Falcon

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What is Falcon?

- Falcon stands for

  Fast Fourier lattice-based compact signatures over NTRU

- Falcon is a:
  - Signature scheme
  - Based on the GPV framework [GPV08]
  - Relying on NTRU lattices [HHGP+03]

- The main design principle:

  Compactness: to minimize $|\text{pk}| + |\text{sig}|$

Falcon in a Nutshell

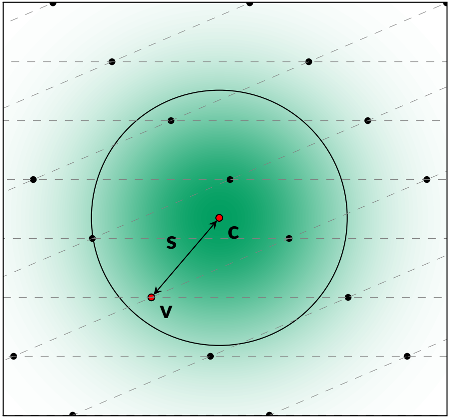
We work over the cyclotomic ring $\mathcal{R} = \mathbb{Z}_q[x]/(x^n + 1)$.

- **Keygen()**
  1. Generate matrices **A**, **B** with coefficients in $\mathcal{R}$ such that
     - **BA** $= 0$
     - **B** has small coefficients
  2. pk $\leftarrow$ **A**
  3. sk $\leftarrow$ **B**

- **Sign(m,sk)**
  1. Compute **c** such that **cA** $= H(\mathrm{m})$
  2. **v** $\leftarrow$ "a vector in the lattice $\Lambda(\mathbf{B})$, close to **c**"
  3. **s** $\leftarrow$ **c** $-$ **v**

  The signature sig is **s** $= (s_1, s_2)$

$\Rightarrow$



- **Verify(m,pk sig)**
  Accept iff:
  1. **s** is short
  2. **sA** $= H(\mathrm{m})$

Parameters and performances

NIST level	n	q	$\|pk\|$ (bytes)	$\|sig\|$ (bytes)	Sign/sec.	Verify/sec.
1	512	$12 \cdot 1024 + 1$	897	618	6082	37175
4-5	1024	$12 \cdot 1024 + 1$	1793	1233	3073	17697

Timings measured on an Intel Skylake @ 3.3Ghz.

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A few remarks:

- Falcon is the most compact of *all post-quantum signature schemes*
- Falcon is also quite fast
- Sign is the most delicate part to implement (*Fast Fourier Sampling*)
- Falcon includes a third set of parameters, which might be discarded in the future

Timings measured on an Intel Skylake @ 3.3Ghz.

Modes of operation

Falcon offers a few modes of operation:

Mode	**Classical**	**Message-recovery**	**Key-recovery** New!
pk	pk $= h$	pk $= h$	pk $= H(h)$
sig	sig $= s_2$	sig $= (s_1, s_2)$	sig $= (s_1, s_2)$
Verify	Recover s_1 from m and s_2. Accept iff $\|(s_1, s_2)\|$ is small.	Extract m from sig, using techniques from [dPLP16]. Accept iff $\|(s_1, s_2)\|$ is small.	Compute pk′ from m and sig. Accept iff $\|(s_1, s_2)\|$ is small and pk $=$ pk′.
Advantage	Simple, balanced.	Embed up to $n \log q$ bits of m in the signature.	Minimizes $\|\text{pk}\|$, *and* h may be recovered from one signature.
$\|$pk$\|$ (LV5)	1793	1793	40
$\|$sig$\|$ (LV5)	1233	706*	2466

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Mode	**Classical**	**Message-recovery**	**Key-recovery** New!
pk sig	pk $= h$ sig $= s_2$	pk $= h$ sig $= (s_1, s_2)$	pk $= H(h)$ sig $= (s_1, s_2)$
Verify	Recover s_1 from m and s_2. Accept iff $\|(s_1, s_2)\|$ is small.	Extract m from sig, using techniques from [dPLP16]. Accept iff $\|(s_1, s_2)\|$ is small.	Compute pk$'$ from m and sig. Accept iff $\|(s_1, s_2)\|$ is small and pk $=$ pk$'$.
Advantage	Simple, balanced.	Embed up to $n \log q$ bits of m in the signature.	Minimizes $\vert$pk$\vert$, *and* h may be recovered from one signature.
$\vert$pk$\vert$ (LV5) $\vert$sig$\vert$ (LV5)	1793 1233	1793 706*	40 2466

Falcon can also be turned into a full-fledged **identity-based encryption scheme** [DLP14], and more.

Possible attacks

Key recovery

- Lattice reduction (the most effective)
- Combinatorial attacks [HG07, BKW00] $\Rightarrow$ not a threat AFAWK (*as far as we know*)
- *Overstretched NTRU* attacks [ABD16, CJL16, KF17] $\Rightarrow$ not a threat AFAWK
- Other algebraic attacks? [CDPR16, CDW17] $\Rightarrow$ not a threat AFAWK
- Learning attacks [NR06, DN12] $\Rightarrow$ not a threat AFAWK

Forgery

- Lattice reduction + enumeration

Side-channel attacks

- Remains to be studied

Key takeaways

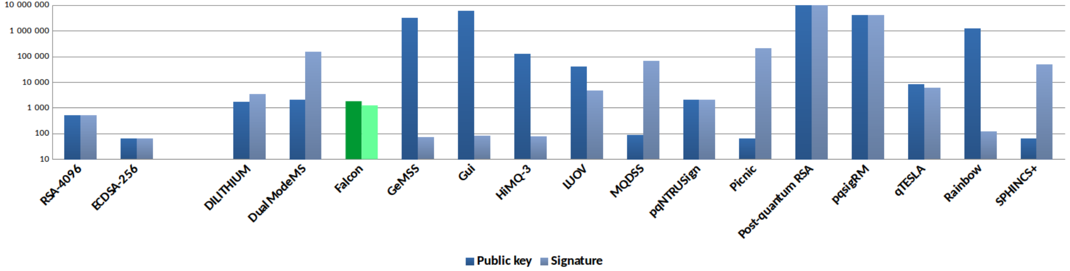
Advantages:

- ✓ Compact
- ✓ Fast
- ✓ GPV framework proven secure in the ROM [GPV08] and QROM [BDF+11]
- ✓ Several modes of operations

Limitations:

- ⚠ Non-trivial to understand and implement
- ⚠ Floating-point arithmetic
- ⚠ Side-channel resistance?

Comparison with other signature schemes at NIST level 5 (sizes in bytes):



Resources

Resources can be found on our website: `https://falcon-sign.info/`

- Specification
- Reference implementation in C
- **New!** Additional implementation in Python
- **New!** Slides presenting various aspects of Falcon

Thank you for your attention!

Thanks to Fabrice Mouhartem for the Falcon origami!

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