Quantum Resistant Public Key Encryption Scheme RLCE

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Outline

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McEliece Scheme

McEliece Scheme (1978)

`Mc.KeySetup`: An $(n, k, 2t+1)$ linear Goppa code $\mathcal{C}$ with $k \times n$ generator matrix G_s. Public key: $G = SG_sP$. Private key: G_s Where S is random and P is permutation.

`Mc.Enc`$(G, \mathbf{m}, \mathbf{e})$. For a message $\mathbf{m} \in \{0, 1\}^k$, choose a random vector $\mathbf{e} \in \{0, 1\}^n$ of weight t. The cipher text $\mathbf{c} = \mathbf{m}G + \mathbf{e}$

`Mc.Dec`$(S, G_s, P, \mathbf{c})$. For a received ciphertext $\mathbf{c}$, first compute $\mathbf{c}' = \mathbf{c}P^{-1} = \mathbf{m}SG$. Next use an error-correction algorithm to recover $\mathbf{m}' = \mathbf{m}S$ and compute the message $\mathbf{m}$ as $\mathbf{m} = \mathbf{m}'S^{-1}$.

McEliece Security

- Broken ones: Niederreiter's scheme with Generalized Reed-Solomon Code Broken
- Broken ones: Wild Goppa code based McEliece, GRS-McEliece with random columns
- Unbroken ones: Original McEliece, MDPC/LDPC McEliece, Wang's RLCE

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RLCE Key setup

`RLCE.KeySetup`. Let G_s be a $k \times n$ generator matrix for an $[n, k, d]$ linear code $\mathcal{C}$correcting at least t errors and $w \leq n$. Let $G_s P_1 = [\mathbf{g}_0, \cdots, \mathbf{g}_{n-1}]$ for a random permutation P_1

1. Let $G_1 = [\mathbf{g}_0, \cdots, \mathbf{g}_{n-w}, \mathbf{r}_0, \cdots, \mathbf{g}_{n-1}, \mathbf{r}_{w-1}]$ be a $k \times (n + w)$ matrix where $\mathbf{r}_i \in GF(q)^k$ are random
2. Let $A_i \in GF(q)^{2\times 2}$ be random 2×2 matrices. Let $A = \text{diag}[I_{n-w}, A_0, \cdots, A_{w-1}]$ be an $(n + w) \times (n + w)$ non-singular matrix.
3. The public key: $k \times (n + w)$ matrix $G = SG_1AP_2$ and the private key: (S, G_s, P_1, P_2, A) where S is random $k \times k$ matrix and P_2 is a permutation.

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RLCE Encryption/Decryption

`RLCE.Enc`$(G, \mathbf{m}, \mathbf{e})$. For a message $\mathbf{m} \in GF(q)^k$, choose $\mathbf{e} \in GF(q)^{n+w}$ of weight at most t. The cipher: $\mathbf{c} = \mathbf{m}G + \mathbf{e}$.

`RLCE.Dec`$(S, G_s, P_1, P_2, A, \mathbf{c})$. For a cipher text $\mathbf{c}$, compute

$$\mathbf{c}P_2^{-1}A^{-1} = \mathbf{m}SG_1 + \mathbf{e}P_2^{-1}A^{-1} = [c'_0, \ldots, c'_{n+w-1}].$$

Let $\mathbf{c}' = [c'_0, c'_1, \cdots, c'_{n-w}, c'_{n-w+2}, \cdots, c'_{n+w-2}] \in GF(q)^n$. Then $\mathbf{c}'P_1^{-1} = \mathbf{m}SG_s + \mathbf{e}'$ for some $\mathbf{e}' \in GF(q)^n$ of weight at most t. Using an efficient decoding algorithm, one can recover $\mathbf{m}SG_s$ from $\mathbf{c}'P_1^{-1}$. Let D be a $k \times k$ inverse matrix of SG'_s where G'_s is the first k columns of G_s. Then $\mathbf{m} = \mathbf{c}_1 D$ where $\mathbf{c}_1$ is the first k elements of $\mathbf{m}SG_s$.

Why RLCE?

- The problem of decoding random linear codes is **NP**-hard
- Though challenging to show that decoding RLCE is **NP**-hard, the mixed random columns could hide all structures of underlying linear code
- Goppa-McEliece assumes Goppa codes behave like random codes while RLCE does not requires such kind of assumption
- Other McEliece variants are based on stronger assumption that certain structured codes are hard to decode.
- Reed-Solomon codes has wide industry experience
- **Limitation**: RLCE public key sizes are larger though smaller than Goppa-McEliece

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Systematic RLCE

- Decryption for systematic RLCE could be more efficient.
- In the RLCE, one recovers $\mathbf{m}SG_s$ first.
- Let $\mathbf{m}SG_sP_1 = (d_0, \cdots, d_{n-1})$ and $\mathbf{c}_d = (d'_0, \cdots, d'_{n+w}) = (d_0, d_1, \cdots, d_{n-w}, \perp, d_{n-w+1}, \perp, \cdots, d_{n-1}, \perp)P_2$ be a length $n + w$ vector.
- For each $i < k$ such that $d'_i = d_j$ for some $j < n - w$, we have $m_i = d_j$. Let

$$I_R = \{i : m_i \text{ is recovered via } \mathbf{m}SG_s\} \text{ and } \bar{I}_R = \{0, \cdots, k-1\} \setminus I_R.$$

Assume that $|\bar{I}_R| = u$. It suffices to recover the remaining u message symbols m_i with $i \in \bar{I}_R$.

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Assume that $|\bar{I}_R| = u$. It suffices to recover the remaining u message symbols m_i with $i \in \bar{I}_R$.

Decoding algorith 1

The message symbols with indices in $\bar{I}_R$ could be recovered by solving the linear equation system

$$\mathbf{m}\left[\mathbf{g}_{i_0},\cdots,\mathbf{g}_{i_{u-1}}\right]=[d'_{i_0},\cdots,d'_{i_{u-1}}]$$

where $\mathbf{g}_{i_0},\cdots,\mathbf{g}_{i_{u-1}}$ are the corresponding columns in the public key. Choose P such that $\mathbf{m}P=(\mathbf{m}_{I_R},\mathbf{m}_{\bar{I}_R})$. Then

$$(\mathbf{m}_{I_R},\mathbf{m}_{\bar{I}_R})P^{-1}\left[\mathbf{g}_{i_0},\cdots,\mathbf{g}_{i_{u-1}}\right]=[d'_{i_0},\cdots,d'_{i_{u-1}}]$$

Let $P^{-1}\left[\mathbf{g}_{i_0},\cdots,\mathbf{g}_{i_{u-1}}\right]=\begin{pmatrix}V\\W\end{pmatrix}$. Then

$$\mathbf{m}_{\bar{I}_R}W=[d'_{i_0},\cdots,d'_{i_{u-1}}]-\mathbf{m}_{I_R}V.$$

$$\mathbf{m}_{\bar{I}_R}=\left([d'_{i_0},\cdots,d'_{i_{u-1}}]-\mathbf{m}_{I_R}V\right)W^{-1}.$$

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Defeating side-channel attacks

For the decoding algorithms 1, the value u is dependent on the choice of the private permutation P_2. Though the leakage of the size of u is not sufficient for the adversary to recover P_2 or to carry out other attacks against RLCE scheme, this kind of side-channel information leakage could be easily defeated by requiring u be smaller than u_0 in the following Table for selected P_2.

RLCE ID	0	1	2	3	4	5	6
u_0	200	123	303	190	482	309	7

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Two groups of parameters

- Group 1: $w < n - w$: This group is insecure due to the recent analysis by Alain Couvreur, Matthieu Lequesne, and Jean-Pierre Till
- Group 2: $w = n - k$: This one should be used

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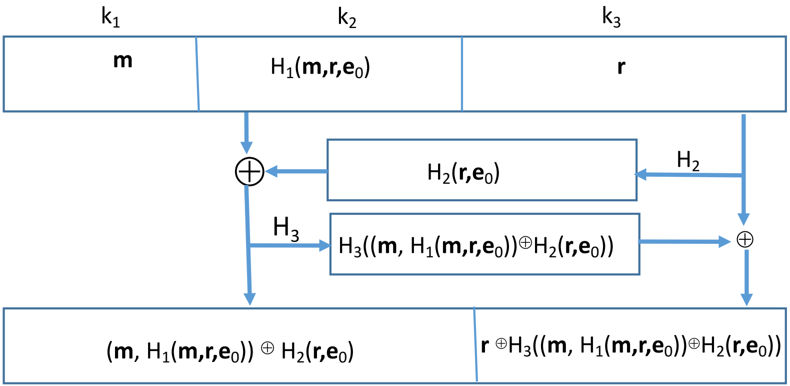
Recommended parameters

ID	κ_c, κ_q	LD	n	k	t	w	m	sk	cipher	pk
0	**128, 80**	$\perp$	630	470	80	160	**10**	310116	988	188001
2	**192,110**	$\perp$	1000	764	118	236	**10**	747393	1545	450761
4	**256,144**	$\perp$	1360	800	280	560	**11**	1773271	2640	1232001
6	22,22	$\perp$	40	20	10	5	**10**	1059	57	626
7	**128, 80**	(13,6663,14)	612	466	76	146	**10**	284636	948	170091
9	**192, 110**	(11,9317,12)	1000	790	108	210	**10**	703371	1513	414751
11	**256,144**	(26,23350,34)	1200	700	280	500	**11**	1382314	2338	926501
13	24,24	(3, 68,4)	40	20	11	5	**10**	1059	57	626
14	25,25	(10, 262,14)	40	20	12	5	**10**	1059	57	626

RLCE Padding: RLCEpad



Questions

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Information-set decoding (ISD)

- Information-set decoding (ISD) is one of the most important message recovery attacks on McEliece encryption schemes.
- For the RLCE encryption scheme, the ISD attack is based on the number of columns in the public key G instead of the number of columns in the private key G_s.
- The cost of ISD attack on an $[n, k, t; w]$-RLCE scheme is equivalent to the cost of ISD attack on an $[n + w, k; t]$-McEliece scheme.

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Naive ISD

- Uniformly selects k columns from the public key and checks whether it is invertible.
- If it is invertible, one multiplies the inverse with the corresponding ciphertext values in these coordinates that correspond to the k columns of the public key.
- If these coordinates contain no errors in the ciphertext, one recovers the plain text.

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Quantum ISD

- For a function $f : \{0,1\}^l \to \{0,1\}$ with the property that there is an $x_0 \in \{0,1\}^l$ such that $f(x_0) = 1$ and $f(x) = 0$ for all $x \neq x_0$, Grover's algorithm finds the value x_0 using $\frac{\pi}{4}\sqrt{2^l}$ Grover iterations and $O(l)$ qubits.
- Grover's algorithm converts the function f to a reversible circuit C_f and calculates

$$|x\rangle \xrightarrow{C_f} (-1)^{f(x)}|x\rangle$$

in each of the Grover iterations. Thus the total steps for Grover's algorithm is bounded by $\frac{\pi|C_f|}{4}\sqrt{2^l}$.

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Quantum ISD against RLCE

Thus Grover's quantum algorithm requires approximately

$$7\left((n+w)k + k^{2.807} + k^2\right)(\log_2 q)^{1.585}\sqrt{\frac{\binom{n+w}{k}}{\binom{n+w-t}{k}}}$$

steps for the simple ISD algorithm against RLCE encryption scheme.

ISD for systematic RLCE schemes

- One uniformly selects $k = k_1 + k_2$ columns from the public key where k_1 columns are from the first k columns of the public key.
- Assume that first k_1 columns have no error. Simplify the computation process for ISD

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Insecure ciphertexts for systematic RLCE schemes

- For a systematic RLCE, if a small number of errors were added to the first k components of the ciphertext, one may be able to exhaustively search these errors.
- Let

$$\gamma_l = \max_{l \le i \le t} \left\{ \frac{\binom{k-l}{k-i}}{q^i \binom{k}{i}} \right\}$$

The RLCE produces an insecure ciphertext in case that the ciphertext contains at most l errors within the first k components of the ciphertext and $\gamma_l > 2^{-\kappa_c}$ where κ_c is the security parameter.

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Sidelnikov-Shestakov's attack

- If $w \geq n - k$, not enough equations for Sidelnikov-Shestakov's attack
- If $w < n - k$, one need to guess some values to establish enough equations. The guess space is normally too big to be successful.

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Known non-randomized column attack

- What happens if the positions of non-randomized $n-w$ GRS columns are known to the adversary?
- Possibility one: guess the remaining w columns of the GRS generator matrix. Search space too big
- Use Sidelnikov-Shestakov attack to calculate a private key for the punctured $[n-w, k]$ GRS_k code consisting of the non-randomized GRS columns and then list-decode the punctured $[n-w, k]$ GRS_k code.

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Filtration attacks

- For two codes $\mathcal{C}_1$ and $\mathcal{C}_2$ of length n, the star product code $\mathcal{C}_1 * \mathcal{C}_2$ is the vector space spanned by $\mathbf{a} * \mathbf{b}$ for all pairs $(\mathbf{a}, \mathbf{b}) \in \mathcal{C}_1 \times \mathcal{C}_2$ where $\mathbf{a} * \mathbf{b} = [a_0 b_0, a_1 b_1, \cdots, a_{n-1} b_{n-1}]$.
- For the square code $\mathcal{C}^2 = \mathcal{C} * \mathcal{C}$ of $\mathcal{C}$, we have $\dim \mathcal{C}^2 \le \min\left\{n, \binom{\dim \mathcal{C}+1}{2}\right\}$.
- For an $[n, k]$ GRS code $\mathcal{C}$, let $\mathbf{a}, \mathbf{b} \in \mathrm{GRS}_k(\mathbf{x}, \mathbf{y})$ where $\mathbf{a} = (y_0 p_1(x_0), \cdots, y_{n-1} p_1(x_{n-1}))$ and $\mathbf{b} = (y_0 p_2(x_0), \cdots, y_{n-1} p_2(x_{n-1}))$. Then $\mathbf{a} * \mathbf{b} = (y_0^2 p_1(x_0) p_2(x_0), \cdots, y_{n-1}^2 p_1(x_{n-1}) p_2(x_{n-1}))$. Thus $\mathrm{GRS}_k(\mathbf{x}, \mathbf{y})^2 \subseteq \mathrm{GRS}_{2k-1}(\mathbf{x}, \mathbf{y} * \mathbf{y})$ where we assume $2k - 1 \le n$.

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- For an $[n, k]$ GRS code $\mathcal{C}$, let $\mathbf{a}, \mathbf{b} \in \mathrm{GRS}_k(\mathbf{x}, \mathbf{y})$ where $\mathbf{a} = (y_0 p_1(x_0), \cdots, y_{n-1} p_1(x_{n-1}))$ and $\mathbf{b} = (y_0 p_2(x_0), \cdots, y_{n-1} p_2(x_{n-1}))$. Then $\mathbf{a} * \mathbf{b} = (y_0^2 p_1(x_0) p_2(x_0), \cdots, y_{n-1}^2 p_1(x_{n-1}) p_2(x_{n-1}))$. Thus $\mathrm{GRS}_k(\mathbf{x}, \mathbf{y})^2 \subseteq \mathrm{GRS}_{2k-1}(\mathbf{x}, \mathbf{y} * \mathbf{y})$ where we assume $2k - 1 \le n$.

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Filtration attacks

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Filtration attacks against GRS-RLCE

- G is public key for an (n, k, d, t, w) GRS-RLCE scheme.
- Let $\mathcal{C}$ be the code generated by the rows of G.
- Let $\mathcal{D}_1$ be the code with a generator matrix D_1 obtained from G by replacing the randomized $2w$ columns with all-zero columns and let $\mathcal{D}_2$ be the code with a generator matrix D_2 obtained from G by replacing the $n - w$ non-randomized columns with zero columns.
- Since $\mathcal{C} \subset \mathcal{D}_1 + \mathcal{D}_2$ and the pair $(\mathcal{D}_1, \mathcal{D}_2)$ is an orthogonal pair, we have $\mathcal{C}^2 \subset \mathcal{D}_1^2 + \mathcal{D}_2^2$. It follows that

$$2k - 1 \leq \dim \mathcal{C}^2 \leq \min\{2k - 1, n - w\} + 2w \qquad (1)$$

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Filtration attacks against GRS-RLCE: $k \geq n - w$

- Assume that the $2w$ randomized columns in $\mathcal{D}_2$ behave like random columns in the filtration attacks
- We have $\dim \mathcal{C}^2 = \mathcal{D}_1^2 + \mathcal{D}_2^2 = n - w + \mathcal{D}_2^2 = n + w$.
- For any code $\mathcal{C}'$ of length n' that is obtained from $\mathcal{C}$ using code puncturing and code shortening, we have $\dim \mathcal{C}'^2 = n'$.
- Thus filtration techniques could not be used to recover any non-randomized columns in D_1.

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Running times for RLCE with Decoding Algorithm 1 (in milliseconds)

ID	key	encryption		decryption	
		RLCEspad	RLCEpad	RLCEspad	RLCEpad
0	340.616	0.565	0.538	1.574	1.509
2	1253.926	1.255	1.166	3.034	2.937
4	3215.791	2.836	2.796	13.092	12.925

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RLCE CPU cycles

ID	key generation	encryption	decryption
0	1011071617	1805010	4646941
2	3829675407	3331234	8668186
4	9612380645	8184051	36705481

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RLCE peak memory usage (bytes)

ID	Mul. Table	key generation	encryption	decryption
0	N	2,536,704	798,288	1,335,280
0	Y	4,648,656	2,437,320	2,856,584
2	N	6,178,744	1,906,576	3,178,688
2	Y	8,287,312	2,865,400	3,825,112
4	N	11,561,352	4,829,968	7,010,368
4	Y	19,975,040	10,258,112	12,227,384

Questions

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