NTRU Prime: Reducing Attack Surface at Low Cost

Daniel J. Bernstein, Chitchanok Chuengsatiansup,
Tanja Lange, and Christine van Vredendaal

Technische Universiteit Eindhoven

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Idea:

a field-based system that reduces (potential) attack surface, while still being fast and compact

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Whether paranoia, or valid concern; what can we do about it?

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- Choose prime p, set $F = x^p - x - 1$ (tool 4).
- Choose prime q such that F is irreducible mod q (tools 2, 3)
- NTRU Prime works over the field

$$(\mathbb{Z}/q)[x]/(x^p - x - 1),$$

- with Galois group S_p of size $p!$ (tool 5).

Streamlined NTRU Prime

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$$g = g_0 + \cdots + g_{p-1}x^{p-1} \text{ with } g_i \in \{-1, 0, 1\}$$

  No weight restriction on g, only size restriction on coefficients; g required to be invertible in $\mathcal{R}/3$.

- Pick t-small $f \in \mathcal{R}$

$$f = f_0 + \cdots + f_{p-1}x^{p-1} \text{ with } f_i \in \{-1, 0, 1\} \text{ and } \sum |f_i| = 2t$$

  Since $\mathcal{R}/q$ is a field, f is invertible.

- Compute public key $h = g/(3f)$ in $\mathcal{R}/q$.
- Private key is f and $1/g \in \mathcal{R}/3$.

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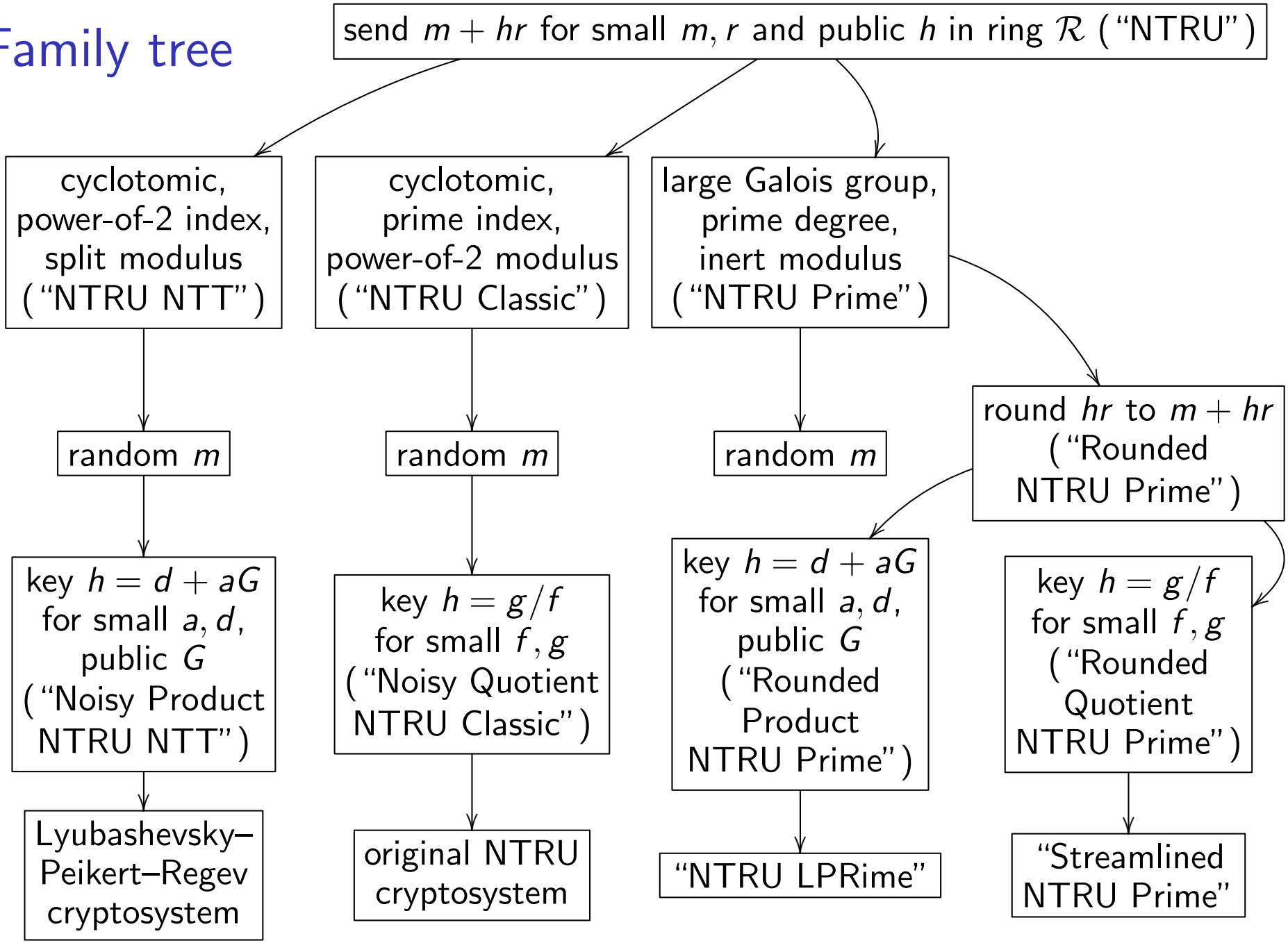
Streamlined NTRU Prime is a Key Encapsulation Mechanism (KEM), combine with Data Encapsulation Mechanism (DEM) to send messages.

Family tree


send $m + hr$ for small m, r and public h in ring $\mathcal{R}$ ("NTRU")
cyclotomic, power-of-2 index, split modulus ("NTRU NTT")
cyclotomic, prime index, power-of-2 modulus ("NTRU Classic")
large Galois group, prime degree, inert modulus ("NTRU Prime")
round hr to $m + hr$ ("Rounded NTRU Prime")
random m
random m
random m
key $h = d + aG$ for small a, d, public G ("Noisy Product NTRU NTT")
key $h = g/f$ for small f, g ("Noisy Quotient NTRU Classic")
key $h = d + aG$ for small a, d, public G ("Rounded Product NTRU Prime")
key $h = g/f$ for small f, g ("Rounded Quotient NTRU Prime")
Lyubashevsky–Peikert–Regev cryptosystem
original NTRU cryptosystem
"NTRU LPRime"
"Streamlined NTRU Prime"

Streamlined NTRU Prime Security I

- What we know so far:

	original NTRU	**New Hope**	**Streamlined NTRU Prime**
Polynomial P	$x^p - 1$	$x^p + 1$	$x^p - x - 1$
Degree p	prime	power of 2	prime
Modulus q	2^d	prime	prime
# factors of P in $\mathcal{R}/q$	> 1	p	1
# proper subfields	> 1	many	1
Every m encryptable	✗	✓	✓
No decryption failures	✗	✗	✓

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- Because of the last 2 ✓'s the analysis is simpler than that of original NTRU.

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- Streamlined NTRU Prime 4591^{761} and NTRU LPRime 4591^{761} both use $p = 761$ and $q = 4591$.
- These parameters lead to respectively 248 bits and 225 bits of pre-quantum security. Quantum computers will speed up by less than squareroot.

Performance

- The resulting sizes and Haswell speeds show that reducing the attack surface has very low cost:

Metric	**Streamlined NTRU Prime** 4591^{761}	**NTRU LPRime** 4591^{761}
Public-key size	1218 bytes	1047 bytes
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- This beats cycle counts of New Hope, Kyber and Curve25519
- It does not outperform NTRU KEM (HRSS '17) (48646 cycles enc, 67338 cycles dec)
- NTRU KEM ciphertexts are longer (1281 bytes vs. 1047 bytes)

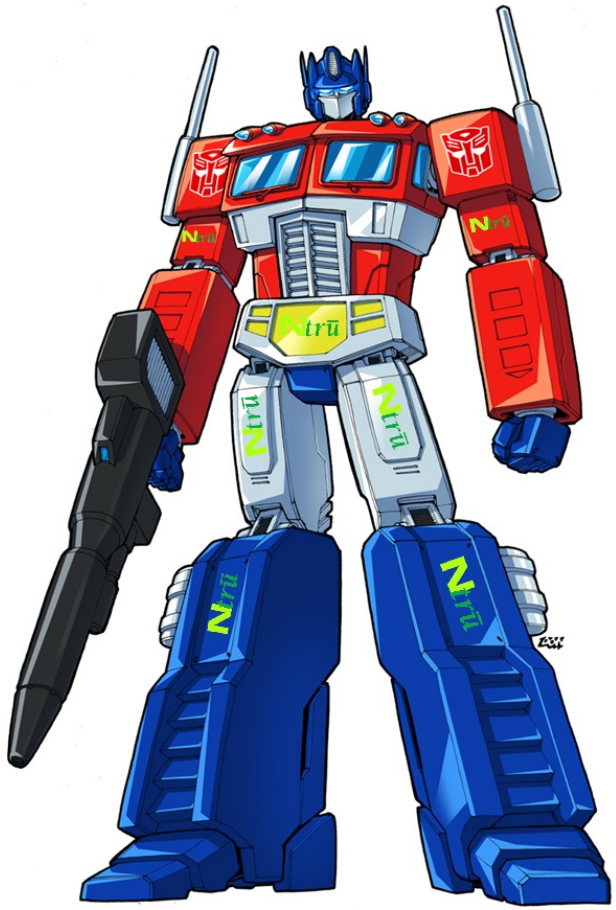

Figure: (Optimus) NTRU Prime.

Thank you!