MQDSS

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In a nutshell..

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  - Fiat-Shamir transform
- Loose reduction from (only!) $\mathcal{MQ}$ problem
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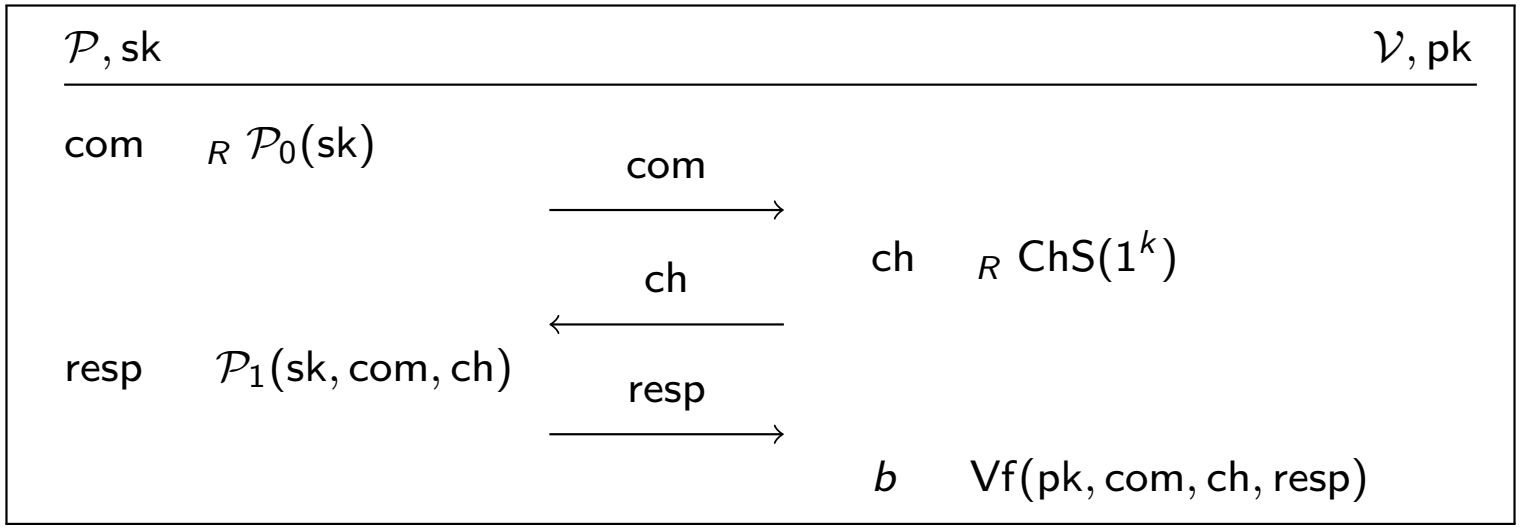
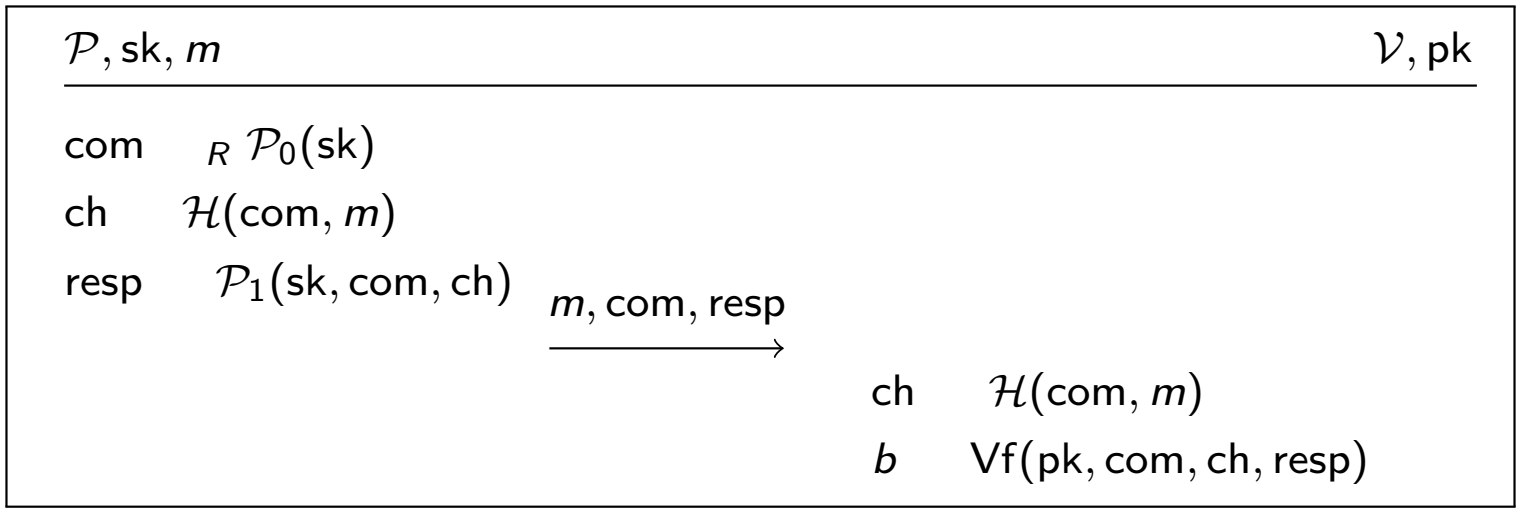
Canonical Identification Schemes



Informally:

1. Prover commits to some (randomized) value derived from sk
2. Verifier picks a challenge 'ch'
3. Prover computes response 'resp'
4. Verifier checks if response matches challenge

Fiat-Shamir transform



- Unpredictably derive ch from m and com
- Repeat to compensate for adversary 'guessing right'

Sakumoto-Shirai-Hiwatari 5-pass IDS [SSH11]

$\mathcal{P}\leftarrow(\mathbf{F}, \mathbf{v}, \mathbf{s})$		$\mathcal{V}\leftarrow(\mathbf{F}, \mathbf{v})$
$\mathbf{r}_0, \mathbf{t}_0 \ {}_R\leftarrow \mathbb{F}_q^n, \mathbf{e}_0 \ {}_R\leftarrow \mathbb{F}_q^m$		
$\mathbf{r}_1 \leftarrow \mathbf{s} - \mathbf{r}_0$		
$c_0 \leftarrow \mathit{Com}(\mathbf{r}_0, \mathbf{t}_0, \mathbf{e}_0)$		
$c_1 \leftarrow \mathit{Com}(\mathbf{r}_1, \mathbf{G}(\mathbf{t}_0, \mathbf{r}_1) + \mathbf{e}_0)$	$\xrightarrow{(c_0, c_1)}$	
	$\xleftarrow{\alpha}$	$\alpha \ {}_R\leftarrow \mathbb{F}_q$
$\mathbf{t}_1 \leftarrow \alpha\mathbf{r}_0 - \mathbf{t}_0$		
$\mathbf{e}_1 \leftarrow \alpha\mathbf{F}(\mathbf{r}_0) - \mathbf{e}_0$	$\xrightarrow{\mathsf{resp}_1 = (\mathbf{t}_1, \mathbf{e}_1)}$	
	$\xleftarrow{\mathsf{ch}_2}$	$\mathsf{ch}_2 \ {}_R\leftarrow \{0,1\}$
If $\mathsf{ch}_2 = 0$, $\mathsf{resp}_2 \leftarrow \mathbf{r}_0$		
Else $\mathsf{resp}_2 \leftarrow \mathbf{r}_1$	$\xrightarrow{\mathsf{resp}_2}$	
		If $\mathsf{ch}_2 = 0$, Parse $\mathsf{resp}_2 = \mathbf{r}_0$, check
		$c_0 \stackrel{?}{=} \mathit{Com}(\mathbf{r}_0, \alpha\mathbf{r}_0 - \mathbf{t}_1, \alpha\mathbf{F}(\mathbf{r}_0) - \mathbf{e}_1)$
		Else Parse $\mathsf{resp}_2 = \mathbf{r}_1$, check
		$c_1 \stackrel{?}{=} \mathit{Com}(\mathbf{r}_1, \alpha(\mathbf{v} - \mathbf{F}(\mathbf{r}_1)) - \mathbf{G}(\mathbf{t}_1, \mathbf{r}_1) - \mathbf{e}_1)$

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(evaluating $\mathbf{G} \approx$ evaluating $\mathbf{F}$)

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  - Expand $\mathcal{S}_F$ to $\mathbf{F}$, compute $\mathbf{pk} = \mathbf{F}(\mathbf{sk})$ $\Rightarrow (\mathcal{S}_F, \mathbf{pk})$

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  - Perform r parallel rounds of transformed IDS
    - Sample r vectors $\mathbf{r}$, $\mathbf{t}$ and $\mathbf{e}$
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    - $2r$ $\mathcal{MQ}$ evaluations

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- Parameters: k, n, m, $\mathbb{F}_q$, Com, hash functions, PRGs

Hardness of $\mathcal{MQ}$

- Assume $m \geqslant n$, $m \in \mathcal{O}(n)$
- HybridF5 [BFS15], BooleanSolve [BFSS13], Crossbred [JV17]
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- Analyze both classically and using Grover
  - Classical gates, quantum gates, circuit depth

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- SHAKE-256 for commitments / hashes
  - Match output length to k

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Magali Bardet, Jean-Charles Faugère, and Bruno Salvy.

On the complexity of the F5 Gröbner basis algorithm.

Journal of Symbolic Computation, 70(Supplement C):49 – 70, 2015.

https://arxiv.org/pdf/1312.1655.pdf.

Magali Bardet, Jean-Charles Faugère, Bruno Salvy, and Pierre-Jean Spaenlehauer.

On the complexity of solving quadratic boolean systems.

Journal of Complexity, 29(1):53–75, 2013.

www-polsys.lip6.fr/~jcf/Papers/BFSS12.pdf.

Ming-Shing Chen, Andreas Hülsing, Joost Rijneveld, Simona Samardjiska, and Peter Schwabe.

From 5-pass $\mathcal{MQ}$-based identification to $\mathcal{MQ}$-based signatures.

In Jung Hee Cheon and Tsuyoshi Takagi, editors, *Advances in Cryptology – ASIACRYPT 2016*, volume 10032 of *LNCS*, pages 135–165. Springer, 2016.

http://eprint.iacr.org/2016/708.

References II

Antoine Joux and Vanessa Vitse.

A crossbred algorithm for solving boolean polynomial systems.

Cryptology ePrint Archive, Report 2017/372, 2017.

http://eprint.iacr.org/2017/372.

Koichi Sakumoto, Taizo Shirai, and Harunaga Hiwatari.

Public-key identification schemes based on multivariate quadratic polynomials.

In Phillip Rogaway, editor, *Advances in Cryptology – CRYPTO 2011*, volume 6841 of *LNCS*, pages 706–723. Springer, 2011.

https://www.iacr.org/archive/crypto2011/68410703/68410703.pdf.