NTRUEncrypt and pqNTRUSign

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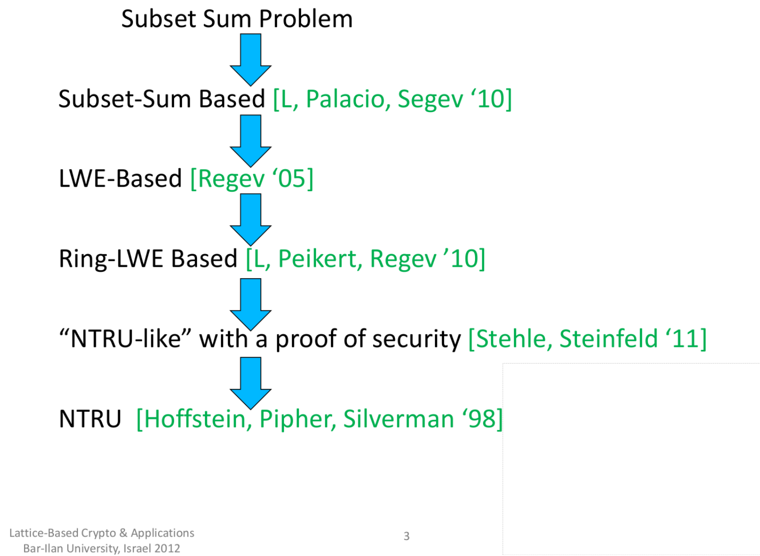
NTRU

One of the first lattice based cryptosystems; 20 years old.

Through the years we heard

- It doesn't have security proof!
- It only focuses on practicality!
- It uses an ad-hoc ring!
- It uses a sparse trinary polynomial!
- It has decryption errors!



How lattice based encryption should have been developed - Vadim Lyubashevsky

An alternate universe

- What if NTRU was not proposed 22 years ago?

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- We wouldn't have seen the failure of NTRUSign.
- Luckily, we still have FALCON.

An alternate universe

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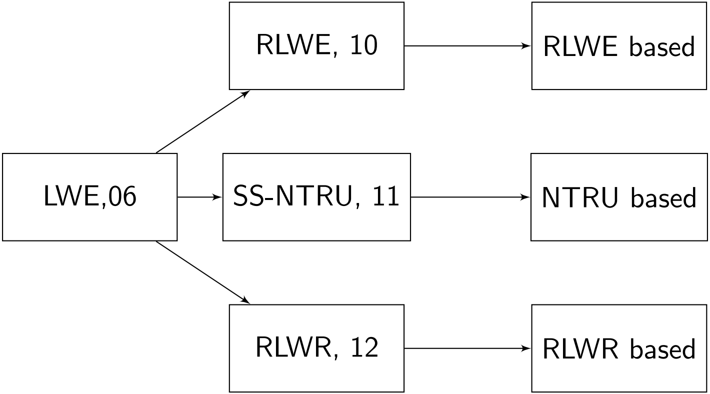

An alternate universe

What if NTRU was not proposed 22 years ago, but now?

Earth 1

- It doesn't have security proof!
- It only focuses on practicality!
- It uses an ad-hoc ring!
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Earth 2

- It stems from a provable secure design;
- and is practical!
- Ring is not restricted to $x^{2^p} + 1$!
- It uses a sparse trinary polynomial!
- Decrypt errors are negligible!

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NTRU APPEARS more popular if it wasn't invented 22 years ago!

What about (provable) security?

- Just find parameters secure from BKZ (+ sieving)
  - We did it with (R)-LWE based KEX anyway ...

An alternate universe

What if NTRU was not proposed 22 years ago?

Let's do a clean slate comparison

- NTRU uses a trapdoored lattice; RLWE/RLWR uses a generic lattice
- NTRU relies on uSVP - unique shortest vector is sparse trinary;
- Practical RLWE/RLWR rely on BDD - distance vector MAY be sparse trinary;
- The rest are all tunable parameters (in practice)
  - Both can be instantiated with the same ring; same noise distribution

Fundamental difference: Trapdoor

- NTRU lattices are more useful in PKE and Signatures
- RLWE/RLWR have the advantages in KEX

NTRU lattice

NTRU assumption

- Decisional: given two small ring elements f and g; it is hard to distinguish $h = f/g$ from a uniformly random ring element;
- Computational: given h, find f and g.

NTRU lattice with unique shortest vectors (g, f)

$$\begin{bmatrix} qI_N & 0 \\ H & I_N \end{bmatrix} := \begin{bmatrix} q & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & q & \dots & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & q & 0 & 0 & \dots & 0 \\ h_0 & h_1 & \dots & h_{N-1} & 1 & 0 & \dots & 0 \\ h_{N-1} & h_0 & \dots & h_{N-2} & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ h_1 & h_2 & \dots & h_0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

NTRUEncrypt

A CCA-2 secure encryption scheme based on NTRU assumption

Enc $(h = g/f, p = 3, \mathcal{R}, m \in \{-1, 0, 1\}^N)$

- Find a random ring element r;
- Compute $e = p \times r \cdot h + m$;

Dec $(f, p = 3, \mathcal{R}, e)$

- Compute $c = e \cdot f = p \times r \cdot g + m \cdot f$;
- Reduce $c \bmod p = m \cdot f \bmod p$
- Recover $m = c \cdot f^{-1} \bmod p$

NTRUEncrypt

A CCA-2 secure encryption scheme based on NTRU assumption

Enc $(h = g/f, p = 3, f \equiv 1 \bmod p, \mathcal{R}, m \in \{-1, 0, 1\}^N)$

- Find a random ring element r;
- Compute $e = p \times r \cdot h + m$;

Dec $(f \equiv 1 \bmod p, p = 3, \mathcal{R}, e)$

- Compute $c = e \cdot f = p \times r \cdot g + m \cdot f$;
- Reduce $c \bmod p = m \cdot f \bmod p = m$

NTRUEncrypt

A CCA-2 secure encryption scheme based on NTRU assumption

Enc $(h = g/f, p = 3, f \equiv 1 \bmod p, \mathcal{R}, m \in \{-1, 0, 1\}^k)$

- Find a random string b; $r = \text{hash}(h|b)$
- $m' = r \otimes \langle m|b \rangle$
- Compute $e = p \times r \cdot h + m'$;

Dec $(f \equiv 1 \bmod p, g, p = 3, \mathcal{R}, e)$

- Compute $c = e \cdot f = p \times r \cdot g + m' \cdot f$;
- Reduce $c \bmod p = m' \cdot f \bmod p = m'$
- Compute $r' = p^{-1} \times (c - m' \cdot f) \cdot g^{-1}$
- Extract m, b from $m' \otimes r'$, compute $r = \text{hash}(h|b)$;
- Output m if $r = r'$.

Modular Lattice Signatures

The core idea

- Given a lattice $\mathcal{L}$ with a trapdoor T, a message m, find a vector v
  - $v \in \mathcal{L}$
  - $v \equiv \text{hash}(m) \bmod p$

- Can be instantiated via any trapdoored lattice
  - SIS, R-SIS, etc
- pqNTRUSign is an efficient instantiation using the NTRU lattice

pqNTRUSign

Sign $(f, g, h = g/f, p = 3, \mathcal{R}, m)$

- Hash message into a "mod p" vector $\langle v_p, u_p \rangle = hash(m|h)$
- Repeat with rejection sampling:
  - Sample v_0 from certain distribution; compute $v_1 = p \times v_0 + v_p$
  - Find a random lattice vector $\langle v_1, u_1 \rangle = v_1 \cdot \langle I, h \rangle$
    - "v-side" meets the congruent condition.
  - Micro-adjust "u-side" using trapdoor f and g
    - Compute $a = (u_1 - u_p) \cdot g^{-1} \bmod p$
    - Compute $\langle v_2, u_2 \rangle = a \cdot \langle p \times f, g \rangle$
    - Compute $\langle v, u \rangle = \langle v_1, u_1 \rangle + \langle v_2, u_2 \rangle$
- Output v as signature

Remark

$$v = v_1 + v_2 = (p \times v_0 + v_p) + p \times a \cdot f = p \times (v_0 + a \cdot f) + v_p$$

pqNTRUSign

Verify $(h, p = 3, \mathcal{R}, m, v)$

- Hash message into a "mod p" vector $\langle v_p, u_p \rangle = hash(m|h)$
- Reconstruct the lattice vector $\langle v, u \rangle = v \cdot \langle I, h \rangle$
- Check $\langle v_p, u_p \rangle = hash(m|h)$

pqNTRUSign

- Public key security: recover f and g from h;
- Forgery: as hard as solving an approx.-SVP in an intersected lattice;
- Transcript security - achieved via rejection sampling.

Forgery

- Forgery: as hard as solving an approx.-SVP in an intersected set: $\mathcal{L}' := \mathcal{L}_h \cap (p\mathbb{Z}^{2N} + \langle v_p, u_p \rangle)$

- $\det(\mathcal{L}_h \cap p\mathbb{Z}^{2N}) = p^{2N} q^N \quad \longrightarrow \quad$ Gaussian heuristic length $= \sqrt{\frac{p^2 qN}{\pi e}}$
- Target vector length $\| \langle v, u \rangle \| \leq \sqrt{2N}\frac{q}{2}$
- Approx.-SVP with root Hermite factor $\gamma = \sqrt{\frac{q\pi e}{2p^2}}^{\frac{1}{\dim}} = \left(\frac{q\pi e}{2p^2}\right)^{\frac{1}{4N}}$

Transcript Security and Rejection Sampling

Consider $b := v_0 + a \cdot f$

- "large" v_0 drawn from uniform or Gaussian;
- "small" a drawn from sparse trinary/binary;
- sparse trinary/binary f is the secret.

RS on b

- b follows certain publicly known distribution independent from f;
- for two secret keys f_1, f_2 and a signature b, one is not able to tell which key signs b.

Performance

Param	PK size	CTX size	KeyGen	Encryption	Decryption
ntrukem-743	8184 bits	8184 bits	1017 μs	140 μs	210 μs
ntrupke-743	8184 bits	8184 bits	990 μs	121 μs	195 μs

Table: NTRUEncrypt

Param	PK size	RSig size	KeyGen	Signing	Verifying
Gaussian-1024	16384 bits	$\approx$ 11264 bits	47.8 ms	120 ms	0.96 ms
Uniform-1024	16384 bits	16384 bits	48.9 ms	289 ms	0.97 ms

Table: pqNTRUSign

Feedback we have received so far

Bugs in the code

- mask function was incorrectly implemented for NTRUEncrypt with Gaussian secret
- Gauss sampler took smaller deviation than required for NTRUEncrypt with Gaussian secret
- Rejection sampling on ag is missing for pqNTRUSign

Mistakes in the algorithm

- Parameter for the bound of v-side was incorrect

Signature simulations

- Attacker learns more information on the lattice vs simulator
- Can be fixed via message randomization or deterministic signing.