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Combinatorial Testing

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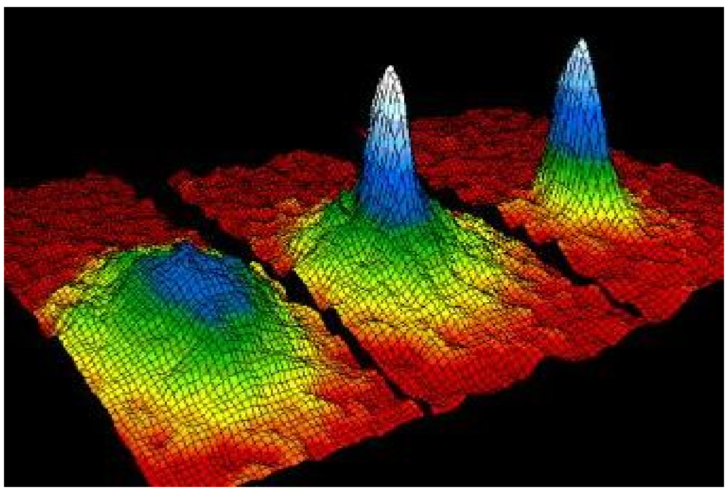
Tutorial Overview

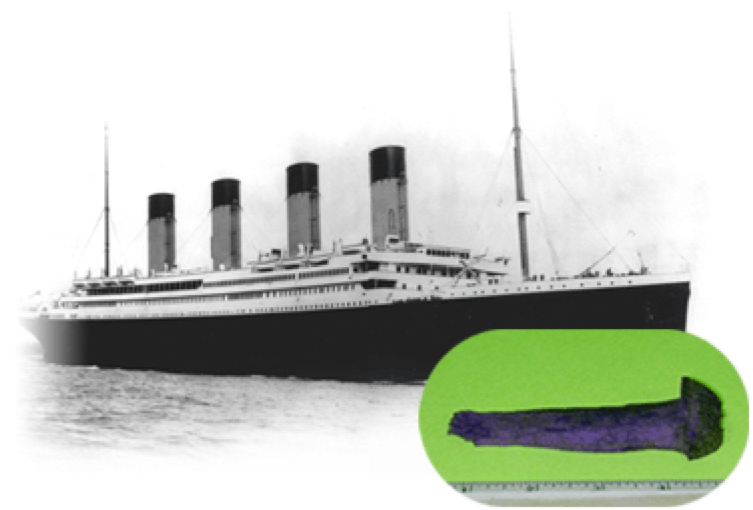
1. Why are we doing this?
2. What is combinatorial testing?
3. How is it used and how long does it take?
4. What tools are available?
5. What's next?

What is NIST and why are we doing this?

- A US Government agency
- The nation's measurement and testing laboratory – 3,000 scientists, engineers, and support staff including 3 Nobel laureates



Research in physics, chemistry, materials, manufacturing, computer science



Analysis of engineering failures, including buildings, materials, **and ...**

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Software Failure Analysis

- We studied software failures in a variety of fields including 15 years of FDA medical device recall data
- What causes software failures?
  - logic errors?
  - calculation errors?
  - interaction faults?
  - inadequate input checking?   Etc.
- What testing and analysis would have prevented failures?
- Would statement coverage, branch coverage, all-values, all-pairs etc. testing find the errors?

Interaction faults: e.g., failure occurs if
pressure < 10   (1-way interaction <= all-values testing catches)
pressure < 10 & volume > 300 (2-way interaction <= all-pairs testing catches)

Software Failure Internals

- How does an interaction fault manifest itself in code?

Example: pressure < 10 & volume > 300 (2-way interaction)

```
if (pressure < 10) {
    // do something
    if (volume > 300)  { faulty code!  BOOM! }
    else { good code, no problem}
}
else {
    // do something else
}
```

Pairwise testing is popular, but is it enough?

- Pairwise testing commonly applied to software
- Intuition: some problems only occur as the result of an interaction between parameters/components
- Pairwise testing finds about 50% to 90% of flaws
  - Cohen, Dalal, Parelius, Patton, 1995 – 90% coverage with pairwise, all errors in small modules found
  - Dalal, et al. 1999 – effectiveness of pairwise testing, no higher degree interactions
  - Smith, Feather, Muscetolla, 2000 – 88% and 50% of flaws for 2 subsystems

Finding 90% of flaws is pretty good,right?

"Relax, our engineers found 90 percent of the flaws."

I don't think I want to get on that plane.

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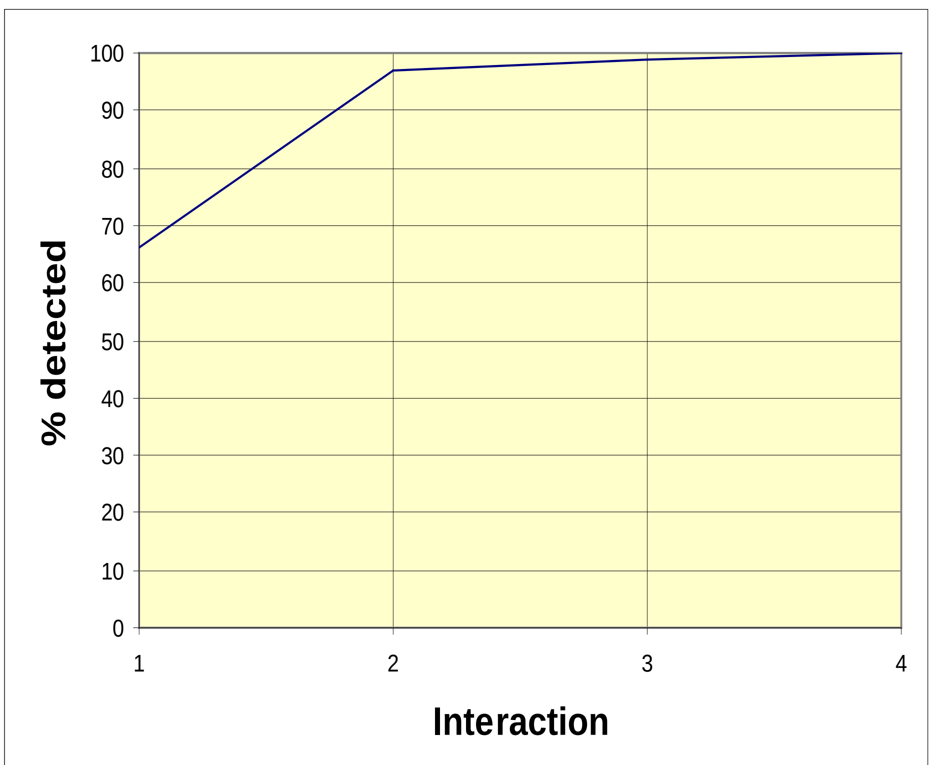
How about hard-to-find flaws?

•Interactions   e.g., failure occurs if

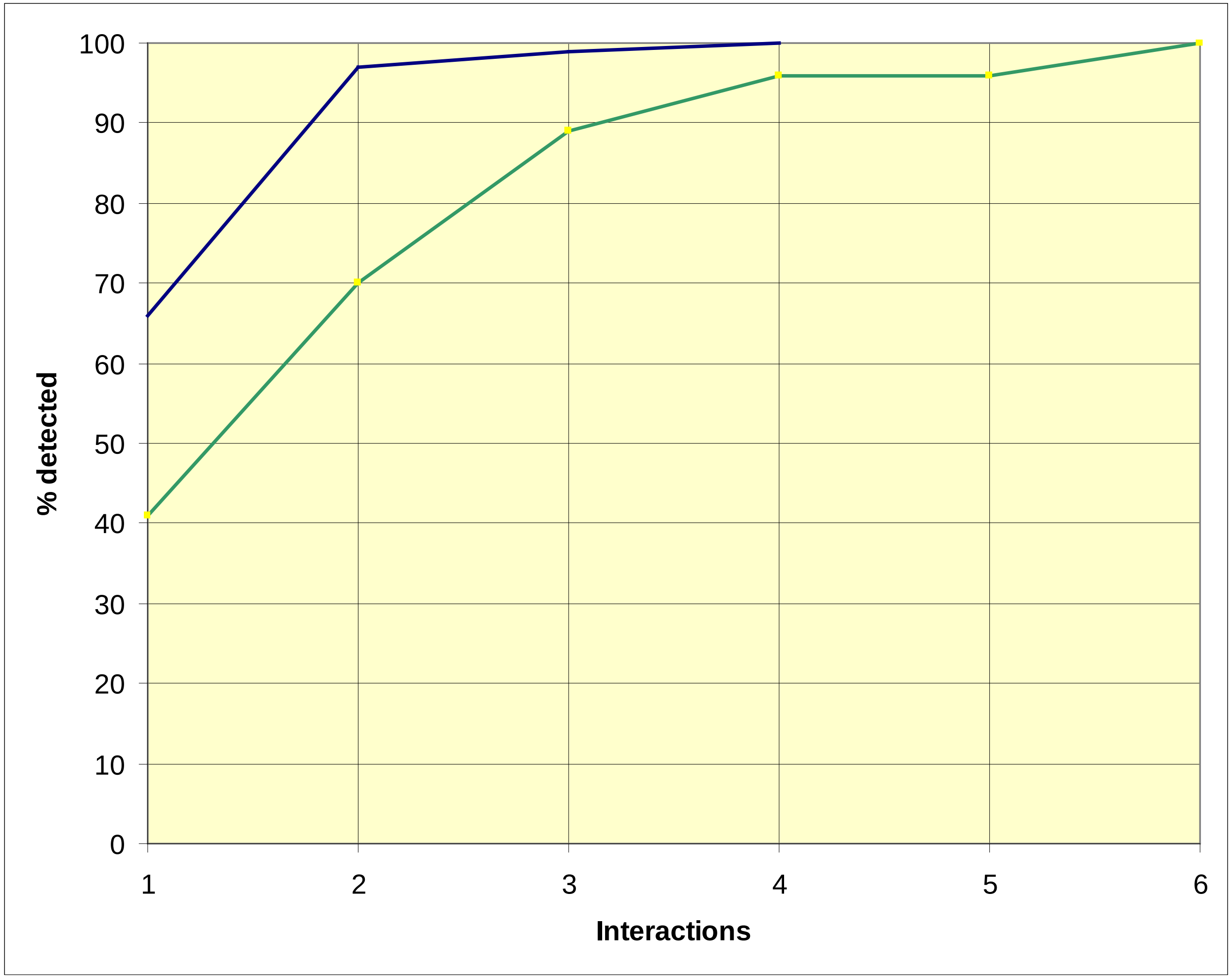
- pressure < 10   (1-way interaction)
- pressure < 10 & volume > 300 (2-way interaction)
- pressure < 10 & volume > 300 & velocity = 5
  (3-way interaction)
- The most complex failure reported required
  4-way interaction to trigger



Interesting, but that's just one kind of application.

How about other applications?

Browser (green)

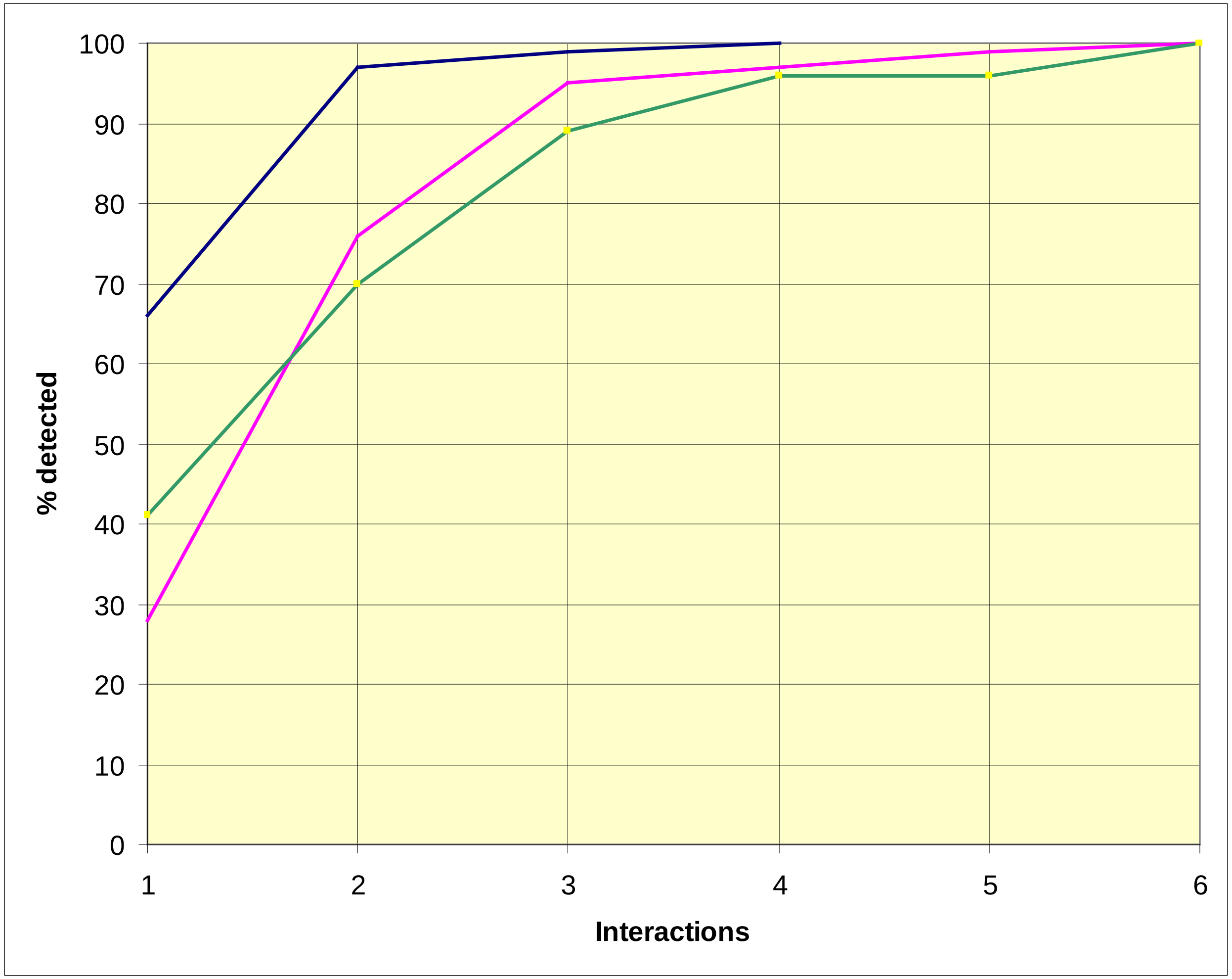


These faults more complex than medical device software!!

Why?

And other applications?

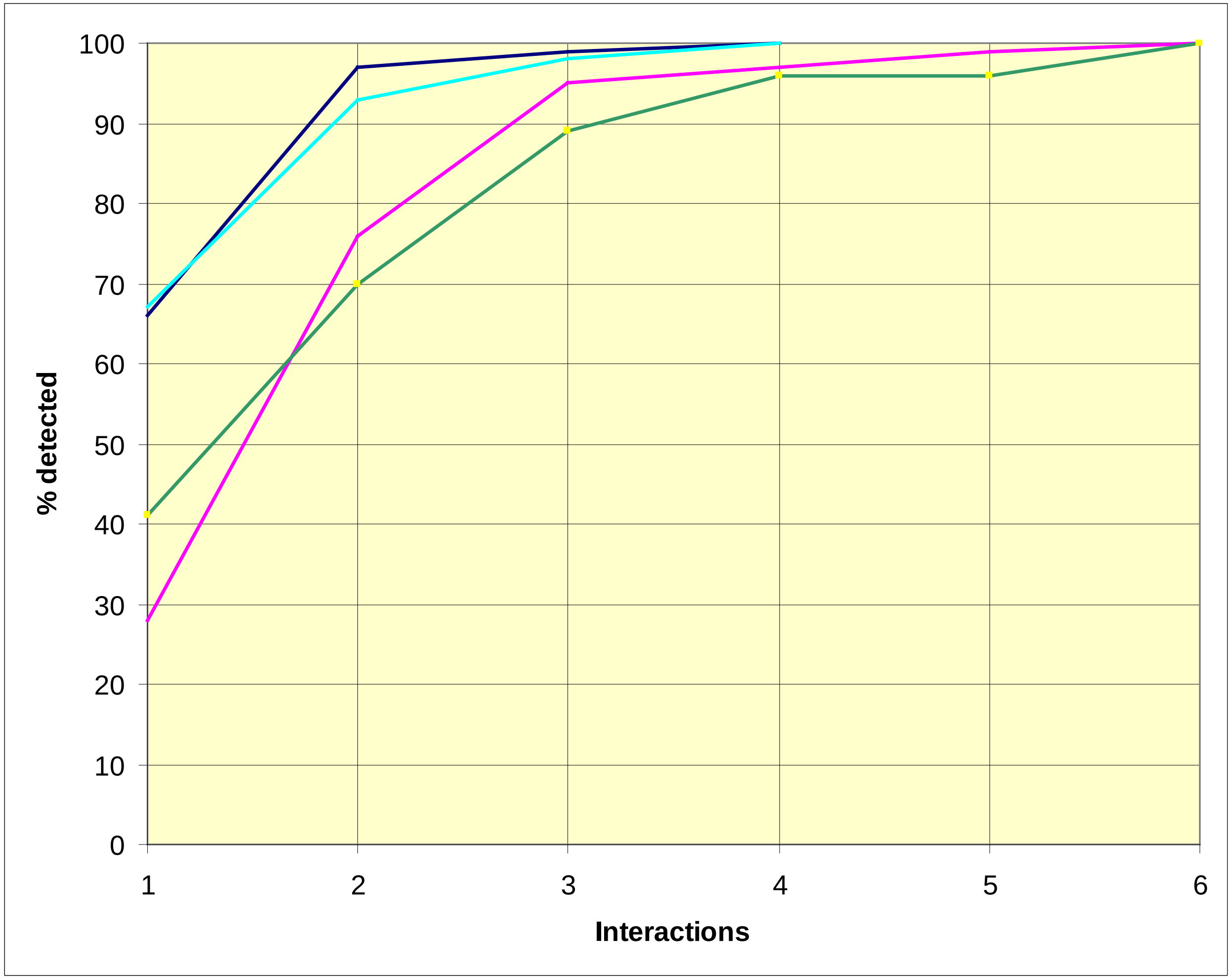
Server (magenta)



Still more?

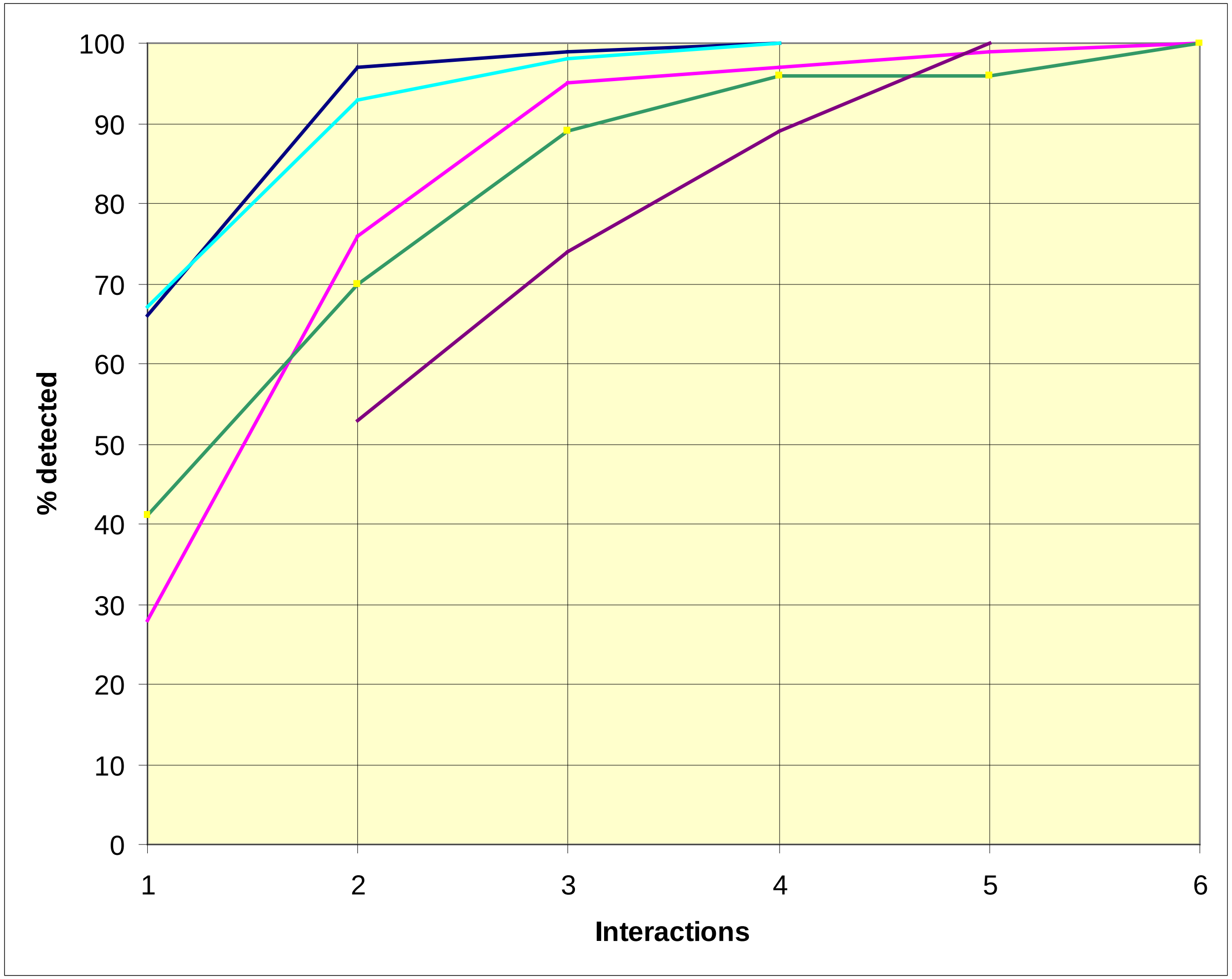
NASA distributed database

(light blue)



Even more?

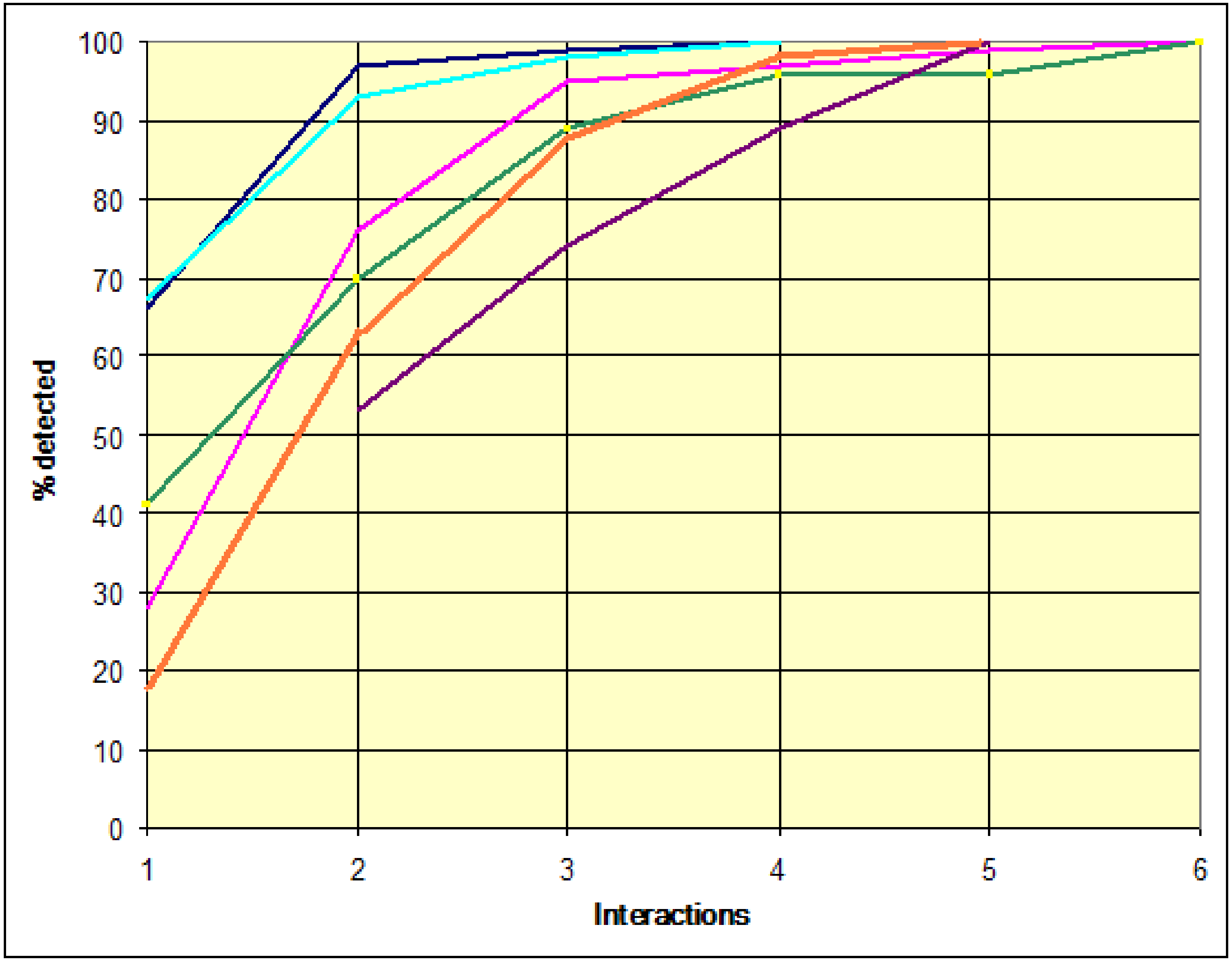
Traffic Collision Avoidance System module (seeded errors) (purple)



Finally

Network security (Bell, 2006)

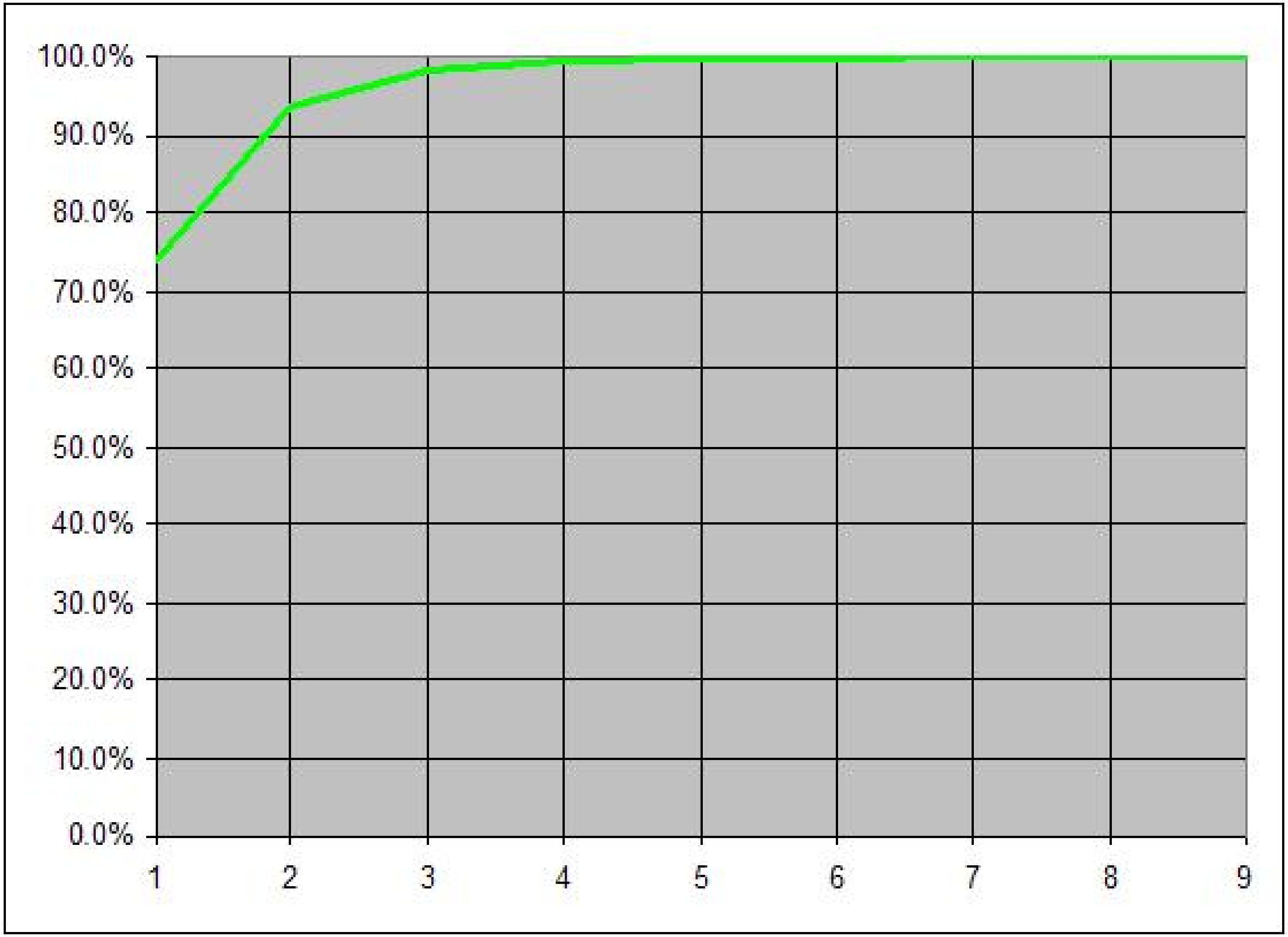
(orange)



Curves appear to be similar across a variety of application domains.

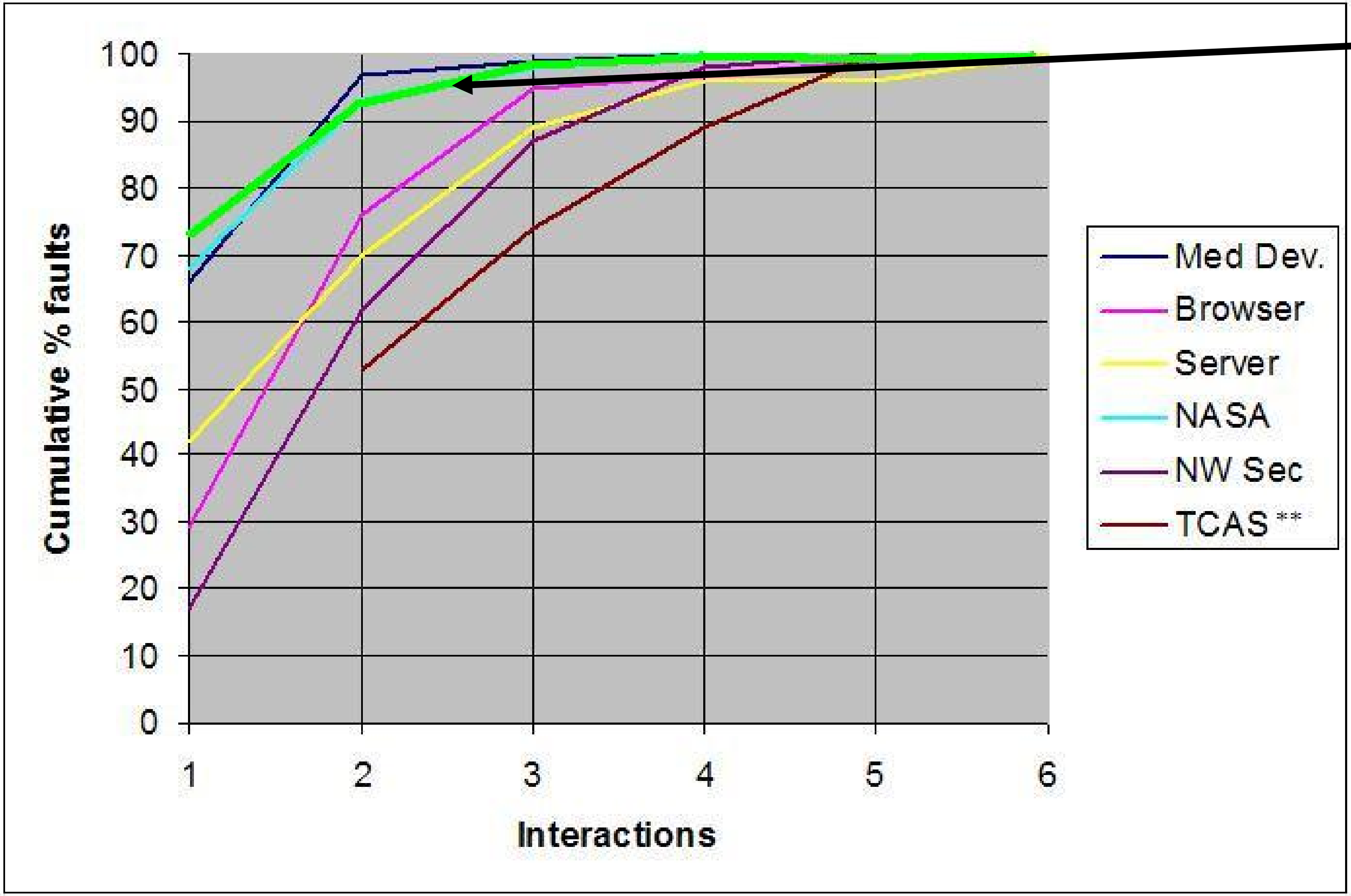
Why this distribution?

What causes this distribution?



One clue: branches in avionics software.
7,685 expressions from *if* and *while* statements

Comparing with Failure Data



Branch statements

So, how many parameters are involved in really tricky faults?

- Maximum interactions for fault triggering for these applications was 6
- Much more empirical work needed
- Reasonable evidence that maximum interaction strength for fault triggering is relatively small

How does it help me to know this?

How does this knowledge help?

Biologists have a “central dogma”, and so do we:

If all faults are triggered by the interaction of *t* or fewer variables, then testing all *t*-way combinations can provide strong assurance

(taking into account: value propagation issues, equivalence partitioning, timing issues, more complex interactions, . . .)

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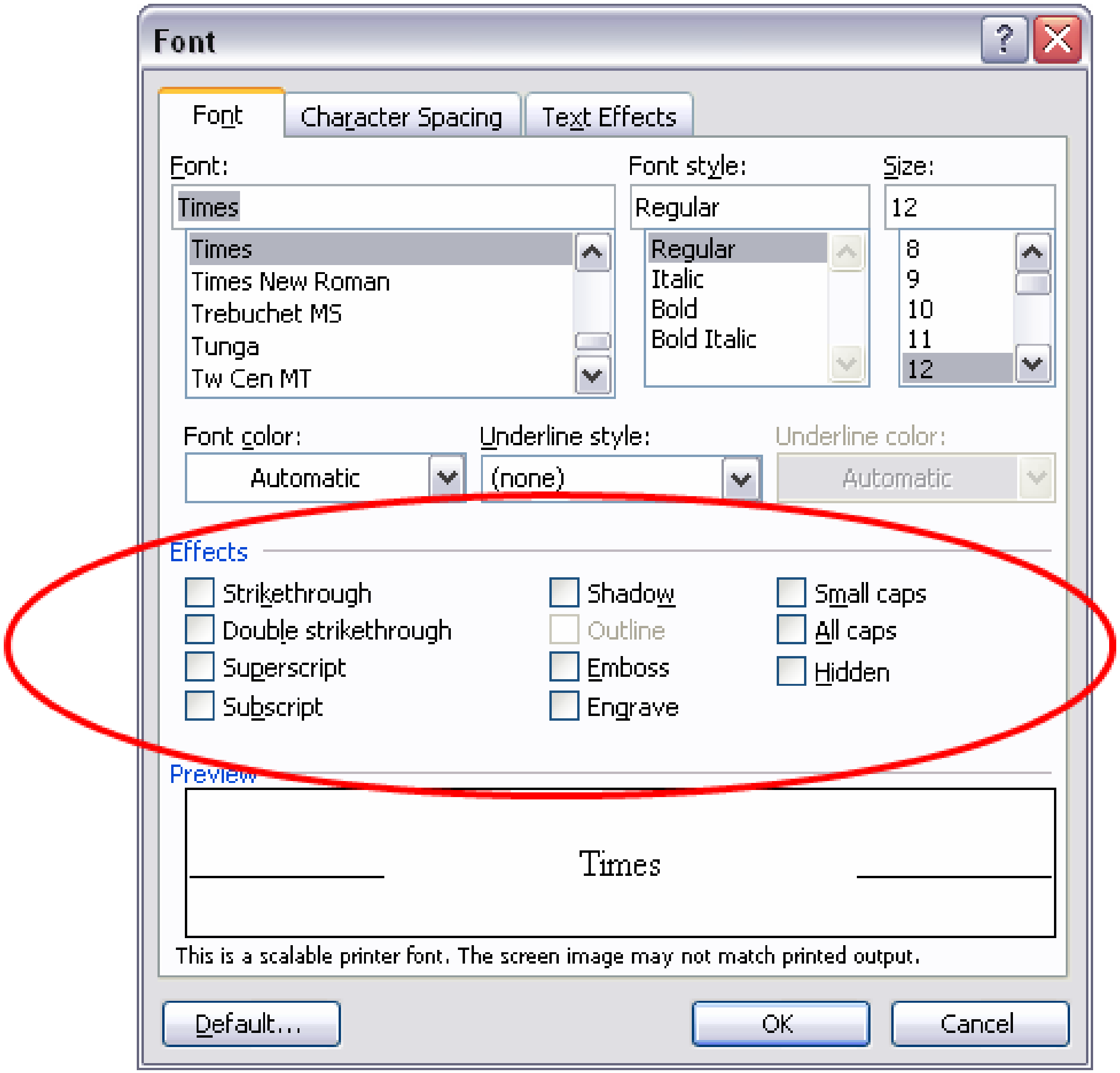
What is combinatorial testing? A simple example



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How Many Tests Would It Take?

- There are 10 effects, each can be on or off
- All combinations is 2^{10} = 1,024 tests
- What if our budget is too limited for these tests?
- Instead, let's look at all 3-way interactions …

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Now How Many Would It Take?

- There are $\binom{10}{3}$ = 120 3-way interactions.
- Naively 120 x 2^3 = 960 tests.
- Since we can pack 3 triples into each test, we need no more than 320 tests.
- Each test exercises many triples:

0 1 1 0 0 0 0 1 1 0

We can pack a lot into one test, so what’s the smallest number of tests we need?

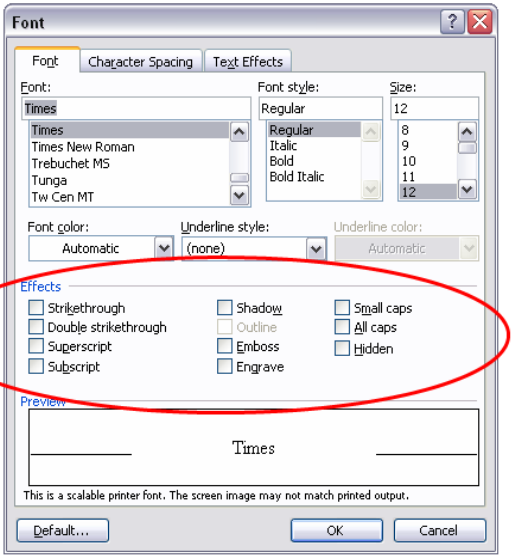
A covering array

All triples in only 13 tests, covering $\binom{10}{3}2^3 = 960$ combinations

Each row is a test:

Each column is a parameter:

0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1
1	1	1	0	1	0	0	0	0	1
1	0	1	1	0	1	0	1	0	0
1	0	0	0	1	1	1	0	0	0
0	1	1	0	0	1	0	0	1	0
0	0	1	0	1	0	1	1	1	0
1	1	0	1	0	0	1	0	1	0
0	0	0	1	1	1	0	0	1	1
0	0	1	1	0	0	1	0	0	1
0	1	0	1	1	0	0	1	0	0
1	0	0	0	0	0	0	1	1	1
0	1	0	0	0	1	1	1	0	1



Each test covers $\binom{10}{3} = 120$ 3-way combinations

<u>Finding covering arrays is NP hard</u>

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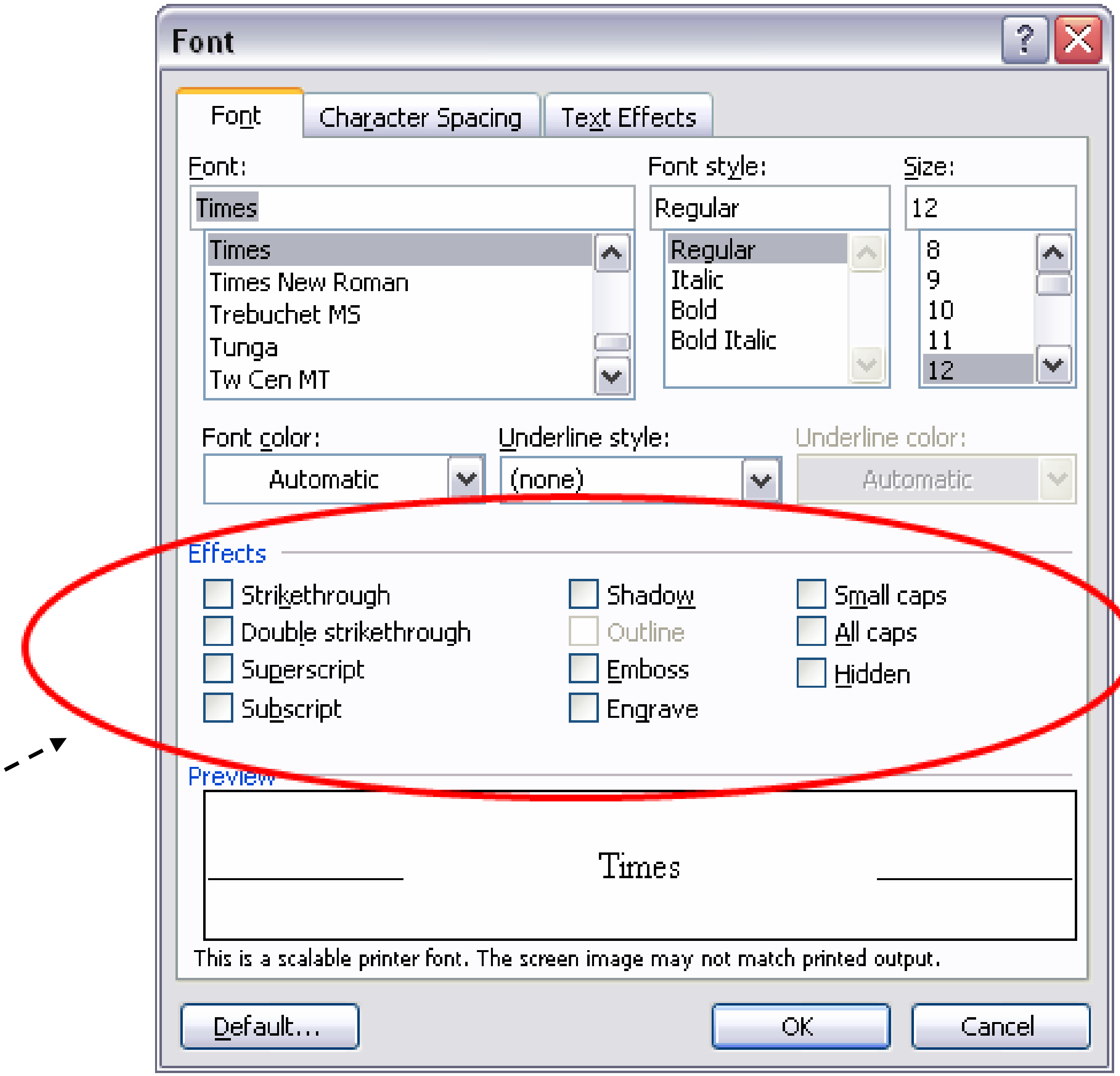
0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1
1	1	1	0	1	0	0	0	0	1
1	0	1	1	0	1	0	1	0	0
1	0	0	0	1	1	1	0	0	0
0	1	1	0	0	1	0	0	1	0
0	0	1	0	1	0	1	1	1	0
1	1	0	1	0	0	1	0	1	0
0	0	0	1	1	1	0	0	1	1
0	0	1	1	0	0	1	0	0	1
0	1	0	1	1	0	0	1	0	0
1	0	0	0	0	0	0	1	1	1
0	1	0	0	0	1	1	1	0	1



0 = effect off
1 = effect on

13 tests for **all 3-way combinations**

2^{10} = **1,024** tests for **all combinations**

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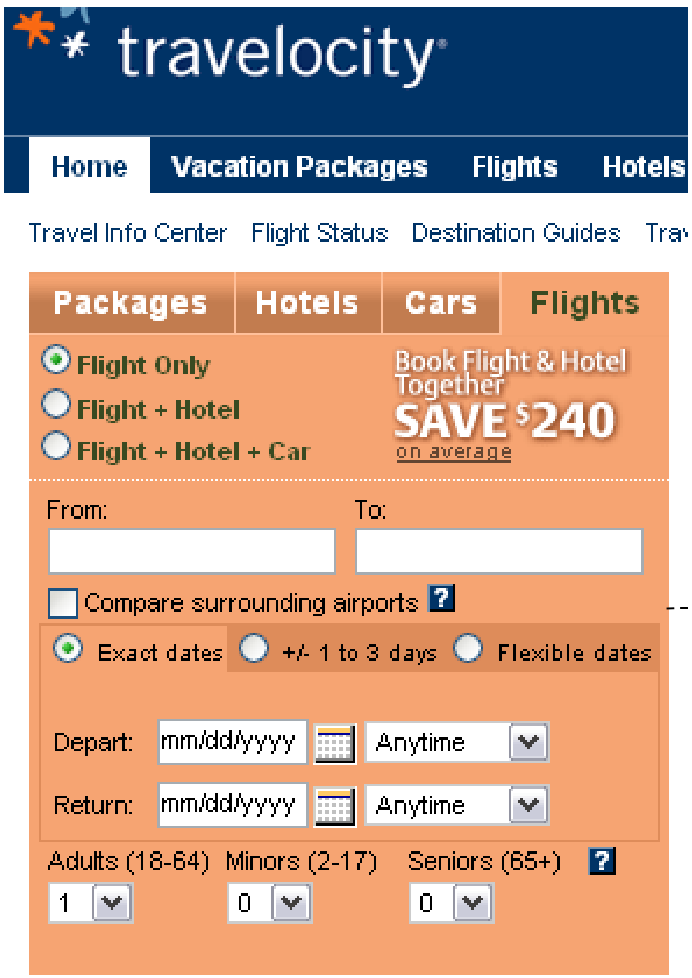
Another familiar example



No silver bullet because:

Many values per variable

Need to abstract values

But we can still increase information per test

Plan: flt, flt+hotel, flt+hotel+car
From: CONUS, HI, Europe, Asia …
To: CONUS, HI, Europe, Asia …
Compare: yes, no
Date-type: exact, 1to3, flex
Depart: today, tomorrow, 1yr, Sun, Mon …
Return: today, tomorrow, 1yr, Sun, Mon …
Adults: 1, 2, 3, 4, 5, 6
Minors: 0, 1, 2, 3, 4, 5
Seniors: 0, 1, 2, 3, 4, 5

A larger example

- Suppose we have a system with on-off switches:

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How do we test this?

- 34 switches = 2^{34} = 1.7 x 10^{10} possible inputs = 1.7 x 10^{10} tests

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What if we knew no failure involves more than 3 switch settings interacting?

- 34 switches = 2^{34} = 1.7 x 10^{10} possible inputs = **1.7 x 10^{10}** tests
- If only 3-way interactions, need only **33** tests
- For 4-way interactions, need only **85** tests

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Two ways of using combinatorial testing

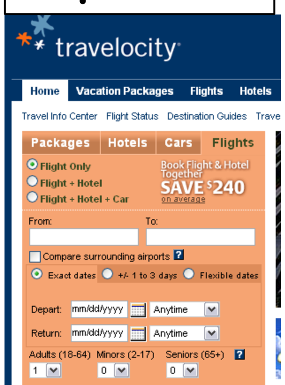
Use combinations here

or here

Test case	OS	CPU	Protocol
1	Windows	Intel	IPv4
2	Windows	AMD	IPv6
3	Linux	Intel	IPv6
4	Linux	AMD	IPv4

Configuration

Test data inputs



System under test

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Testing Configurations

- Example: app must run on any configuration of OS, browser, protocol, CPU, and DBMS
- Very effective for interoperability testing

Test	OS	Browser	Protocol	CPU	DBMS
1	XP	IE	IPv4	Intel	MySQL
2	XP	Firefox	IPv6	AMD	Sybase
3	XP	IE	IPv6	Intel	Oracle
4	OS X	Firefox	IPv4	AMD	MySQL
5	OS X	IE	IPv4	Intel	Sybase
6	OS X	Firefox	IPv4	Intel	Oracle
7	RHL	IE	IPv6	AMD	MySQL
8	RHL	Firefox	IPv4	Intel	Sybase
9	RHL	Firefox	IPv4	AMD	Oracle
10	OS X	Firefox	IPv6	AMD	Oracle

Combinatorial testing with existing test set

1. Use t-way coverage for system configuration values
2. Apply existing tests

Test case	OS	CPU	Protocol
1	Windows	Intel	IPv4
2	Windows	AMD	IPv6
3	Linux	Intel	IPv6
4	Linux	AMD	IPv4

- Common practice in telecom industry

Modeling & Simulation Application

- "Simured" network simulator
  - Kernel of ~ 5,000 lines of C++ (not including GUI)
- Objective: detect configurations that can produce deadlock:
  - Prevent connectivity loss when changing network
  - Attacks that could lock up network
- Compare effectiveness of random vs. combinatorial inputs
- Deadlock combinations discovered
- Crashes in >6% of tests w/ valid values (Win32 version only)

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Simulation Input Parameters

Parameter		Values
1	DIMENSIONS	1,2,4,6,8
2	NODOSDIM	2,4,6
3	NUMVIRT	1,2,3,8
4	NUMVIRTINJ	1,2,3,8
5	NUMVIRTEJE	1,2,3,8
6	LONBUFFER	1,2,4,6
7	NUMDIR	1,2
8	FORWARDING	0,1
9	PHYSICAL	true, false
10	ROUTING	0,1,2,3
11	DELFIFO	1,2,4,6
12	DELCROSS	1,2,4,6
13	DELCHANNEL	1,2,4,6
14	DELSWITCH	1,2,4,6

5x3x4x4x4x4x2x2 x2x4x4x4x4x4 = 31,457,280 configurations

Are any of them dangerous?

If so, how many?

Which ones?

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Network Deadlock Detection

Deadlocks Detected: combinatorial

t	Tests	500 pkts	1000 pkts	2000 pkts	4000 pkts	8000 pkts
2	28	0	0	0	0	0
3	161	2	3	2	3	3
4	752	14	14	14	14	14

Average Deadlocks Detected: random

t	Tests	500 pkts	1000 pkts	2000 pkts	4000 pkts	8000 pkts
2	28	0.63	0.25	0.75	0. 50	0. 75
3	161	3	3	3	3	3
4	752	10.13	11.75	10.38	13	13.25

Network Deadlock Detection

Detected 14 configurations that can cause deadlock:

$14/ 31,457,280 = 4.4 \times 10^{-7}$

Combinatorial testing found more deadlocks than random, including some that might never have been found with random testing

Why do this testing? Risks:

- accidental deadlock configuration: low
- deadlock config discovered by attacker: **much higher**

(because they are looking for it)

Testing inputs

- Traffic Collision Avoidance System (TCAS) module
  - Used in previous testing research
  - 41 versions seeded with errors
  - 12 variables: 7 boolean, two 3-value, one 4-value, two 10-value
  - All flaws found with 5-way coverage
  - Thousands of tests - generated by model checker in a few minutes

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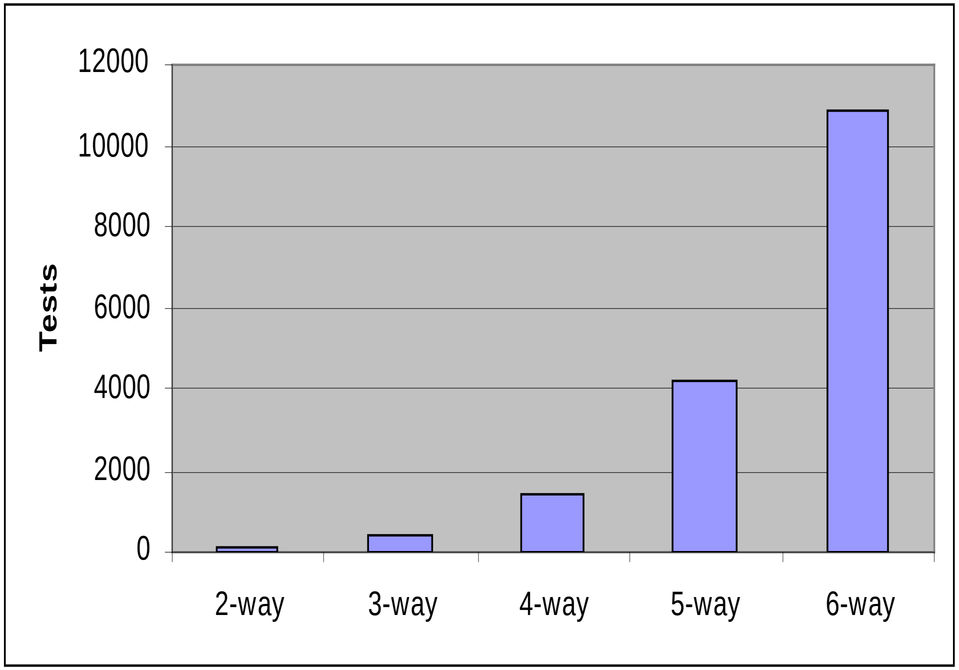
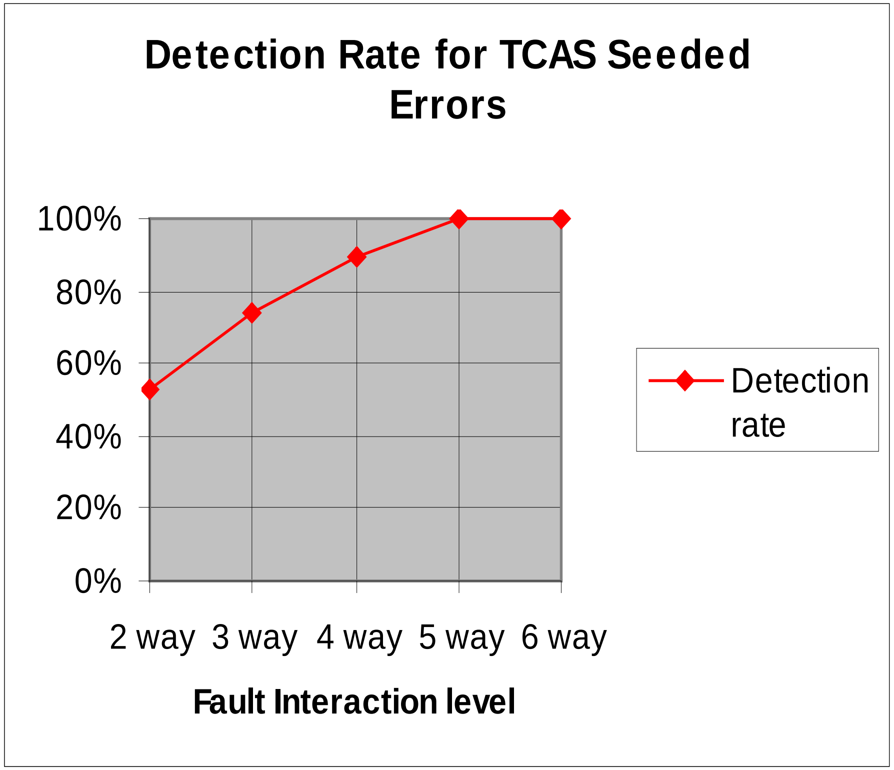
Tests generated

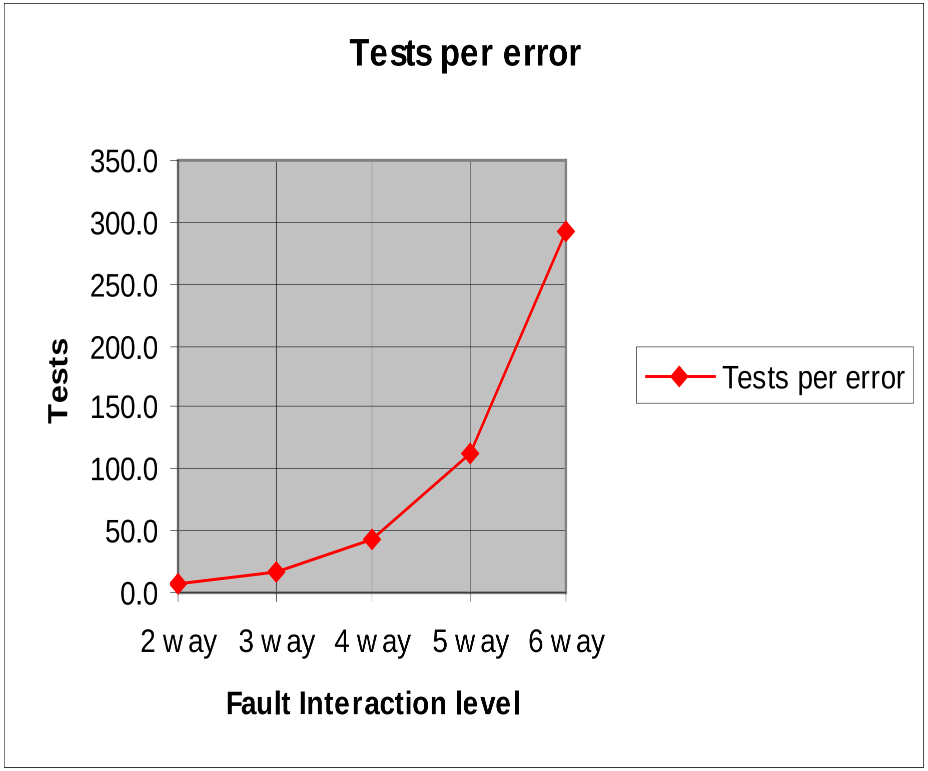
t	Test cases
2-way:	156
3-way:	461
4-way:	1,450
5-way:	4,309
6-way:	11,094



Results

- Roughly consistent with data on large systems
- But errors harder to detect than real-world examples

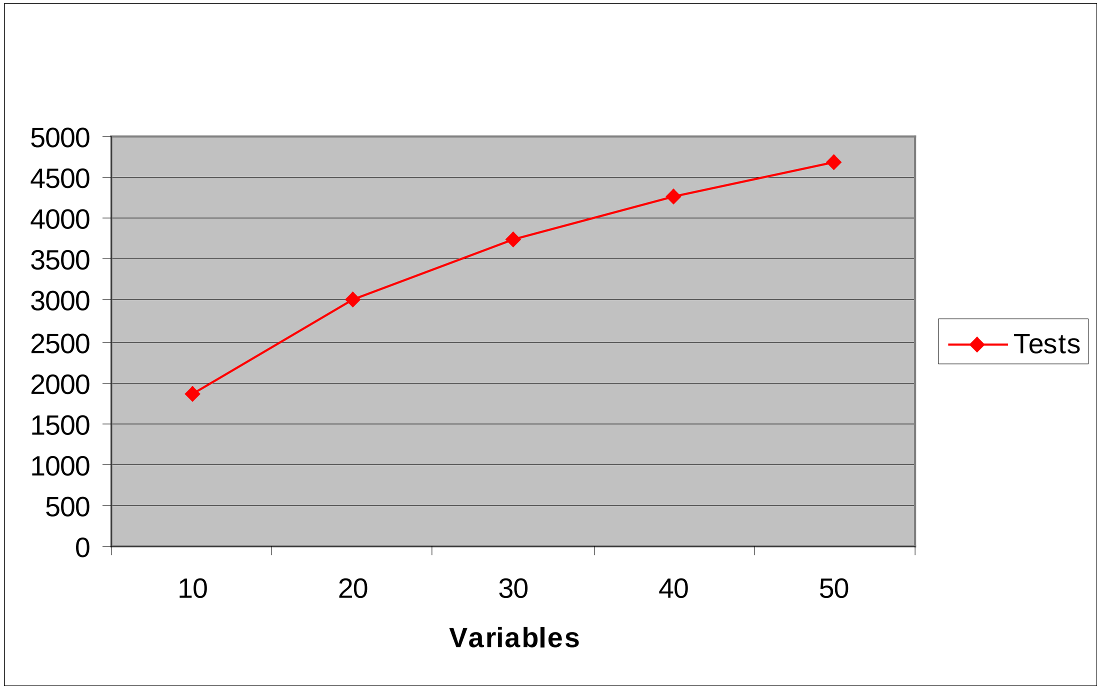




Bottom line for model checking based combinatorial testing:
Expensive but can be highly effective

Cost and Volume of Tests

- Number of tests: proportional to $v^t \log n$ for *v* values, *n* variables, *t*-way interactions
- *Thus:*
  - Tests increase exponentially with interaction strength *t* : BAD, but unavoidable
  - But only logarithmically with the number of parameters : GOOD!
- Example: suppose we want all 4-way combinations of *n* parameters, 5 values each:



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Buffer Overflows

- Empirical data from the National Vulnerability Database
  - Investigated > 3,000 denial-of-service vulnerabilities reported in the NIST NVD for period of 10/06 – 3/07
  - Vulnerabilities triggered by:
    - Single variable – 94.7%
      example: *Heap-based buffer overflow in the SFTP protocol handler for Panic Transmit … allows remote attackers to execute arbitrary code via a long ftps:// URL.*
    - 2-way interaction – 4.9%
      example: *single character search string in conjunction with a single character replacement string, which causes an "off by one overflow"*
    - 3-way interaction – 0.4%
      example: *Directory traversal vulnerability when register_globals is enabled and magic_quotes is disabled and .. (dot dot) in the page parameter*

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Finding Buffer Overflows

```
if (strcmp(conn[sid].dat->in_RequestMethod, "POST")==0) {
        if (conn[sid].dat->in_ContentLength<MAX_POSTSIZE) {
        ……
conn[sid].PostData=calloc(conn[sid].dat->in_ContentLength+1024, sizeof(char));
                ……
        pPostData=conn[sid].PostData;
        do {
                rc=recv(conn[sid].socket, pPostData, 1024, 0);
                  ……
                pPostData+=rc;
                x+=rc;
        } while ((rc==1024)||(x<conn[sid].dat->in_ContentLength));
conn[sid].PostData[conn[sid].dat->in_ContentLength]='\0';
 }
```

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Interaction: request-method="POST", content-length = -1000, data= a string > 24 bytes

```
if (strcmp(conn[sid].dat->in_RequestMethod, "POST")==0) {
        if (conn[sid].dat->in_ContentLength<MAX_POSTSIZE) {
        ……
conn[sid].PostData=calloc(conn[sid].dat->in_ContentLength+1024, sizeof(char));
              ……
        pPostData=conn[sid].PostData;
        do {
                rc=recv(conn[sid].socket, pPostData, 1024, 0);
                  ……
                pPostData+=rc;
                x+=rc;
        } while ((rc==1024)||(x<conn[sid].dat->in_ContentLength));
conn[sid].PostData[conn[sid].dat->in_ContentLength]='\0';
 }
```

Interaction: request-method="POST", content-length = -1000, data= a string > 24 bytes

```
if (strcmp(conn[sid].dat->in_RequestMethod, "POST")==0) {
        if (conn[sid].dat->in_ContentLength<MAX_POSTSIZE) {
        ......
conn[sid].PostData=calloc(conn[sid].dat->in_ContentLength+1024, sizeof(char));
                ......
        pPostData=conn[sid].PostData;
        do {
                rc=recv(conn[sid].socket, pPostData, 1024, 0);
                  ......
                pPostData+=rc;
                x+=rc;
        } while ((rc==1024)||(x<conn[sid].dat->in_ContentLength));
conn[sid].PostData[conn[sid].dat->in_ContentLength]='\0';
 }
```

true branch

Interaction: request-method="POST", content-length = -1000, data= a string > 24 bytes

```
if (strcmp(conn[sid].dat->in_RequestMethod, "POST")==0) {
        if (conn[sid].dat->in_ContentLength<MAX_POSTSIZE) {
        ......
        conn[sid].PostData=calloc(conn[sid].dat->in_ContentLength+1024, sizeof(char));
                ......
        pPostData=conn[sid].PostData;
        do {
                rc=recv(conn[sid].socket, pPostData, 1024, 0);
                ......
                pPostData+=rc;
                x+=rc;
        } while ((rc==1024)||(x<conn[sid].dat->in_ContentLength));
conn[sid].PostData[conn[sid].dat->in_ContentLength]='\0';
 }
```

true branch

Interaction: request-method="POST", content-length = -1000, data= a string > 24 bytes

```
if (strcmp(conn[sid].dat->in_RequestMethod, "POST")==0) {
        if (conn[sid].dat->in_ContentLength<MAX_POSTSIZE) {
        ……
        conn[sid].PostData=calloc(conn[sid].dat->in_ContentLength+1024,
sizeof(char));
              ……
       pPostData=conn[sid].PostData;
       do {
               rc=recv(conn[sid].socket, pPostData, 1024, 0);
                 ……
               pPostData+=rc;
               x+=rc;
       } while ((rc==1024)||(x<conn[sid].dat->in_ContentLength));
conn[sid].PostData[conn[sid].dat->in_ContentLength]='\0';
 }
```

true branch

Allocate -1000 + 1024 bytes = 24 bytes

Interaction: request-method=”POST”, content-length = -1000, data= a string > 24 bytes

```
if (strcmp(conn[sid].dat->in_RequestMethod, "POST")==0) {
        if (conn[sid].dat->in_ContentLength<MAX_POSTSIZE) {
        ……
        conn[sid].PostData=calloc(conn[sid].dat->in_ContentLength+1024,
sizeof(char));
              ……
       pPostData=conn[sid].PostData;
       do {
              rc=recv(conn[sid].socket, pPostData, 1024, 0);
                ……
              pPostData+=rc;
              x+=rc;
       } while ((rc==1024)||(x<conn[sid].dat->in_ContentLength));
    conn[sid].PostData[conn[sid].dat->in_ContentLength]='\0';
     }
```

true branch

Allocate -1000 + 1024 bytes = 24 bytes

Boom!

Ordering Pizza

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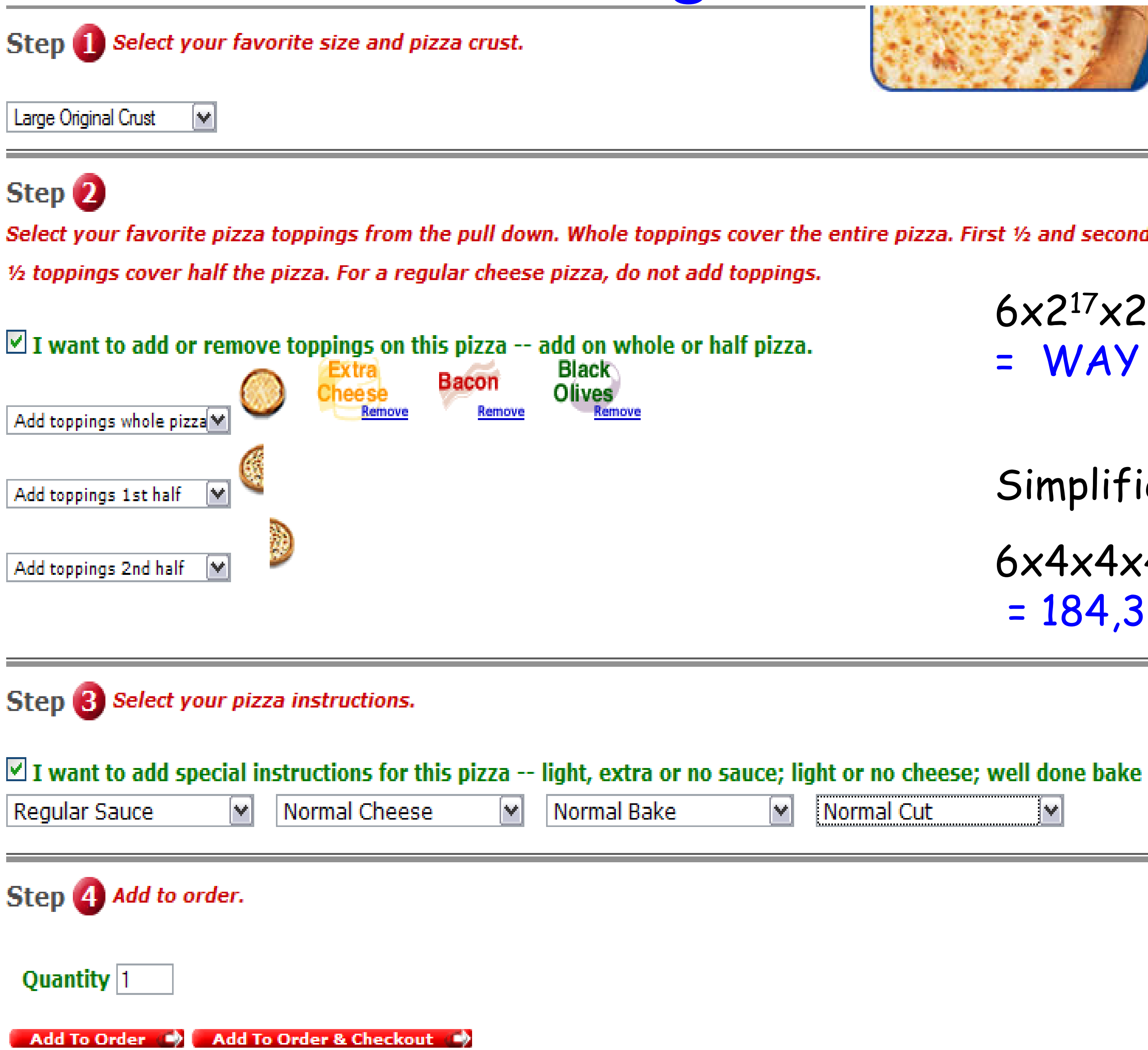

$6 \times 2^{17} \times 2^{17} \times 2^{17} \times 4 \times 3 \times 2 \times 2 \times 5 \times 2$

= WAY TOO MUCH TO TEST

Simplified pizza ordering:

$6 \times 4 \times 4 \times 4 \times 4 \times 3 \times 2 \times 2 \times 5 \times 2$

= 184,320 possibilities

Ordering Pizza Combinatorially

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Simplified pizza ordering:

6x4x4x4x4x3x2x2x5x2
= 184,320 possibilities

2-way tests:	32
3-way tests:	150
4-way tests:	570
5-way tests:	2,413
6-way tests:	8,330

If all failures involve 5 or fewer parameters, then we can have confidence after running all 5-way tests.

So what? Who has time to check 2,413 test results?

How to automate checking correctness of output

- **Creating test data is the easy part!**
- How do we check that the code worked correctly on the test input?
  - **Crash testing** server or other code to ensure it does not crash for any test input (like 'fuzz testing')
    - Easy but limited value
  - **Embedded assertions** – incorporate assertions in code to check critical states at different points in the code, or print out important values during execution
  - **Full scale model-checking** using mathematical model of system and model checker to generate expected results for each input
    - expensive but tractable

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Crash Testing

- Like “fuzz testing” - send packets or other input to application, watch for crashes
- Unlike fuzz testing, input is non-random; cover all t-way combinations
- May be more efficient - random input generation requires several times as many tests to cover the t-way combinations in a covering array

Limited utility, but can detect high-risk problems such as:

- buffer overflows
- server crashes

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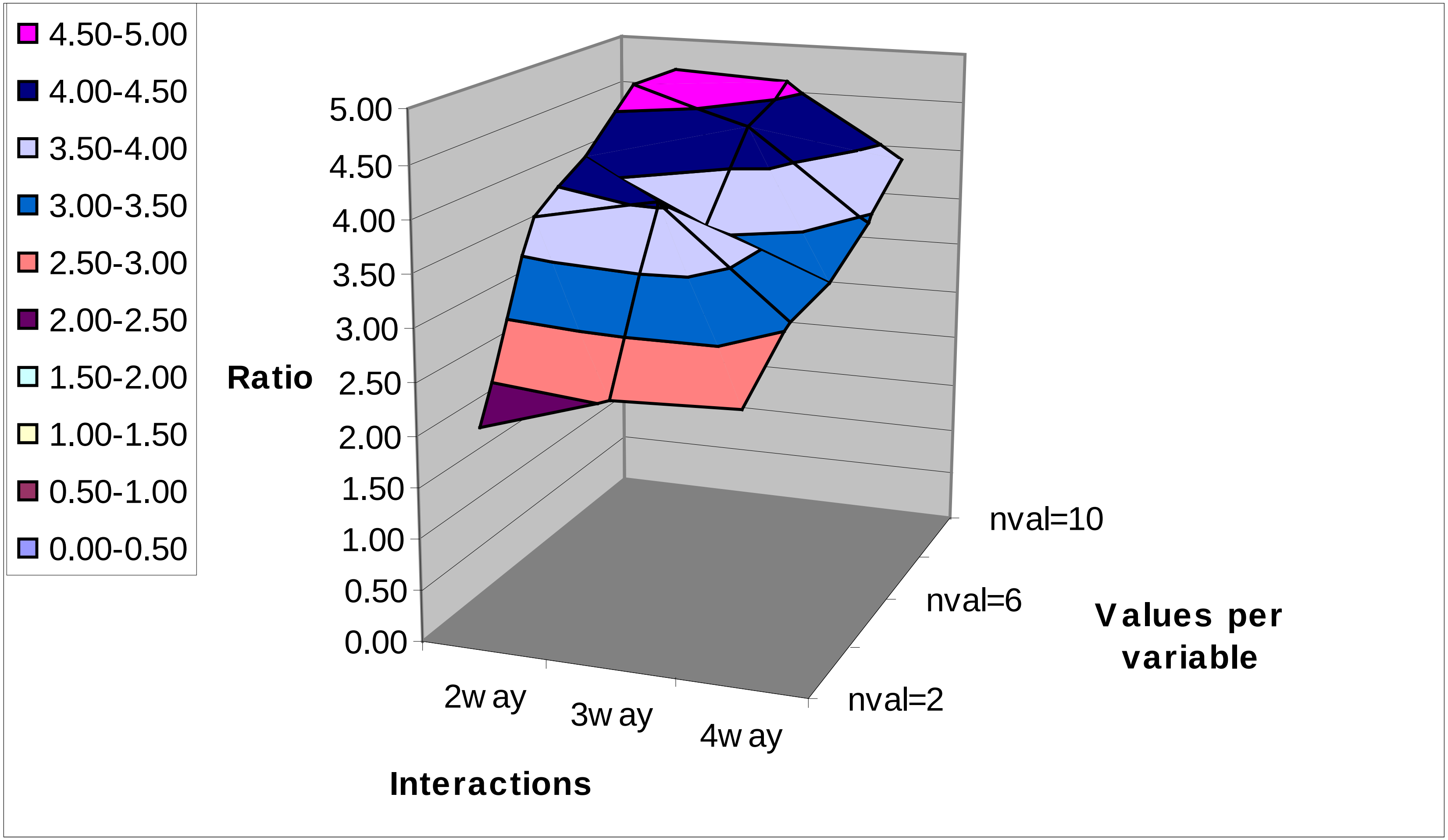
Ratio of Random/Combinatorial Test Set Required to Provide t-way Coverage



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Embedded Assertions

Simple example:

assert(x != 0);   // ensure divisor is not zero

Or pre and post-conditions:

/requires amount >= 0;

/ensures balance == \old(balance) - amount &&
\result == balance;

Embedded Assertions

Assertions check properties of expected result:

```
ensures balance  == \old(balance) - amount
 &&  \result == balance;
```

- Reasonable assurance that code works correctly across the range of expected inputs

- May identify problems with handling unanticipated inputs

- Example: Smart card testing
  - Used Java Modeling Language (JML) assertions
  - Detected 80% to 90% of flaws

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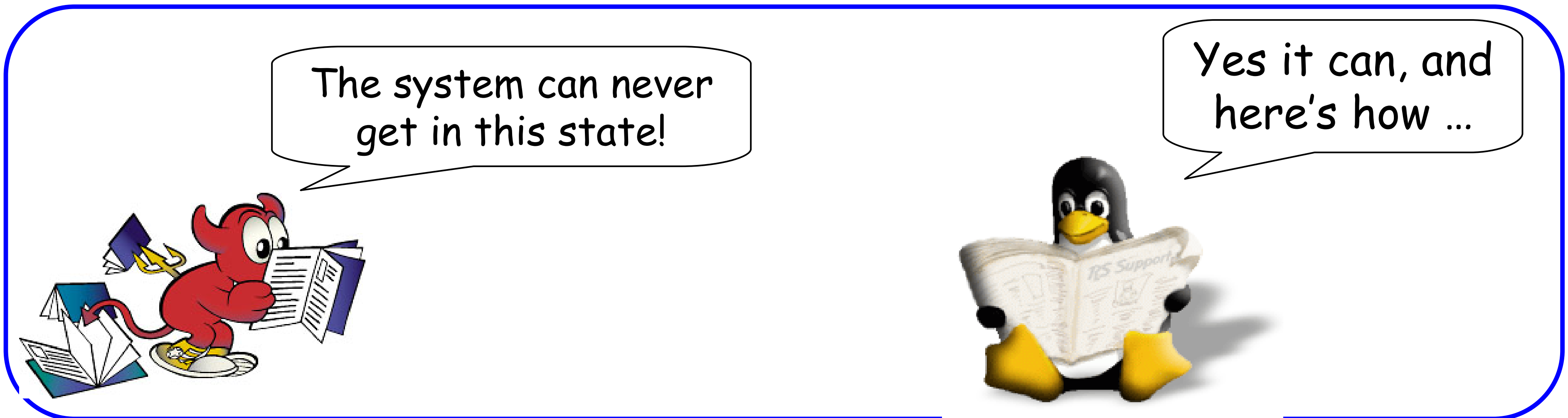
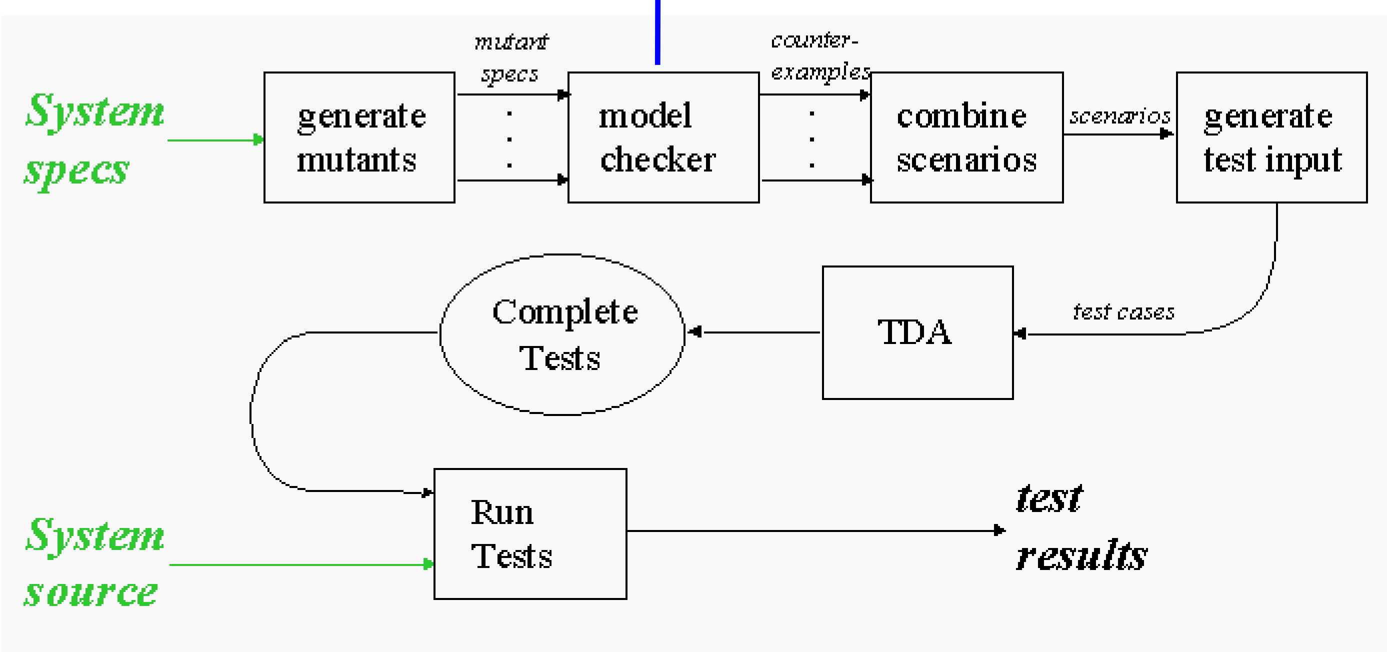
Using model checking to produce tests





- Model-checker test production: if assertion is not true, then a counterexample is generated.

- This can be converted to a test case.

Black & Ammann, 1999

Model checking example

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```
-- specification for a portion of tcas - altitude separation.
-- The corresponding C code is originally from Siemens Corp. Research
-- Vadim Okun 02/2002
MODULE main
VAR
  Cur_Vertical_Sep : { 299, 300, 601 };
  High_Confidence : boolean;
...
init(alt_sep) := START_;
  next(alt_sep) := case
    enabled & (intent_not_known | !tcas_equipped) : case
      need_upward_RA & need_downward_RA : UNRESOLVED;
      need_upward_RA : UPWARD_RA;
      need_downward_RA : DOWNWARD_RA;
      1 : UNRESOLVED;
    esac;
    1 : UNRESOLVED;
  esac;
...
SPEC AG ((enabled & (intent_not_known | !tcas_equipped) &
!need_downward_RA & need_upward_RA) -> AX (alt_sep = UPWARD_RA))
-- "FOR ALL executions,
-- IF enabled & (intent_not_known ....
-- THEN in the next state alt_sep = UPWARD_RA"
```

Computation Tree Logic

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The usual logic operators,plus temporal:

- A φ - All: φ holds on all paths starting from the current state.
- E φ - Exists: φ holds on some paths starting from the current state.
- G φ - Globally: φ has to hold on the entire subsequent path.
- F φ - Finally: φ eventually has to hold
- X φ - Next: φ has to hold at the next state
  [others not listed]

execution paths

states on the execution paths

```
SPEC AG ((enabled & (intent_not_known |
!tcas_equipped) & !need_downward_RA & need_upward_RA)
-> AX (alt_sep = UPWARD_RA))
```

“FOR ALL executions,
IF enabled & (intent_not_known ....
THEN in the next state alt_sep = UPWARD_RA”

What is the most effective way to integrate combinatorial testing with model checking?

- Given **AG(P -> AX(R))**
  "for all paths, in every state,
  if P then in the next state, R holds"

- For k-way variable combinations, **v1 & v2 & ... & vk**

- vi abbreviates "var1 = val1"

- Now combine this constraint with assertion to produce counterexamples. Some possibilities:

1. **AG(v1 & v2 & ... & vk & P -> AX !(R))**
2. **AG(v1 & v2 & ... & vk -> AX !(1))**
3. **AG(v1 & v2 & ... & vk -> AX !(R))**

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What happens with these assertions?

1. `AG(v1 & v2 & ... & vk & P -> AX !(R))`

P may have a negation of one of the v_i, so we get

`0 -> AX !(R))`

always true, so no counterexample, no test.

This is too restrictive!

1. `AG(v1 & v2 & ... & vk -> AX !(1))`

   The model checker makes non-deterministic choices for variables not in v1..vk, so all R values may not be covered by a counterexample.

   This is too loose!

2. `AG(v1 & v2 & ... & vk -> AX !(R))`

   Forces production of a counterexample for each R.

   This is just right!

Tradeoffs

- Advantages
  - Tests rare conditions
  - Produces high code coverage
  - Finds faults faster
  - May be lower overall testing cost
- Disadvantages
  - Very expensive at higher strength interactions (>4-way)
  - May require high skill level in some cases (if formal models are being used)

Tutorial Overview

1. Why are we doing this?
2. What is combinatorial testing?
3. What is it good for?
4. How much does it cost?
5. **What tools are available?**
6. What's next?

New algorithms to make it practical

- **Tradeoffs to minimize calendar/staff time:**
- FireEye (extended IPO) – Lei – roughly optimal, can be used for most cases under 40 or 50 parameters
  - Produces minimal number of tests at cost of run time
  - Currently integrating algebraic methods
- Adaptive distance-based strategies – Bryce – dispensing one test at a time w/ metrics to increase probability of finding flaws
  - Highly optimized covering array algorithm
  - Variety of distance metrics for selecting next test
- PRMI – Kuhn –for more variables or larger domains
  - Parallel, randomized algorithm, generates tests w/ a few tunable parameters; computation can be distributed
  - Better results than other algorithms for larger problems

New algorithms

- Smaller test sets faster, with a more advanced user interface
- First parallelized covering array algorithm
- More information per test

T-Way	IPOG		ITCH (IBM)		Jenny (Open Source)		TConfig (U. of Ottawa)		TVG (Open Source)	
	Size	Time	Size	Time	Size	Time	Size	Time	Size	Time
2	100	0.8	120	0.73	108	0.001	108	>1 hour	101	2.75
3	400	0.36	2388	1020	413	0.71	472	>12 hour	9158	3.07
4	1363	3.05	1484	5400	1536	3.54	1476	>21 hour	64696	127
5	4226	18s	NA	>1 day	4580	43.54	NA	>1 day	313056	1549
6	10941	65.03	NA	>1 day	11625	470	NA	>1 day	1070048	12600

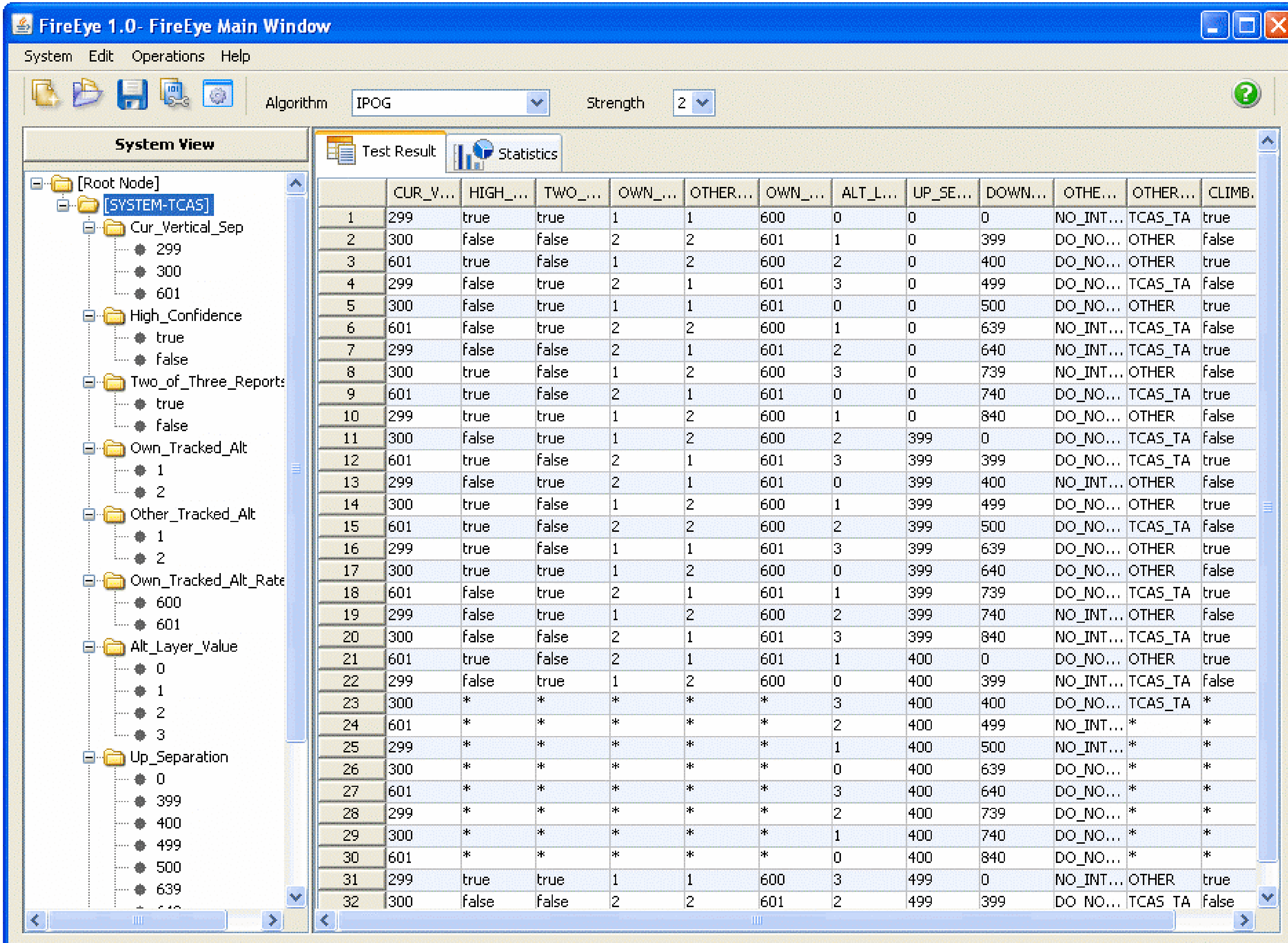
Traffic Collision Avoidance System (TCAS): $2^7 3^2 4^1 10^2$

Times in seconds

That's fast!

Unlike diet plans,
results ARE typical!

ACTS Tool



	CUR_V...	HIGH_...	TWO_...	OWN_...	OTHER...	OWN_...	ALT_L...	UP_SE...	DOWN...	OTHE...	OTHER...	CLIMB.
1	299	true	true	1	1	600	0	0	0	NO_INT...	TCAS_TA	true
2	300	false	false	2	2	601	1	0	399	DO_NO...	OTHER	false
3	601	true	false	1	2	600	2	0	400	DO_NO...	OTHER	true
4	299	false	true	2	1	601	3	0	499	DO_NO...	TCAS_TA	false
5	300	false	true	1	1	601	0	0	500	DO_NO...	OTHER	true
6	601	false	true	2	2	600	1	0	639	NO_INT...	TCAS_TA	false
7	299	false	false	2	1	601	2	0	640	NO_INT...	TCAS_TA	true
8	300	true	false	1	2	600	3	0	739	NO_INT...	OTHER	false
9	601	true	false	2	1	601	0	0	740	DO_NO...	TCAS_TA	true
10	299	true	true	1	2	600	1	0	840	DO_NO...	OTHER	false
11	300	false	true	1	2	600	2	399	0	DO_NO...	TCAS_TA	false
12	601	true	false	2	1	601	3	399	399	DO_NO...	TCAS_TA	true
13	299	false	true	2	1	601	0	399	400	NO_INT...	OTHER	false
14	300	true	false	1	2	600	1	399	499	DO_NO...	OTHER	true
15	601	true	false	2	2	600	2	399	500	DO_NO...	TCAS_TA	false
16	299	true	false	1	1	601	3	399	639	DO_NO...	OTHER	true
17	300	true	true	1	2	600	0	399	640	DO_NO...	OTHER	false
18	601	false	true	2	1	601	1	399	739	DO_NO...	TCAS_TA	true
19	299	false	true	1	2	600	2	399	740	NO_INT...	OTHER	false
20	300	false	false	2	1	601	3	399	840	NO_INT...	TCAS_TA	true
21	601	true	false	2	1	601	1	400	0	DO_NO...	OTHER	true
22	299	false	true	1	2	600	0	400	399	NO_INT...	TCAS_TA	false
23	300	*	*	*	*	*	3	400	400	DO_NO...	TCAS_TA	*
24	601	*	*	*	*	*	2	400	499	NO_INT...	*	*
25	299	*	*	*	*	*	1	400	500	NO_INT...	*	*
26	300	*	*	*	*	*	0	400	639	DO_NO...	*	*
27	601	*	*	*	*	*	3	400	640	DO_NO...	*	*
28	299	*	*	*	*	*	2	400	739	DO_NO...	*	*
29	300	*	*	*	*	*	1	400	740	DO_NO...	*	*
30	601	*	*	*	*	*	0	400	840	DO_NO...	*	*
31	299	true	true	1	1	600	3	499	0	NO_INT...	OTHER	true
32	300	false	false	2	2	601	2	499	399	DO_NO...	TCAS_TA	false

Defining a new system

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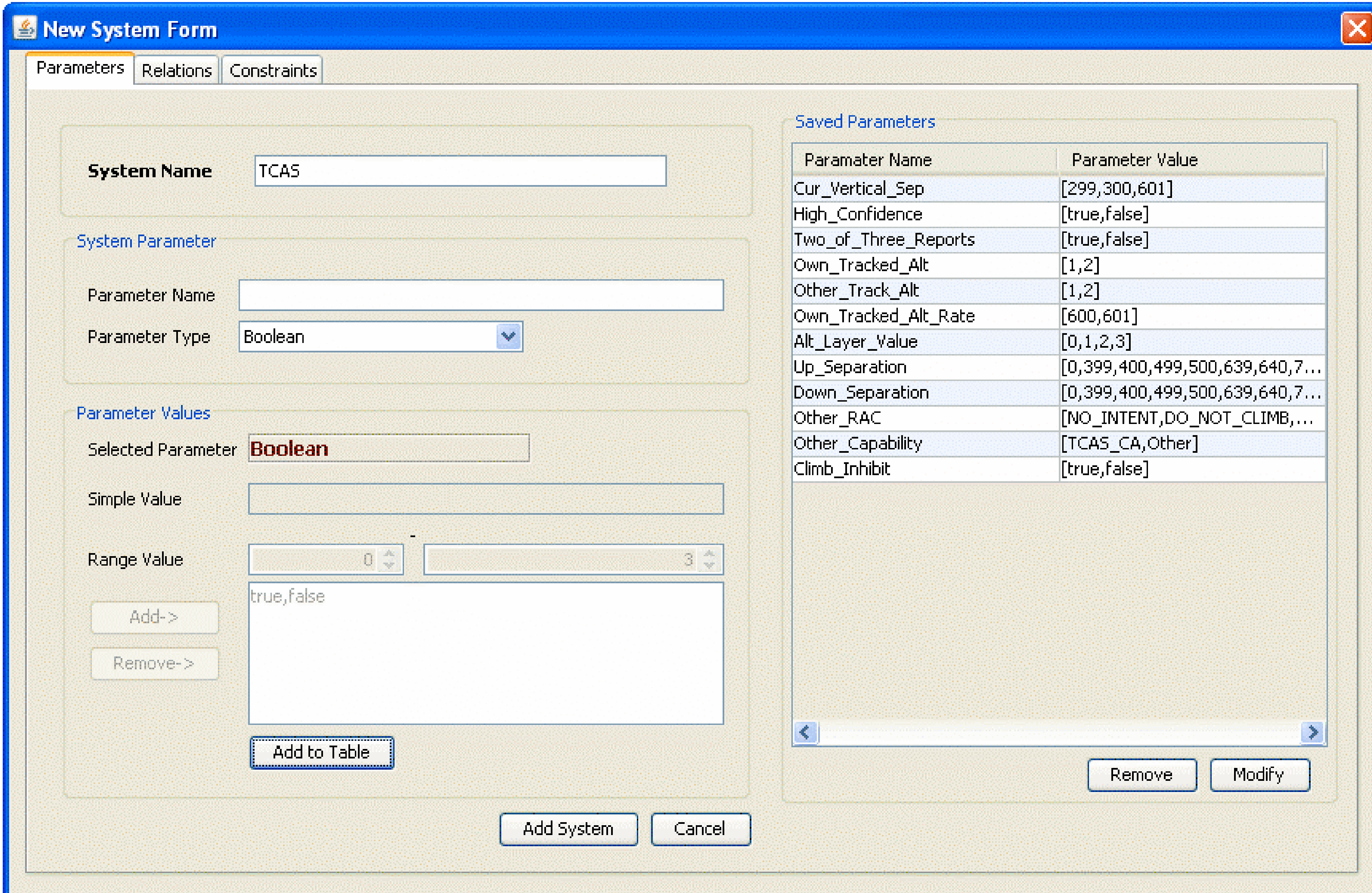
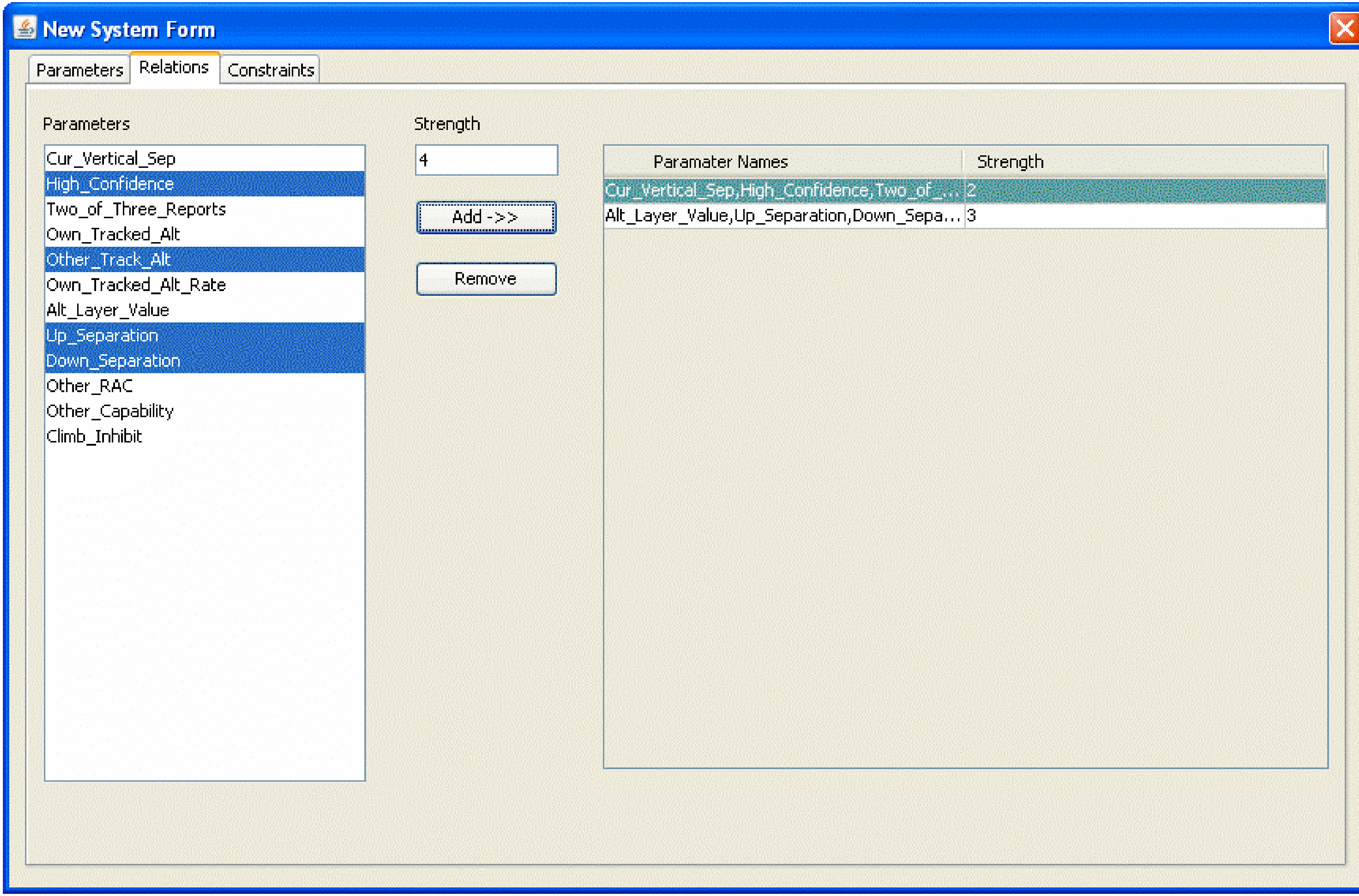
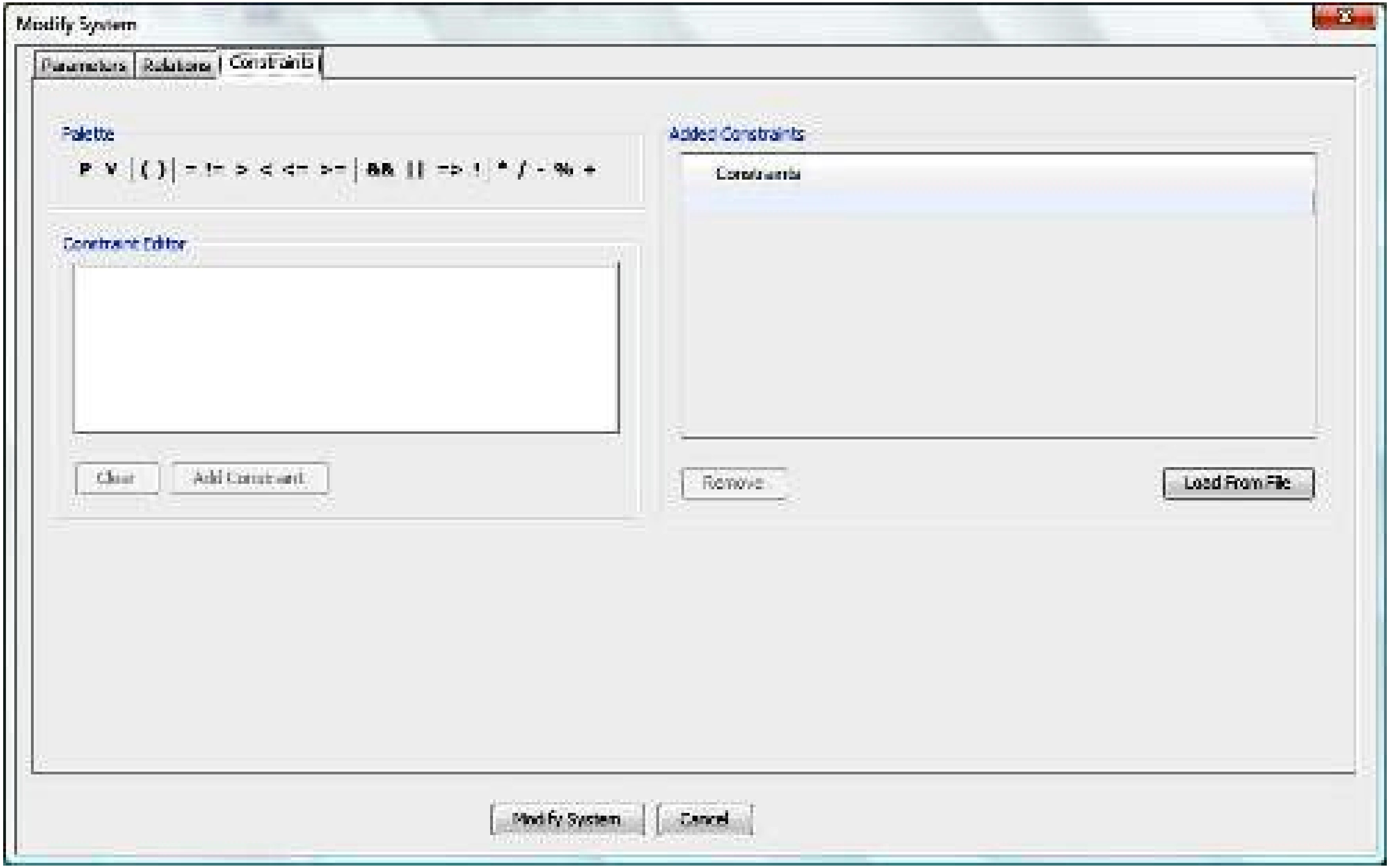


Variable interaction strength



Constraints



Covering array output

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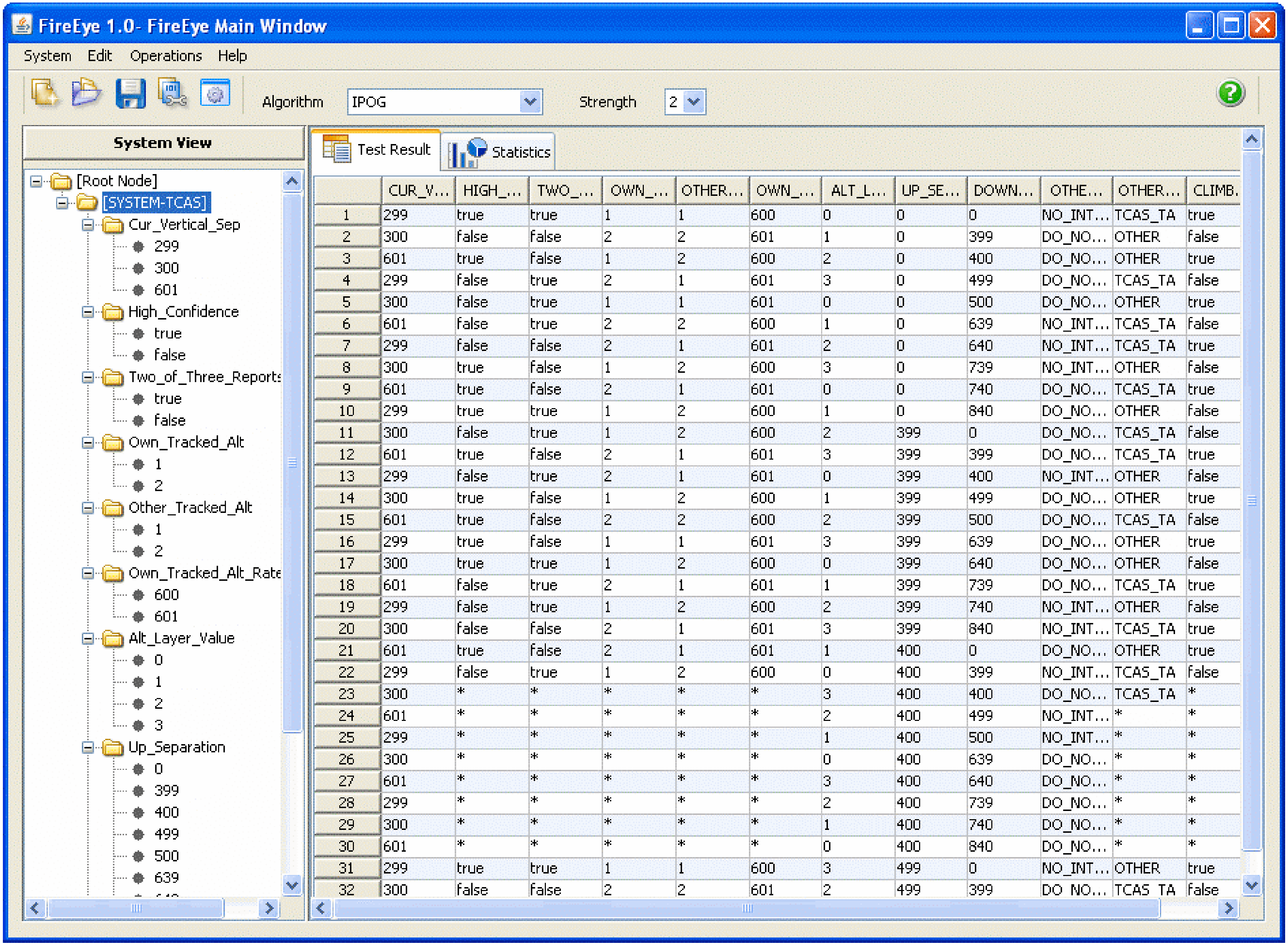

	CUR_V...	HIGH_...	TWO_...	OWN_...	OTHER...	OWN_...	ALT_L...	UP_SE...	DOWN...	OTHE...	OTHER...	CLIMB.
1	299	true	true	1	1	600	0	0	0	NO_INT...	TCAS_TA	true
2	300	false	false	2	2	601	1	0	399	DO_NO...	OTHER	false
3	601	true	false	1	2	600	2	0	400	DO_NO...	OTHER	true
4	299	false	true	2	1	601	3	0	499	DO_NO...	TCAS_TA	false
5	300	false	true	1	1	601	0	0	500	DO_NO...	OTHER	true
6	601	false	true	2	2	600	1	0	639	NO_INT...	TCAS_TA	false
7	299	false	false	2	1	601	2	0	640	NO_INT...	TCAS_TA	true
8	300	true	false	1	2	600	3	0	739	NO_INT...	OTHER	false
9	601	true	false	2	1	601	0	0	740	DO_NO...	TCAS_TA	true
10	299	true	true	1	2	600	1	0	840	DO_NO...	OTHER	false
11	300	false	true	1	2	600	2	399	0	DO_NO...	TCAS_TA	false
12	601	true	false	2	1	601	3	399	399	DO_NO...	TCAS_TA	true
13	299	false	true	2	1	601	0	399	400	NO_INT...	OTHER	false
14	300	true	false	1	2	600	1	399	499	DO_NO...	OTHER	true
15	601	true	false	2	2	600	2	399	500	DO_NO...	TCAS_TA	false
16	299	true	false	1	1	601	3	399	639	DO_NO...	OTHER	true
17	300	true	true	1	2	600	0	399	640	DO_NO...	OTHER	false
18	601	false	true	2	1	601	1	399	739	DO_NO...	TCAS_TA	true
19	299	false	true	1	2	600	2	399	740	NO_INT...	OTHER	false
20	300	false	false	2	1	601	3	399	840	NO_INT...	TCAS_TA	true
21	601	true	false	2	1	601	1	400	0	DO_NO...	OTHER	true
22	299	false	true	1	2	600	0	400	399	NO_INT...	TCAS_TA	false
23	300	*	*	*	*	*	3	400	400	DO_NO...	TCAS_TA	*
24	601	*	*	*	*	*	2	400	499	NO_INT...	*	*
25	299	*	*	*	*	*	1	400	500	NO_INT...	*	*
26	300	*	*	*	*	*	0	400	639	DO_NO...	*	*
27	601	*	*	*	*	*	3	400	640	DO_NO...	*	*
28	299	*	*	*	*	*	2	400	739	DO_NO...	*	*
29	300	*	*	*	*	*	1	400	740	DO_NO...	*	*
30	601	*	*	*	*	*	0	400	840	DO_NO...	*	*
31	299	true	true	1	1	600	3	499	0	NO_INT...	OTHER	true
32	300	false	false	2	2	601	2	499	399	DO_NO...	TCAS_TA	false

Output

- Variety of output formats:
  - XML
  - Numeric
  - CSV
  - Excel

- Separate tool to generate .NET configuration files from ACTS output

- Post-process output using Perl scripts, etc.

Output options

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Mappable values

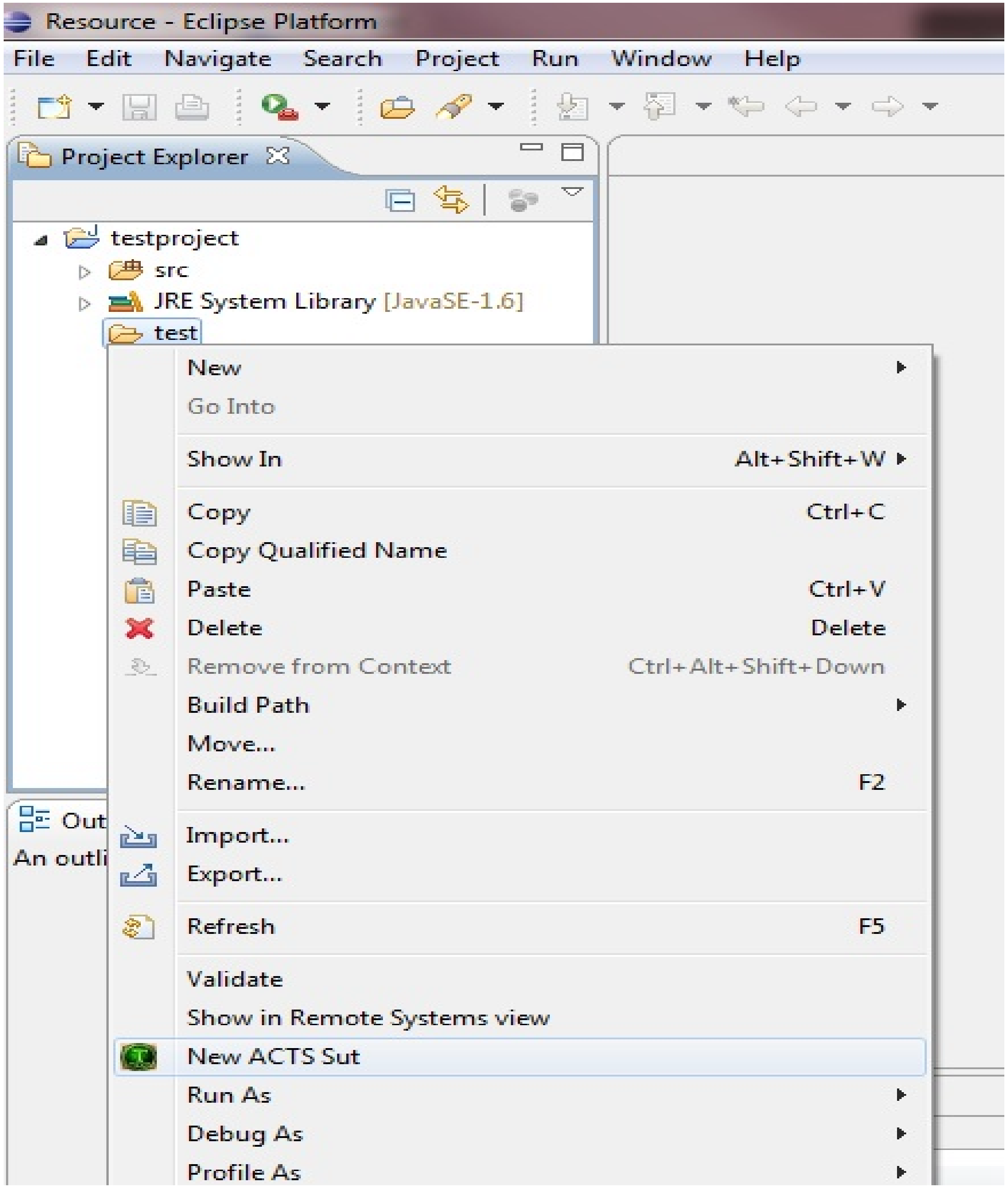
```
Degree of interaction
coverage: 2
Number of parameters: 12
Number of tests: 100

-----------------------------

0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 1 1 1 1 0 1 1 1 1
2 0 1 0 1 0 2 0 2 2 1 0
0 1 0 1 0 1 3 0 3 1 0 1
1 1 0 0 0 1 0 0 4 2 1 0
2 1 0 1 1 0 1 0 5 0 0 1
0 1 1 1 0 1 2 0 6 0 0 0
1 0 1 0 1 0 3 0 7 0 1 1
2 0 1 1 0 1 0 0 8 1 0 0
0 0 0 0 1 0 1 0 9 2 1 1
1 1 0 0 1 0 2 1 0 1 0 1
```

Etc.

Human readable

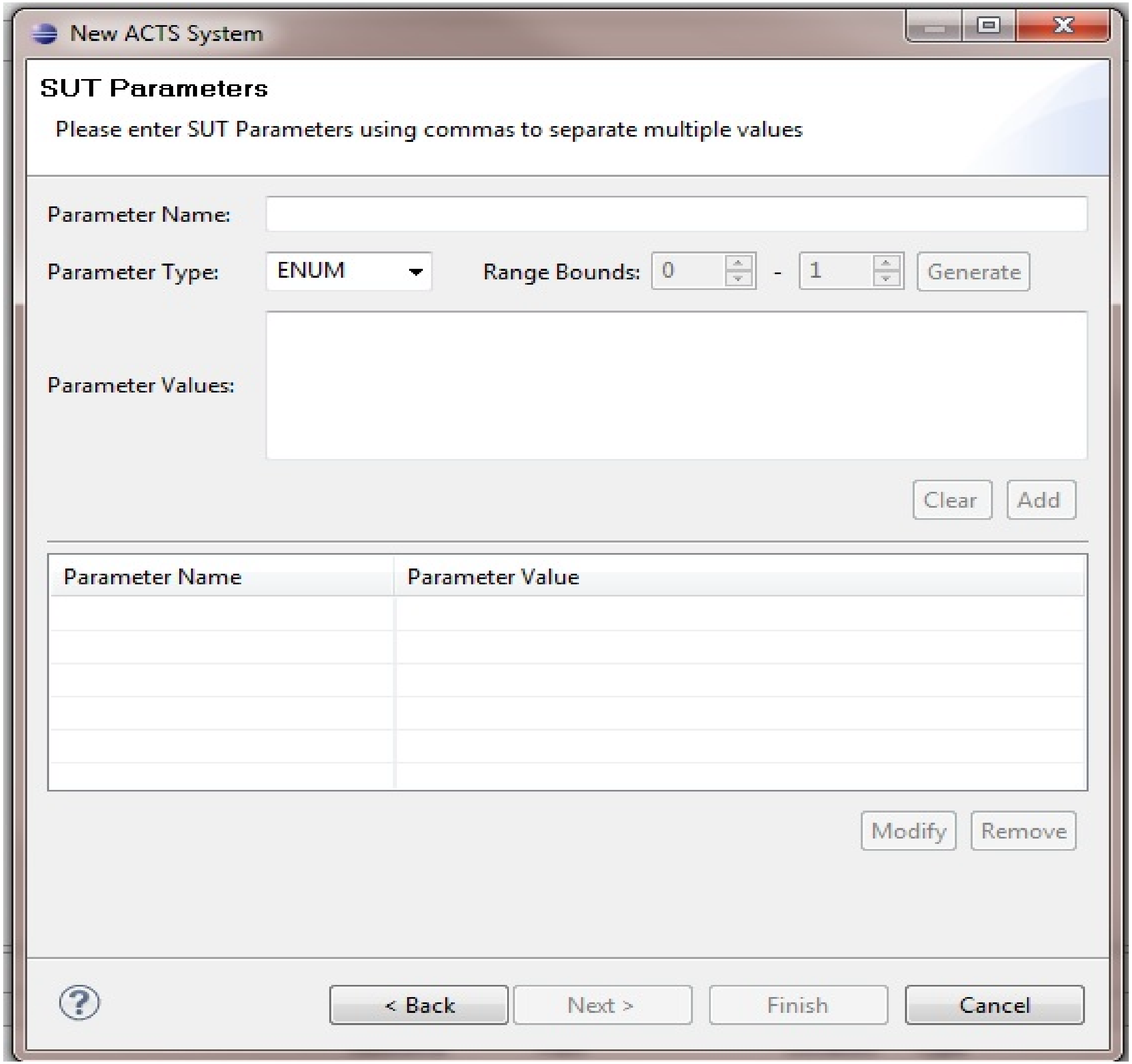
```
Degree of interaction coverage: 2
Number of parameters: 12
Maximum number of values per
parameter: 10
Number of configurations: 100
-----------------------------------
Configuration #1:

1 = Cur_Vertical_Sep=299
2 = High_Confidence=true
3 = Two_of_Three_Reports=true
4 = Own_Tracked_Alt=1
5 = Other_Tracked_Alt=1
6 = Own_Tracked_Alt_Rate=600
7 = Alt_Layer_Value=0
8 = Up_Separation=0
9 = Down_Separation=0
10 = Other_RAC=NO_INTENT
11 = Other_Capability=TCAS_CA
12 = Climb_Inhibit=true
```

Eclipse Plugin for ACTS



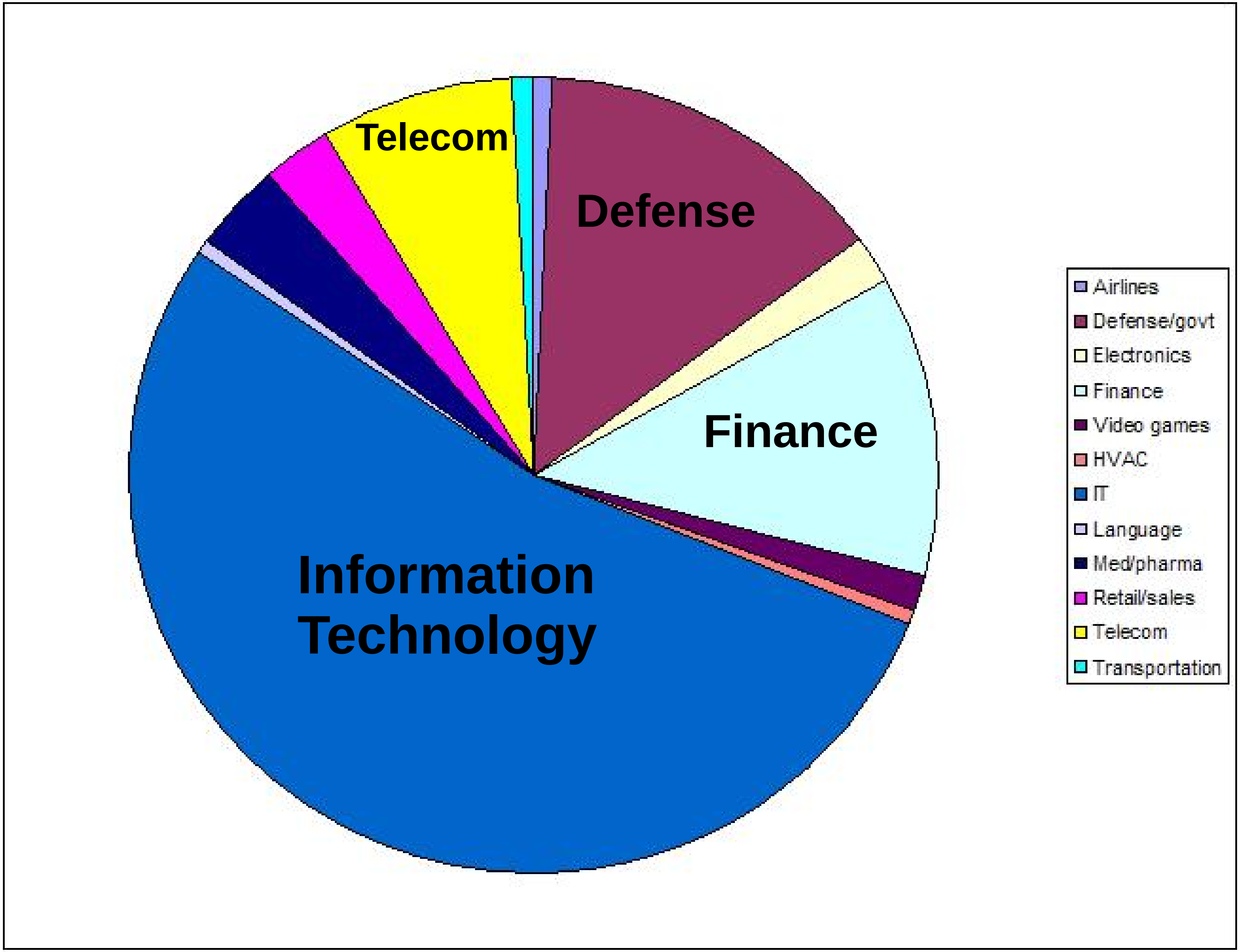
Work in progress

Eclipse Plugin for ACTS



Defining parameters and values

ACTS Users


Telecom
Defense
Finance
Information Technology
Airlines
Defense/govt
Electronics
Finance
Video games
HVAC
IT
Language
Med/pharma
Retail/sales
Telecom
Transportation

Tutorial Overview

1. Why are we doing this?
2. What is combinatorial testing?
3. How is it used and how long does it take?
4. What tools are available?
5. **What's next?**

Combinatorial Coverage Measurement

Tests	Variables			
	a	**b**	**c**	**d**
1	0	0	0	0
2	0	1	1	0
3	1	0	0	1
4	0	1	1	1
5	0	1	0	1
6	1	0	1	1
7	1	0	1	0
8	0	1	0	0

Variable pairs	Variable-value combinations covered	Coverage
ab	00, 01, 10	.75
ac	00, 01, 10	.75
ad	00, 01, 11	.75
bc	00, 11	.50
bd	00, 01, 10, 11	1.0
cd	00, 01, 10, 11	1.0

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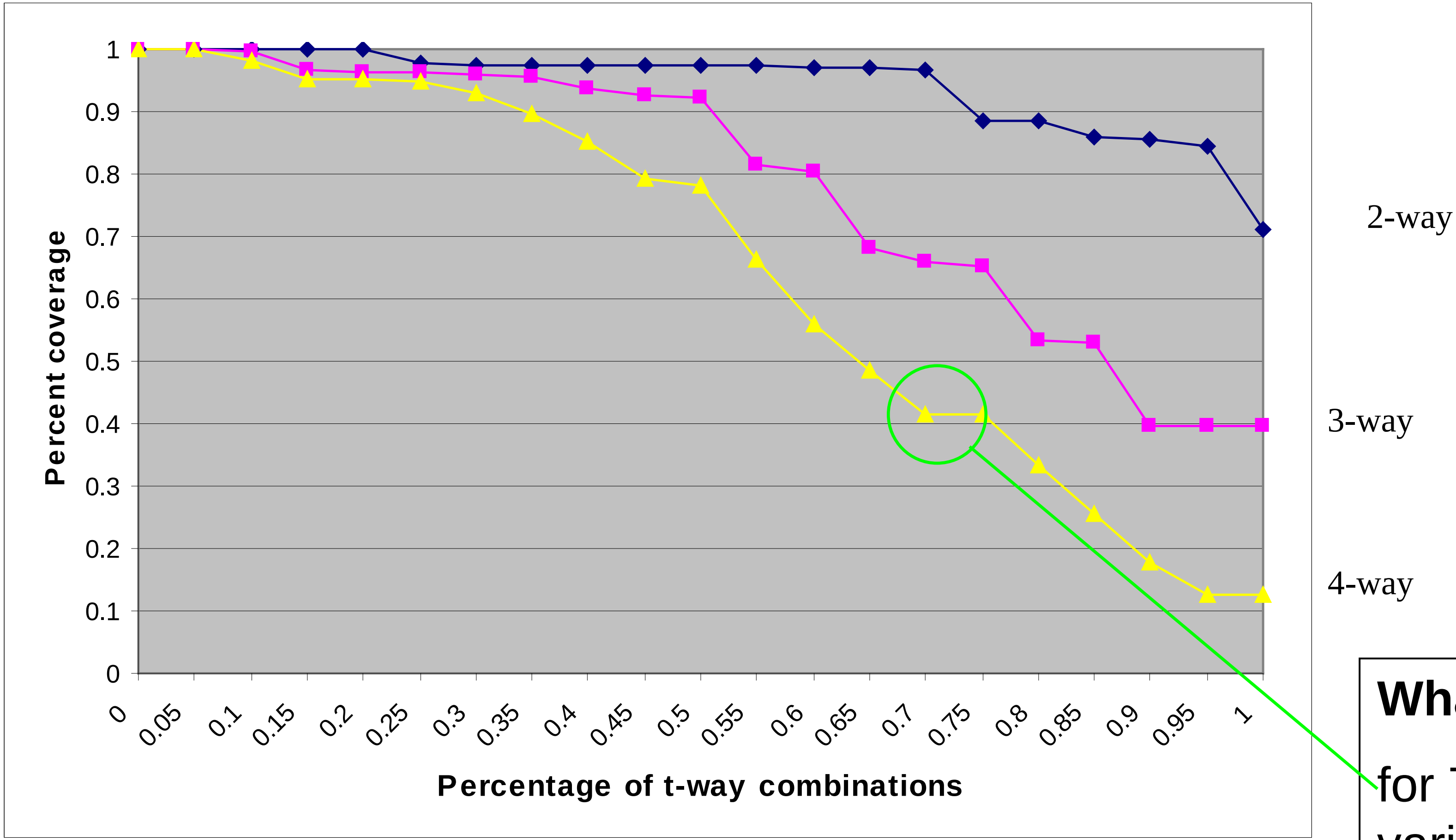
Combinatorial Coverage Measurement



Configuration coverage for $2^{79}3^{1}4^{1}6^{1}9^{1}$ inputs.

What this means:

for 70% of 4-way variable combinations, tests cover at least 40% of variable-value configurations

- Measure coverage provided by existing test sets
- Compare across methodologies

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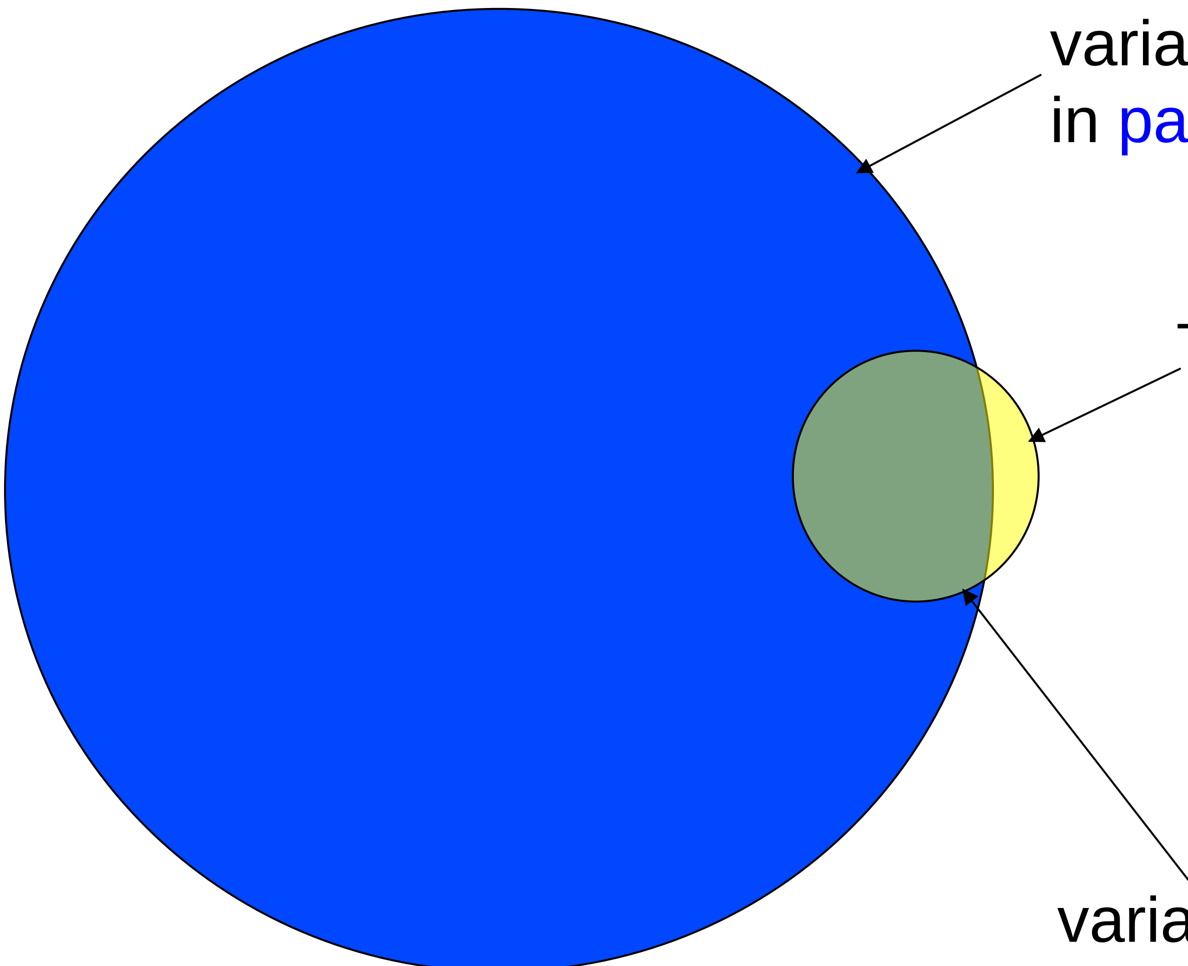
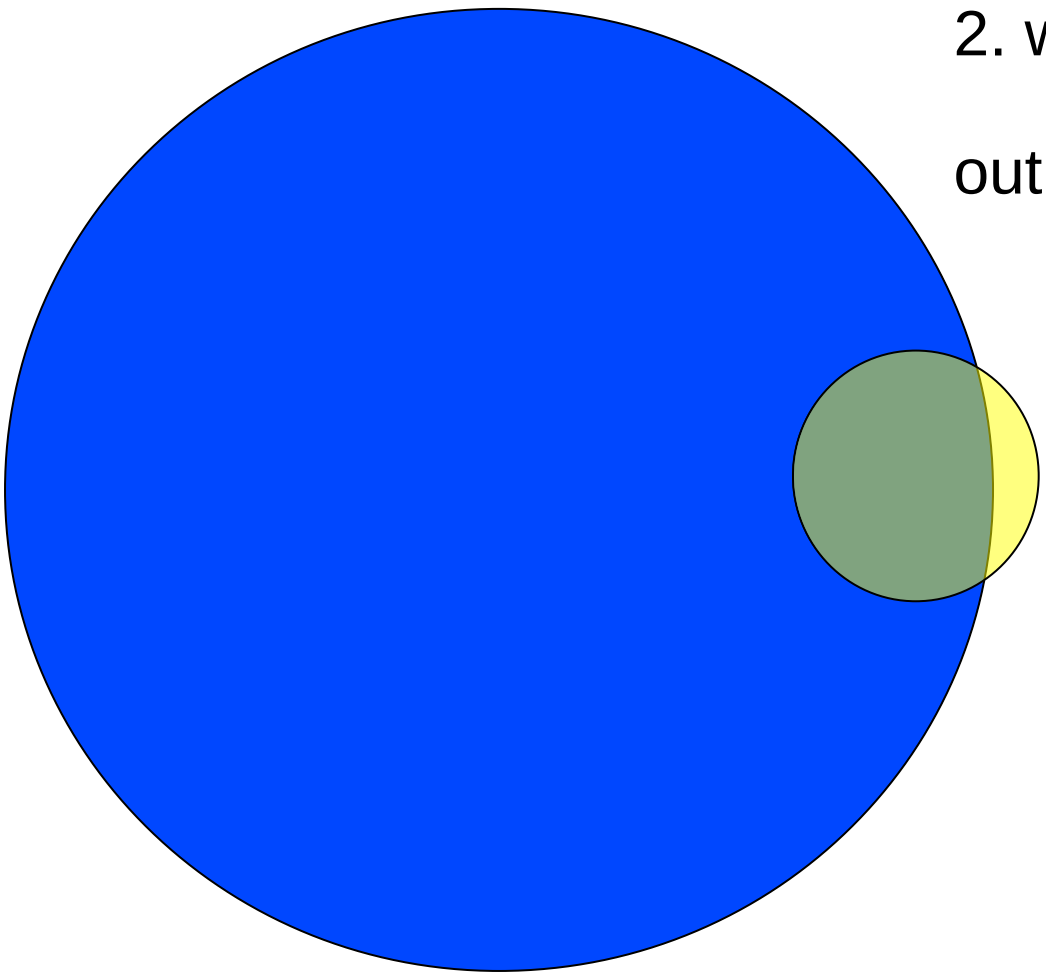
Fault location

Given: a set of tests that the SUT fails, which combinations of variables/values triggered the failure?



Fault location – what's the problem?



If they're in failing set but not in passing set:

1. which ones triggered the failure?
2. which ones don't matter?

out of $v^t\binom{n}{t}$ combinations

Example:
30 variables, 5 values each
= 445,331,250
5-way combinations

142,506 combinations
in each test

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Conclusions

- Empirical research suggests that all software failures caused by interaction of few parameters
- Combinatorial testing can exercise all t-way combinations of parameter values in a very tiny fraction of the time needed for exhaustive testing
- New algorithms and faster processors make large-scale combinatorial testing possible
- Project could produce better quality testing at lower cost for US industry and government
- Beta release of tools available, to be open source
- New public catalog of covering arrays

Future directions

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Real-world examples will help answer these questions

What kinds of software does it work best on?

What kinds of errors does it miss?

- **Other applications:**
  - Modelling and simulation
    - Testing the simulation
    - Finding interesting combinations:

      performance problems, denial of service attacks
- Maybe biotech applications. Others?

Please contact us if you are interested!

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(Or just search “combinatorial testing”. We’re #1!)