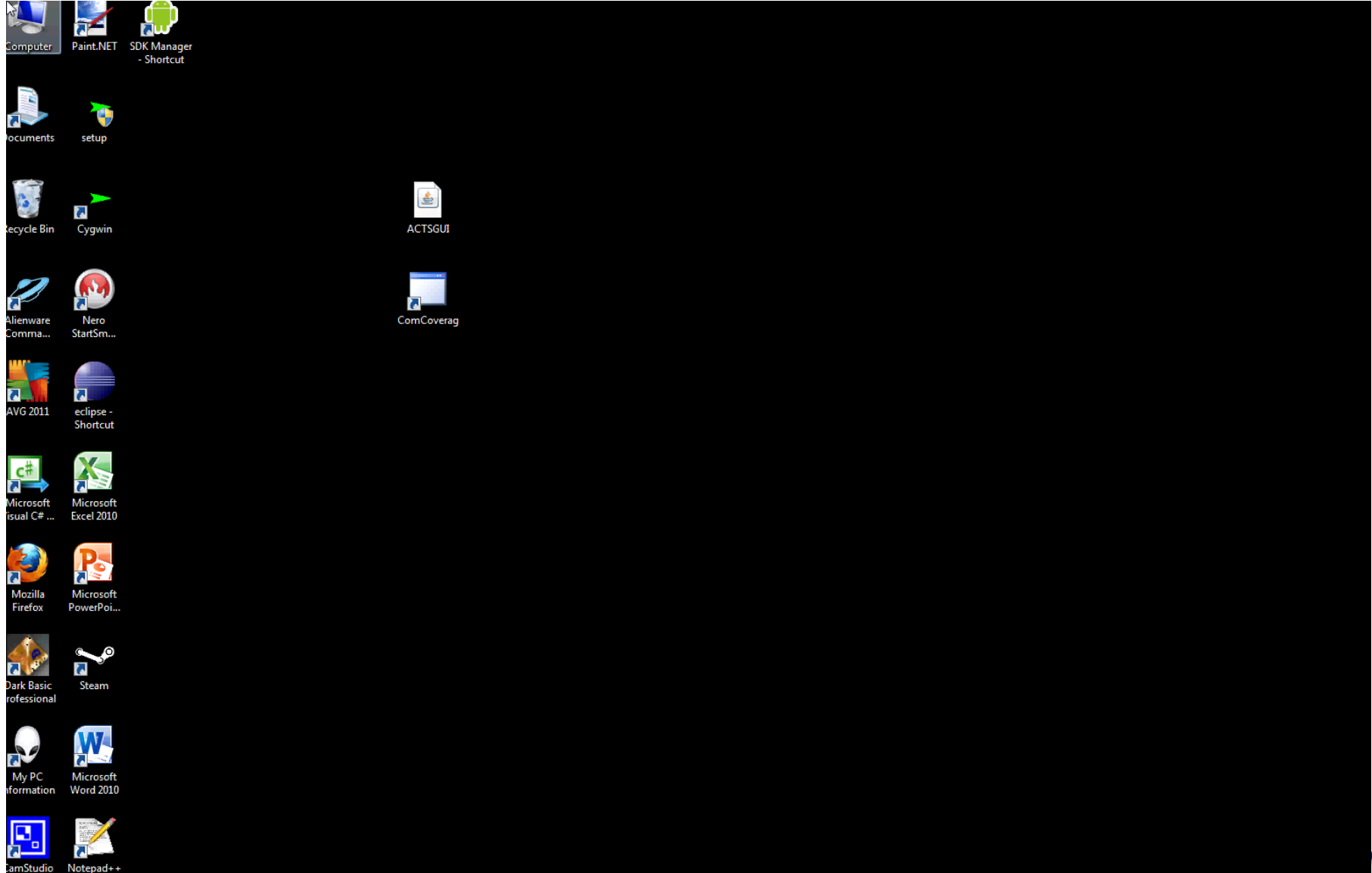


Coverage after adding test [1,0,1,0]  
All combinations covered to 100% level,  
so this is a covering array.

# Combinatorial Coverage Measurement



# Using Coverage Measurement



# Combinatorial Sequences for Testing





# Combinatorial Sequence Testing


- We want to see if a system works correctly regardless of the order of events. How can this be done efficiently?
- Failure reports often say something like: 'failure occurred when A started if B is not already connected'.
- Can we produce compact tests such that all t-way sequences covered (possibly with interleaving events)?

| Event    | Description              |
|----------|--------------------------|
| <i>a</i> | connect flow meter       |
| <i>b</i> | connect pressure gauge   |
| <i>c</i> | connect satellite link   |
| <i>d</i> | connect pressure readout |
| <i>e</i> | start comm link          |
| <i>f</i> | boot system              |



# Sequence Covering Array

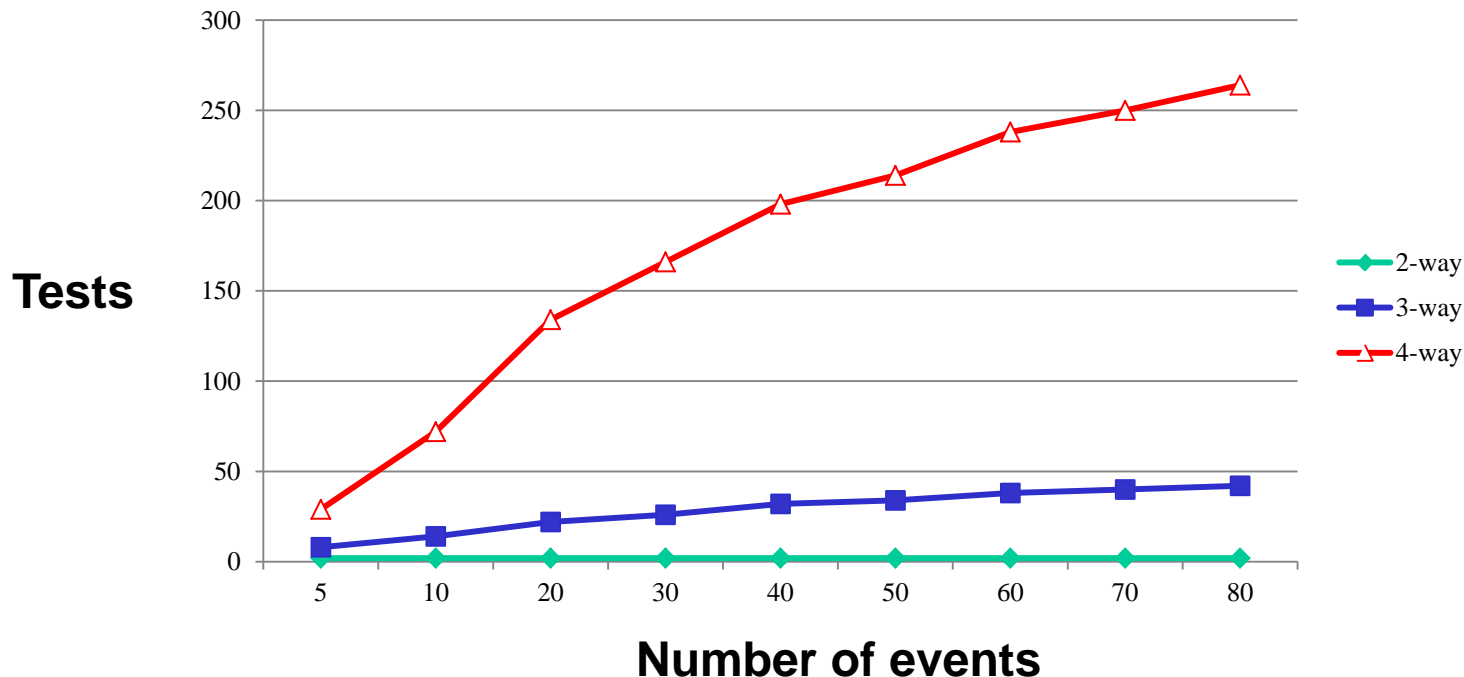
- With 6 events, all sequences =  $6! = 720$  tests
- Only 10 tests needed for all 3-way sequences, results even better for larger numbers of events
- Example: `.*c.*f.*b.*` covered. Any such 3-way seq covered.



| Test | Sequence |   |   |   |   |   |
|------|----------|---|---|---|---|---|
| 1    | a        | b | c | d | e | f |
| 2    | f        | e | d | c | b | a |
| 3    | d        | e | f | a | b | c |
| 4    | c        | b | a | f | e | d |
| 5    | b        | f | a | d | c | e |
| 6    | e        | c | d | a | f | b |
| 7    | a        | e | f | c | b | d |
| 8    | d        | b | c | f | e | a |
| 9    | c        | e | a | d | b | f |
| 10   | f        | b | d | a | e | c |

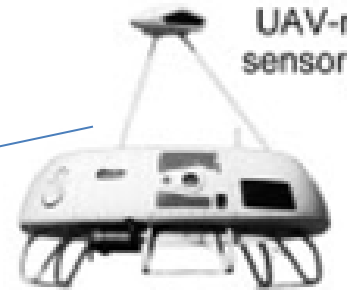
# Sequence Covering Array Properties

- 2-way sequences require only 2 tests  
(write events in any order, then reverse)
- For > 2-way, number of tests grows with  $\log n$ , for  $n$  events
- Simple greedy algorithm produces compact test set



# Example: Laptop application

Problem: connect many peripherals, order of connection may affect application



# Connection Sequences

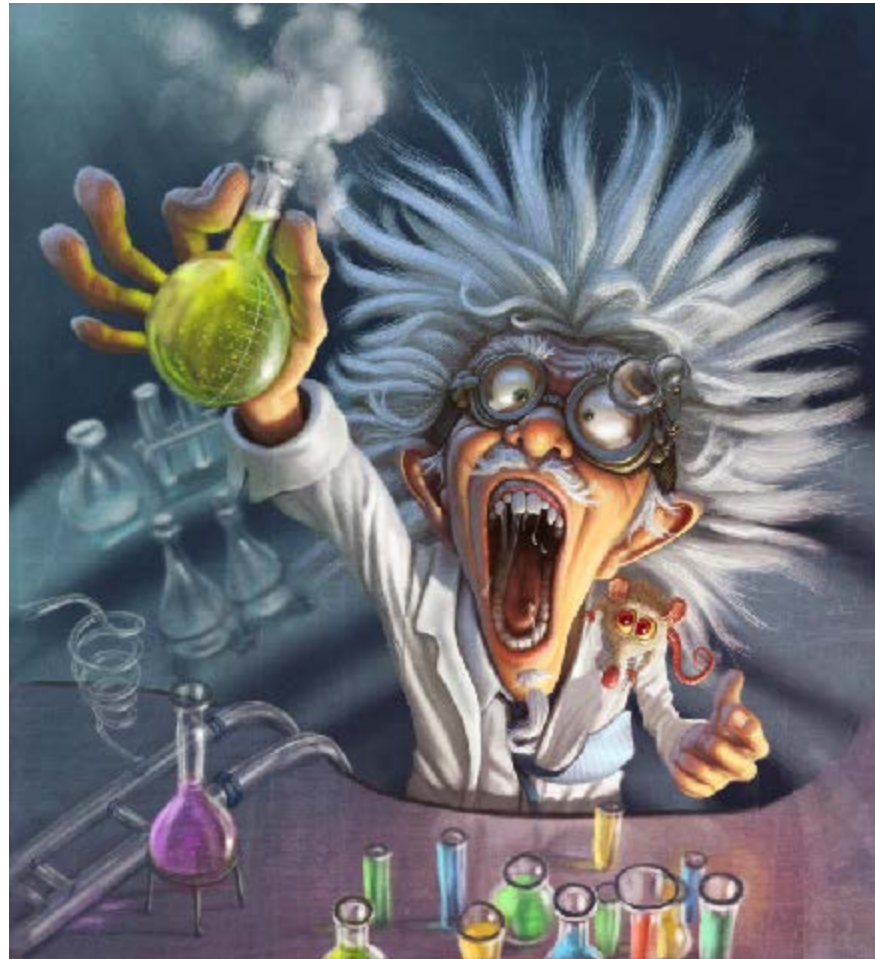
|   |        |                 |                |                |     |                 |                |                |
|---|--------|-----------------|----------------|----------------|-----|-----------------|----------------|----------------|
| 1 | Boot   | P-1 (USB-RIGHT) | P-2 (USB-BACK) | P-3 (USB-LEFT) | P-4 | P-5             | App            | Scan           |
| 2 | Boot   | App             | Scan           | P-5            | P-4 | P-3 (USB-RIGHT) | P-2 (USB-BACK) | P-1 (USB-LEFT) |
| 3 | Boot   | P-3 (USB-RIGHT) | P-2 (USB-LEFT) | P-1 (USB-BACK) | App | Scan            | P-5            | P-4            |
|   | etc... |                 |                |                |     |                 |                |                |

3-way sequence covering  
of connection events

# Results

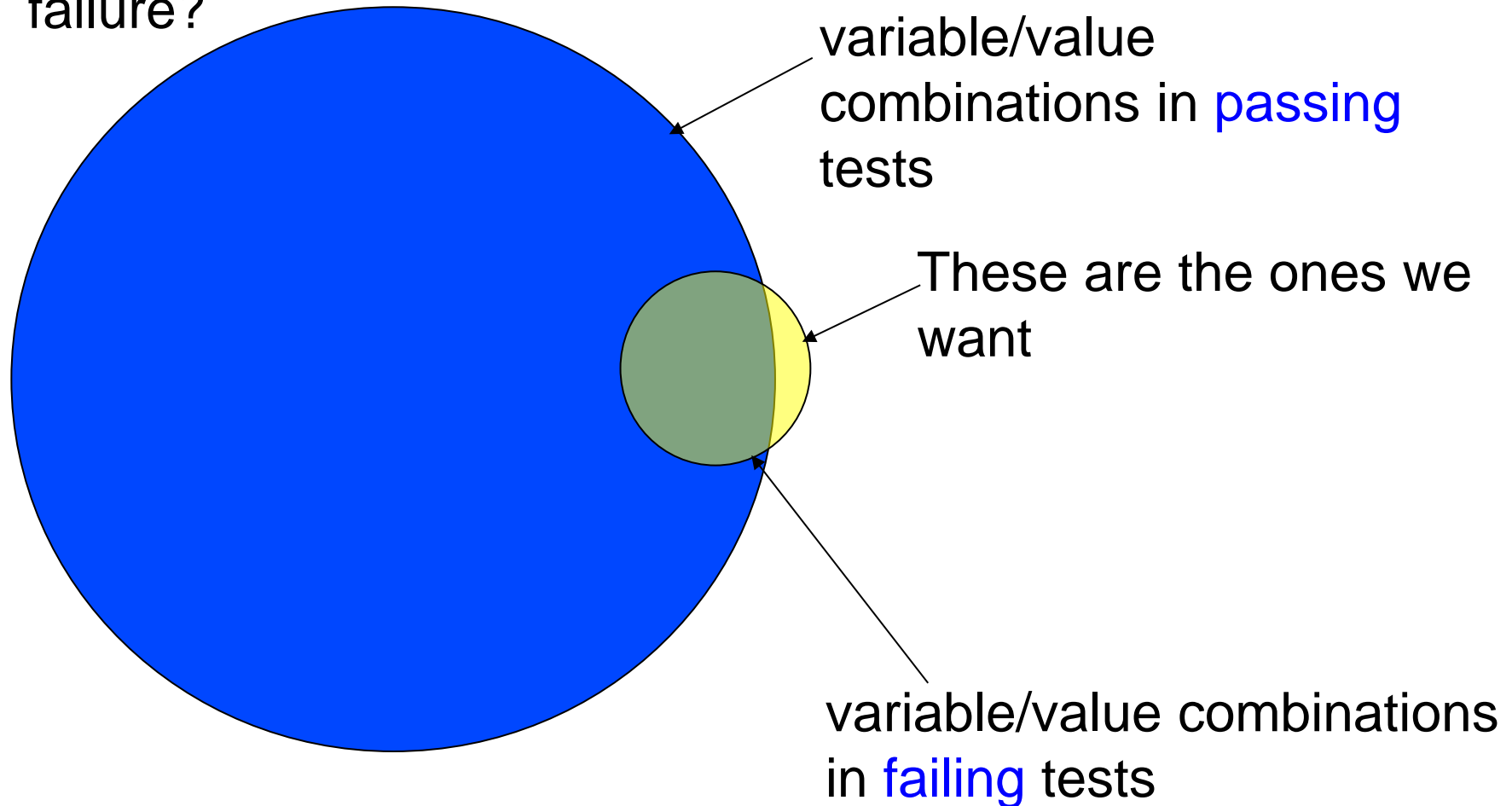
- Tested peripheral connection for 3-way sequences
- Some faults detected that would not have been found with 2-way sequence testing; may not have been found with random
  - Example:
    - If P2-P1-P3 sequence triggers a failure, then a full 2-way sequence covering array would not have found it  
(because 1-2-3-4-5-6-7 and 7-6-5-4-3-2-1 is a 2-way sequence covering array)

# Research Questions



# Fault location

Given: a set of tests that the SUT fails, which combinations of variables/values triggered the failure?





# Fault location – what's the problem?

If they're in failing set but not in passing set:

1. which ones triggered the failure?
2. which ones don't matter?

out of  $v^t \binom{n}{t}$  combinations

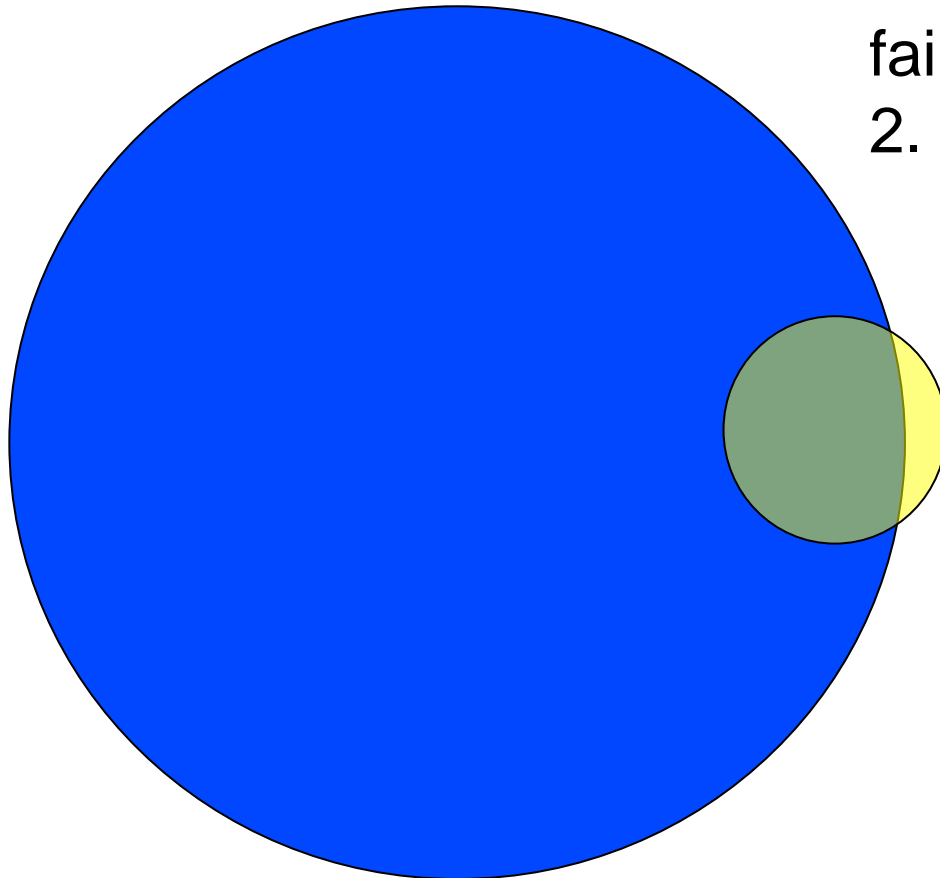
Example:

30 variables, 5 values each

= 445,331,250

5-way combinations

142,506 combinations  
in each test



# Integrating into Testing Program

- Test suite development
  - Generate covering arrays for tests
  - OR
  - Measure coverage of existing tests and supplement
- Training
  - Testing textbooks – Mathur, Ammann & Offutt,
  - Combinatorial testing “textbook” → on ACTS site
  - User manuals
  - Worked examples

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NIST Special Publication 800-142

**NIST**  
National Institute of  
Standards and Technology  
Technology Administration  
U.S. Department of Commerce

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INFORMATION SECURITY

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## PRACTICAL COMBINATORIAL TESTING

D. Richard Kuhn, Raghu N. Kacker, Yu Lei

October, 2010



U.S. Department of Commerce  
Gary Locke, Secretary

National Institute of Standards and Technology  
Patrick Gallagher, Director

# Industrial Usage Reports

- Work with US Air Force on sequence covering arrays, submitted for publication
- World Wide Web Consortium DOM Level 3 events conformance test suite
- Cooperative Research & Development Agreement with Lockheed Martin Aerospace - report to be released 3rd or 4th quarter 2011



# Technology Transfer

- Tools obtained by 700+ organizations; NIST “textbook” on combinatorial testing downloaded 9,000+ times since Oct. 2010
- Collaborations: USAF 46<sup>th</sup> Test Wing, Lockheed Martin, George Mason Univ., Univ. of Maryland Baltimore County, Johns Hopkins Univ. Applied Physics Lab, Carnegie Mellon Univ.
- We are always interested in working with others!

Please contact us if you  
would like more information.



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(Or just search “combinatorial testing”. We’re #1!)