Threshold Schemes for Cryptographic Primitives

A step towards standardization?

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National Institute of Standards and Technology (Gaithersburg, MD, USA)

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Minor updates in August, 2018, for online publication:
NISTIR number and link (8214); comment period;
project webpage; section 4 more succinct.

Contact email: threshold-crypto@nist.gov

Outline

1. Introduction
2. Preliminaries
3. Characterizing features
4. Some numbers
5. Steps (NISTIR, workshop)
6. Final remarks

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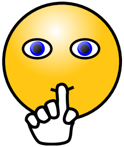

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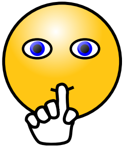

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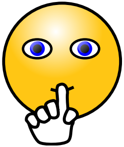

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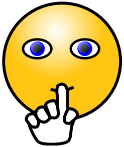

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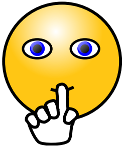

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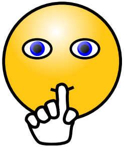
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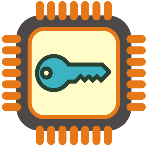
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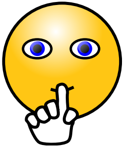
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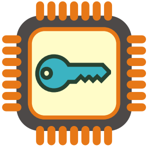
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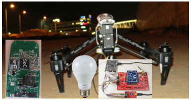

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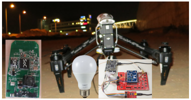

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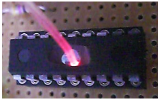

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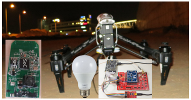

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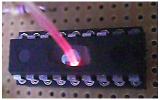

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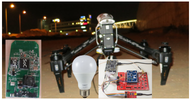

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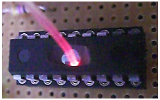

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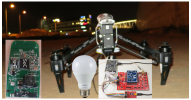

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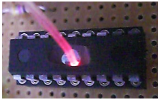

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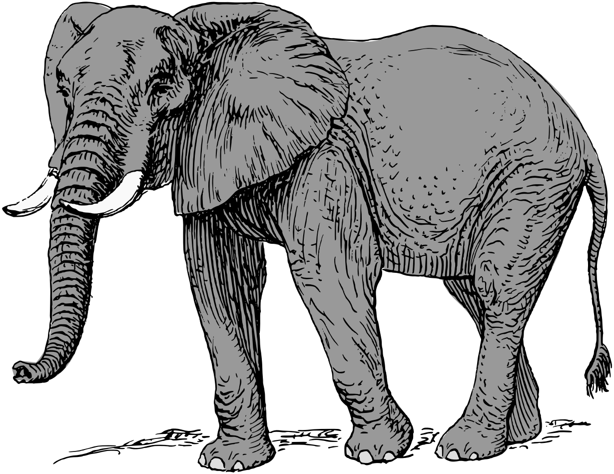
It is essential to have reliable implementations of cryptographic primitives, immune to breaches in the computational environment

Single-Points of Failure!



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Can we standardize
threshold schemes
for cryptographic
primitives
to promote the
security of crypto
implementations

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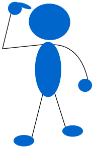

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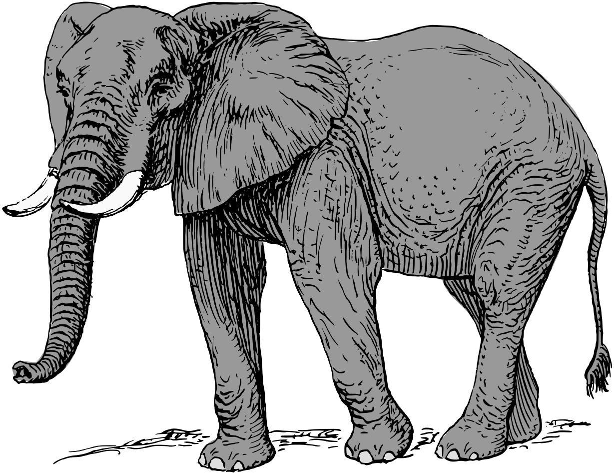

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The threshold approach

High-level idea:
Use redundancy & diversity to mitigate
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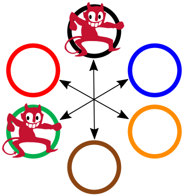

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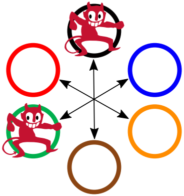

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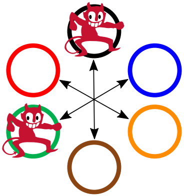

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Note on co-existing notation:

- f-out-of-n: tolerates the compromise of up to f nodes
- k-out-of-n: requires correct participation of at least k nodes

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Split a secret key into n secret “shares” for storage at rest.

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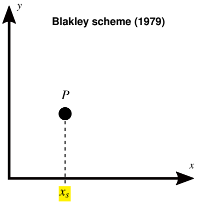

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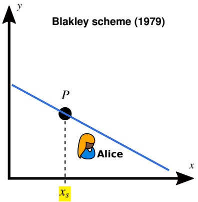

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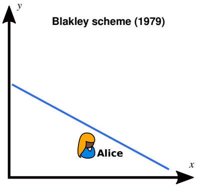

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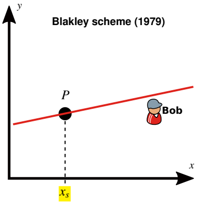

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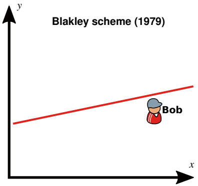

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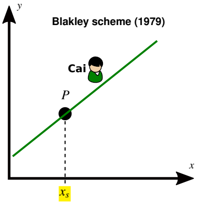

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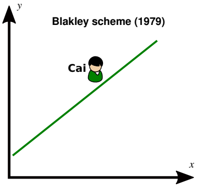

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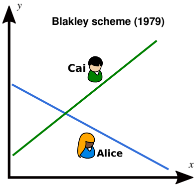

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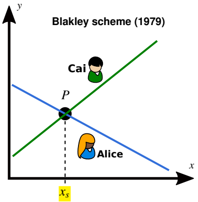

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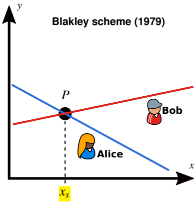

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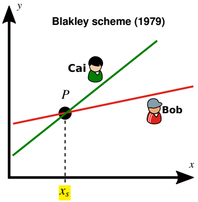

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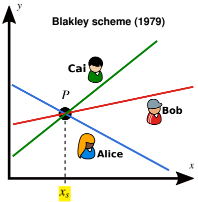

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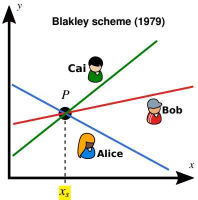

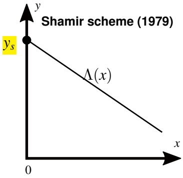

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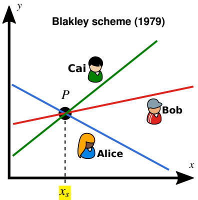

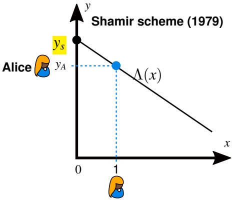

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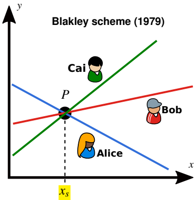

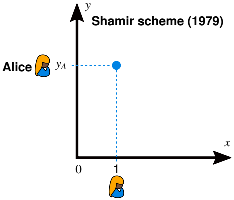

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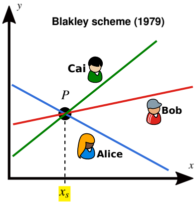

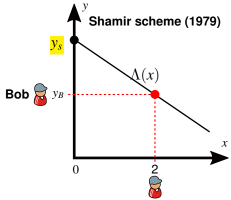

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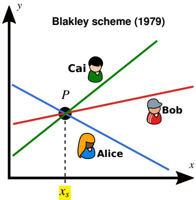

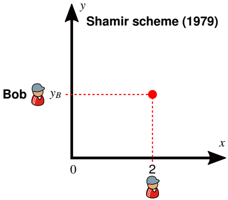

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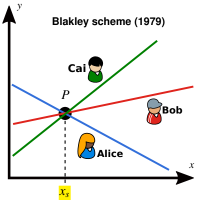

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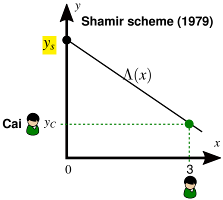

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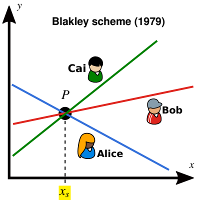

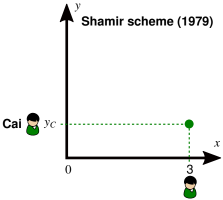

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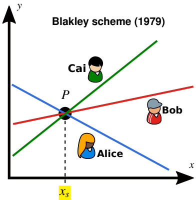

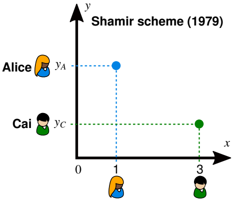

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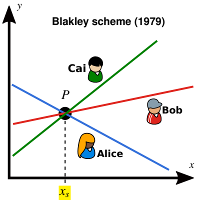

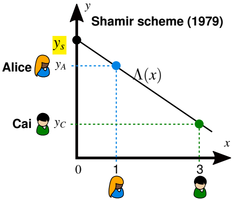

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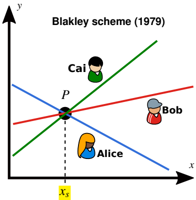

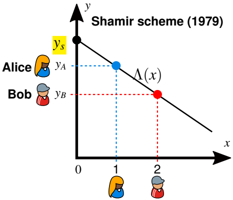

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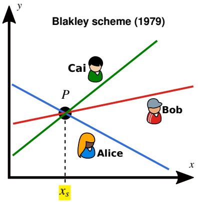

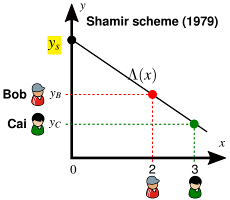

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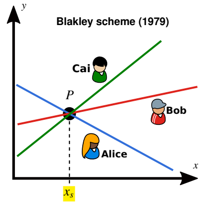

Humanoid cliparts:
clker.com/clipart-*.html
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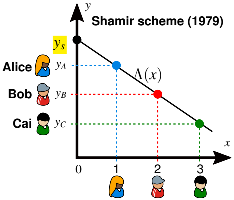

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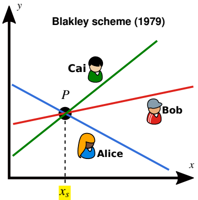

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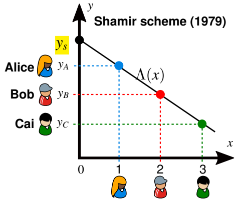

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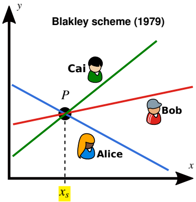

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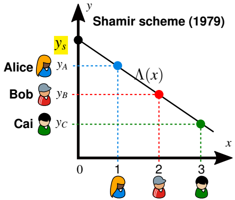

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Use threshold schemes for cryptographic primitives (next)

Our current step

Image adapted from:
openclipart.org/detail/283392

Devise initial questions for discussion
towards standardization and validation of
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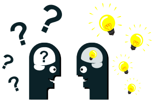

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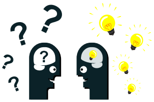

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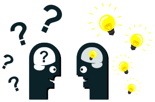

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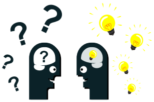
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- Suggest moving forward (with challenges)

Outline

1. Introduction

2. Preliminaries

3. Characterizing features

4. Some numbers

5. Steps (NISTIR, workshop)

6. Final remarks

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Facilitating setting: $\exists$ dealer;

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It is efficient:

- Size: final signature is as original
- Sub-signer computation: original, plus produce 1 NIZKP (2 exps)
- Combiner computation: original, plus 1 ext-GCD and verify NIZKPs ($2 \cdot k$ exps)

A DL-based example: threshold Schnorr signature

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clker.com/clipart-encryption.html

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> "k-out-of-n" or "f-out-of-n" is not a sufficient characterization for a comprehensive security assertion

Reliability — one metric of security

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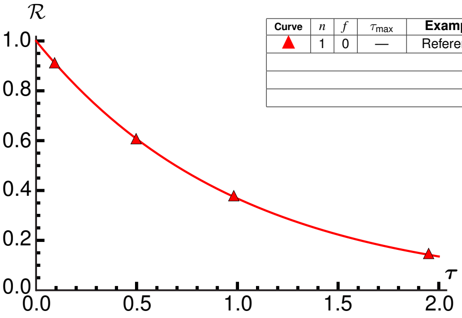

Curve	n	f	$\tau_{\max}$	Example
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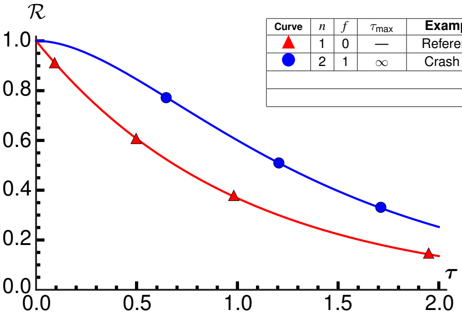

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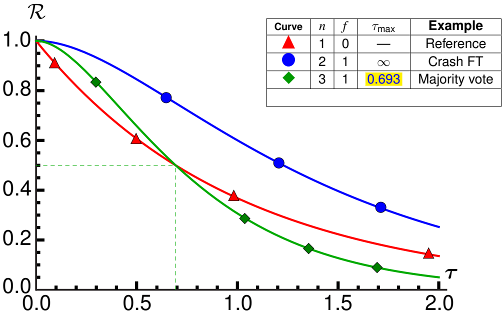

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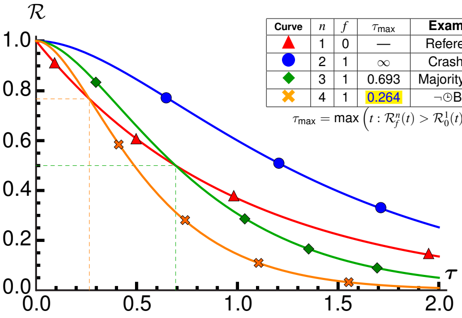

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◆	3	1	0.693	Majority vote
✖	4	1	0.264	¬◷BFT

$\tau_{\max} = \max\left(t : \mathcal{R}_f^n(t) > \mathcal{R}_0^1(t)\right)$

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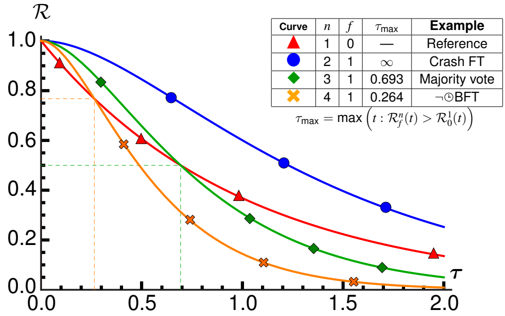

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Reliability can be degraded when increasing the threshold (f), even if nodes fail independently

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Rejuvenation (recovery of nodes): compromised state $\rightarrow$ healthy state

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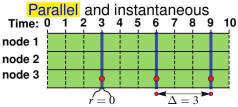

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Rejuvenation modes:

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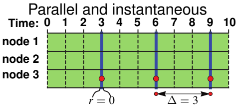

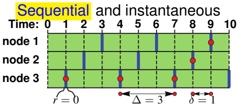

Improve reliability with rejuvenations

Rejuvenation (recovery of nodes): compromised state → healthy state

- Examples: replace device, patch vulnerability, update or reset a state, ...
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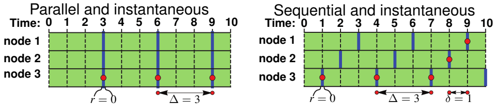

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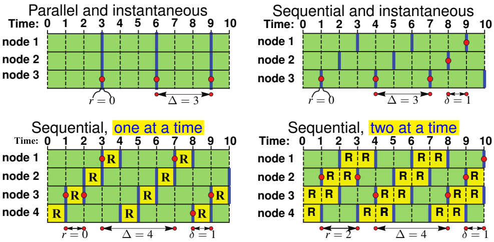

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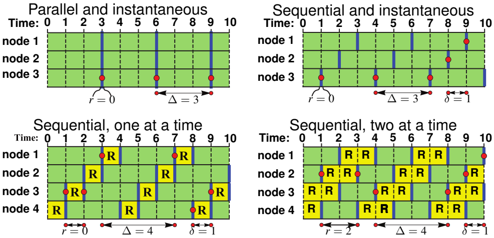

[BB12]

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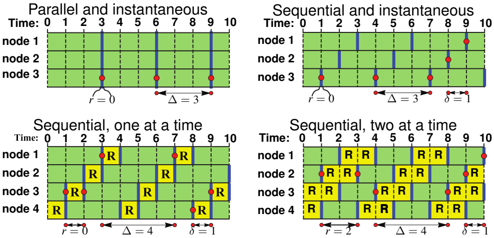

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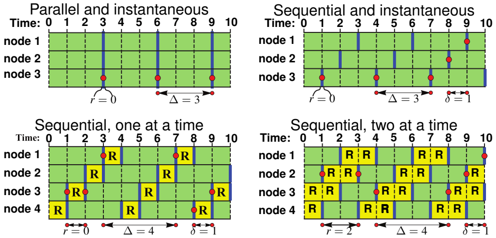

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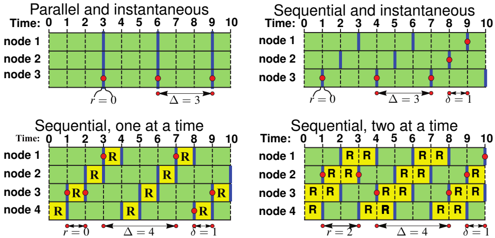

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- sequential rejuvenations may still allow a mobile attacker to persist

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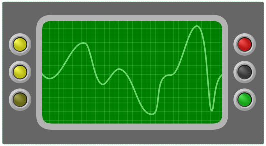

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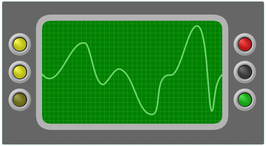

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Challenge questions:

- which models are realistic / match state-of-the-art attacks?
- what concrete parameters (e.g., n) thwart real attacks?

Outline

1. Introduction

2. Preliminaries

3. Characterizing features

4. Some numbers

5. Steps (NISTIR, workshop)

6. Final remarks

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3. Executing platform
4. Setup and maintenance

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A characterization provides a better context for security assertions.

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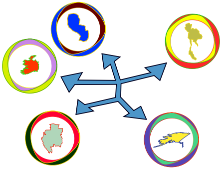

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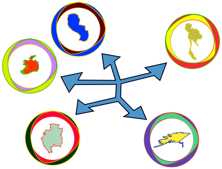

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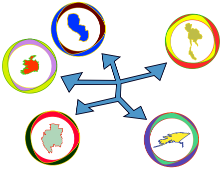
- Variable threshold and number of nodes?
  (changing parameters may need its own protocol)



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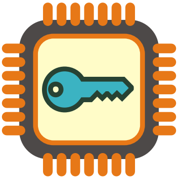

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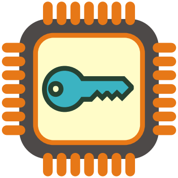

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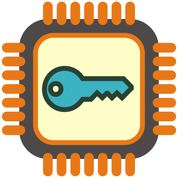
- Additional trusted machinery? (global clock, proxy, RNG, combiner)



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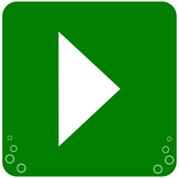

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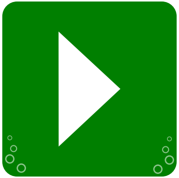

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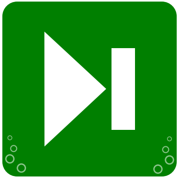

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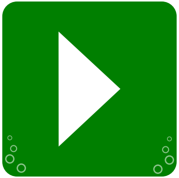

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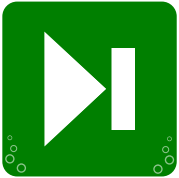

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[Content adapted in online version]

Just for an intuition:
brief notes on
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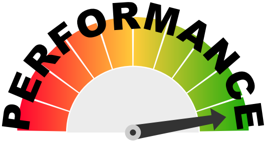

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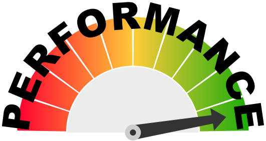

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- threshold AES evaluation (SMPC-based)
- threshold circuit design of symmetric primitives
- threshold random-number generation (coin-tossing)

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Next slides: some representative questions on flexibility and validation challenges

Flexibility of features and parameters

Standard → implementation validation → deployment

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- **Standard.** What flexibility of features & parameters should a threshold-scheme standard allow?
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Answers may to a certain extent depend on what can be assessed by test & validation procedures (some of which to develop)!

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Validation is needed:

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Automation of validation in the CAVP and CMVP:

- Automate CAVP by Fall 2018, based on newly developed ACVP [NIS18]
- Ongoing pilots (Google, Red Hat) on automated module validations

Legend: ACVP (Automated Cryptographic Validation Protocol) CAVP (Cryptographic Algorithm Validation Program); CMVP (Cryptographic Module Validation Program); FIPS (Federal Information Processing Standards).

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- **Adaptation:** how should validation procedures and assertions vary with (or adapt to) threshold features and application context?
  - with/without dealer, executing platform, rejuvenation modes, ...

Modularity

Modularity

Patch-and-revalidate scenario. If a f-out-of-n (for availability) system has *diversity* of implementation across nodes, then:

- a new vulnerability in a node can be patched offline
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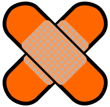

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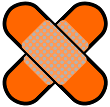

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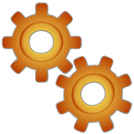
Base primitives. Is it useful to standardize/define certain modules? (composability argument)

- secret sharing
- commitments
- ZK proofs
- oblivious transfer
- ... (other SMPC tools)



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A Workshop?

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Can we do it in an open workshop?

- learn the state-of-the-art and survey the area
- define a criteria for a call proposals for threshold schemes
- tentative month: March 2019?

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Standardizing a threshold scheme would also entail:

- Deciding what remains flexible up to validation and/or deployment phases
- Develop test procedures and security assertions for validation

Moving forward

The end goals:

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- promote good practices of deployment

Meanwhile:

- We would appreciate feedback on the Draft NISTIR (8214).
- We would like to extend an open invitation for you to participate in upcoming steps.

Thanks

Thank you for your attention!

Threshold Schemes for Cryptographic Primitives
A step towards standardization?

Contact us at threshold-crypto@nist.gov

Check updates https://csrc.nist.gov/Projects/Threshold-Cryptography

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