

Anonymous, Robust Post-Quantum Public Key Encryption

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Joint work with Paul Grubbs and Kenneth G. Paterson

[Full version of paper: <https://eprint.iacr.org/2021/708.pdf>]

NIST PQC Finalists

PQC Standardization Process: Third Round Candidate Announcement

July 22, 2020



It has been almost a year and a half since the second round of the NIST PQC Standardization Process began. After careful consideration, NIST would like to announce the candidates that will be moving on to the third round. The seven third-round Finalists are:

Third Round Finalists

Public-Key Encryption/KEMs

Classic McEliece

CRYSTALS-KYBER

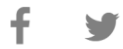
NTRU

SABER

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4.A.2 Security Definition for Encryption/Key-Establishment

NIST intends to standardize one or more schemes that enable “semantically secure” encryption or key encapsulation with respect to adaptive chosen ciphertext attack, for general use. This property is generally denoted *IND-CCA2 security* in academic literature.

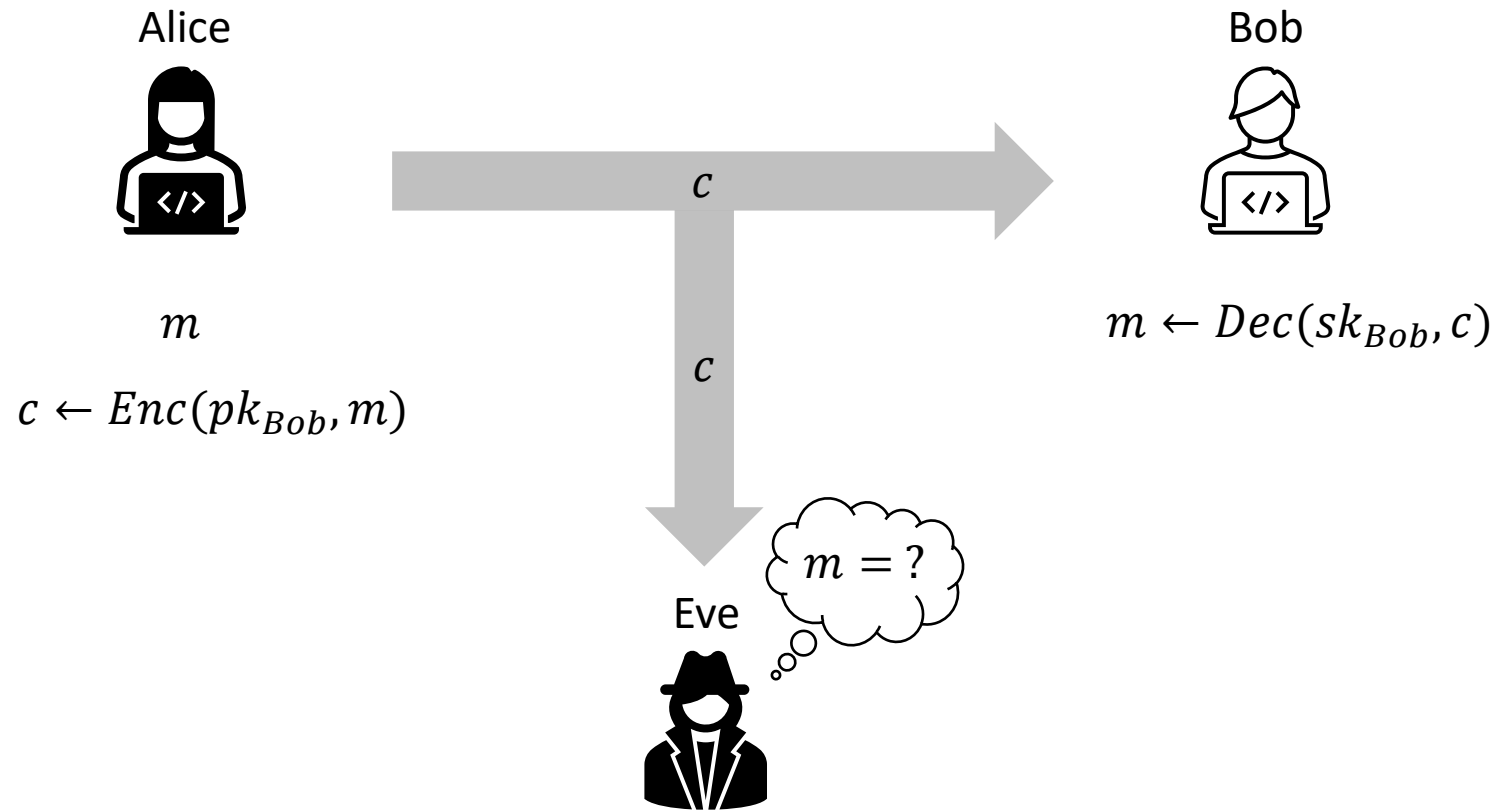
The above security definition should be taken as a statement of what NIST will consider to be a relevant attack. Submitted KEM and encryption schemes will be evaluated based on how well they appear to provide this property, when used as specified by the

(Image taken from <https://www.nist.gov/news-events/news/2020/07/pqc-standardization-process-third-round-candidate-announcement>)

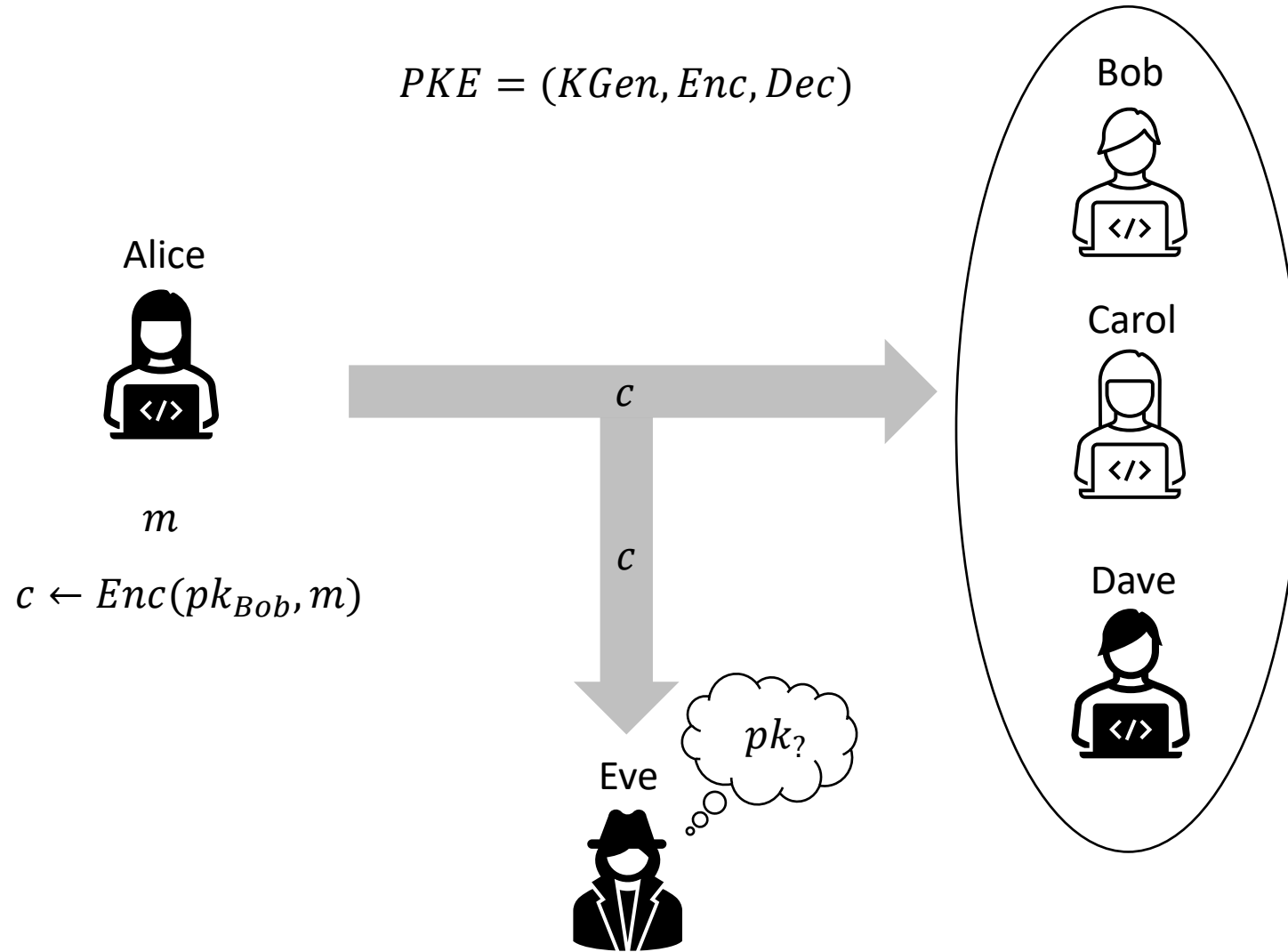
(Image taken from <https://csrc.nist.gov/CSRC/media/Projects/Post-Quantum-Cryptography/documents/call-for-proposals-final-dec-2016.pdf>)

IND-CCA Security

$$PKE = (KGen, Enc, Dec)$$



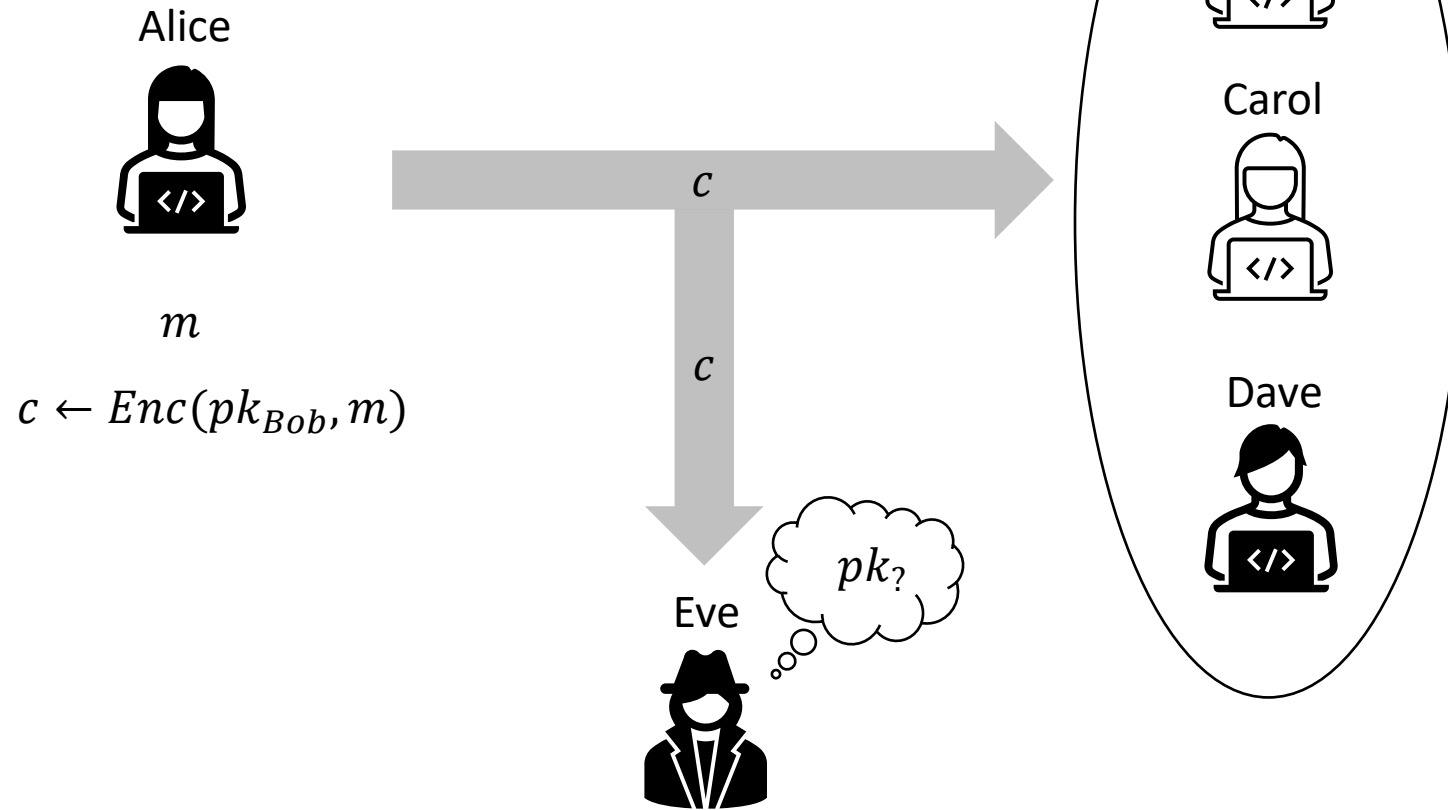
Anonymity (ANO-CCA security)



Anonymity (ANO-CCA security)

Formalized in a public-key setting by [Bellare-Boldyreva-Desai-Pointcheval'01].

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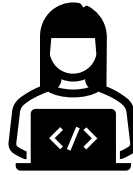


Anonymity (ANO-CCA security)

Formalized in a public-key setting by [Bellare-Boldyreva-Desai-Pointcheval'01].

$$PKE = (KGen, Enc, Dec)$$

Alice



m

$$c \leftarrow Enc(pk_{Bob}, m)$$



c

c

Eve

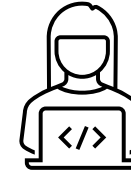


$pk?$

Bob



Carol



Dave



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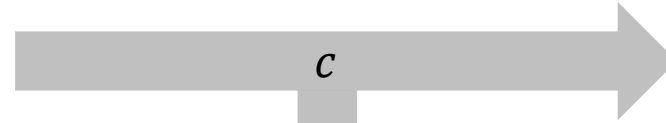
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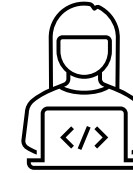
c

c

Bob



Carol



Dave



Eve



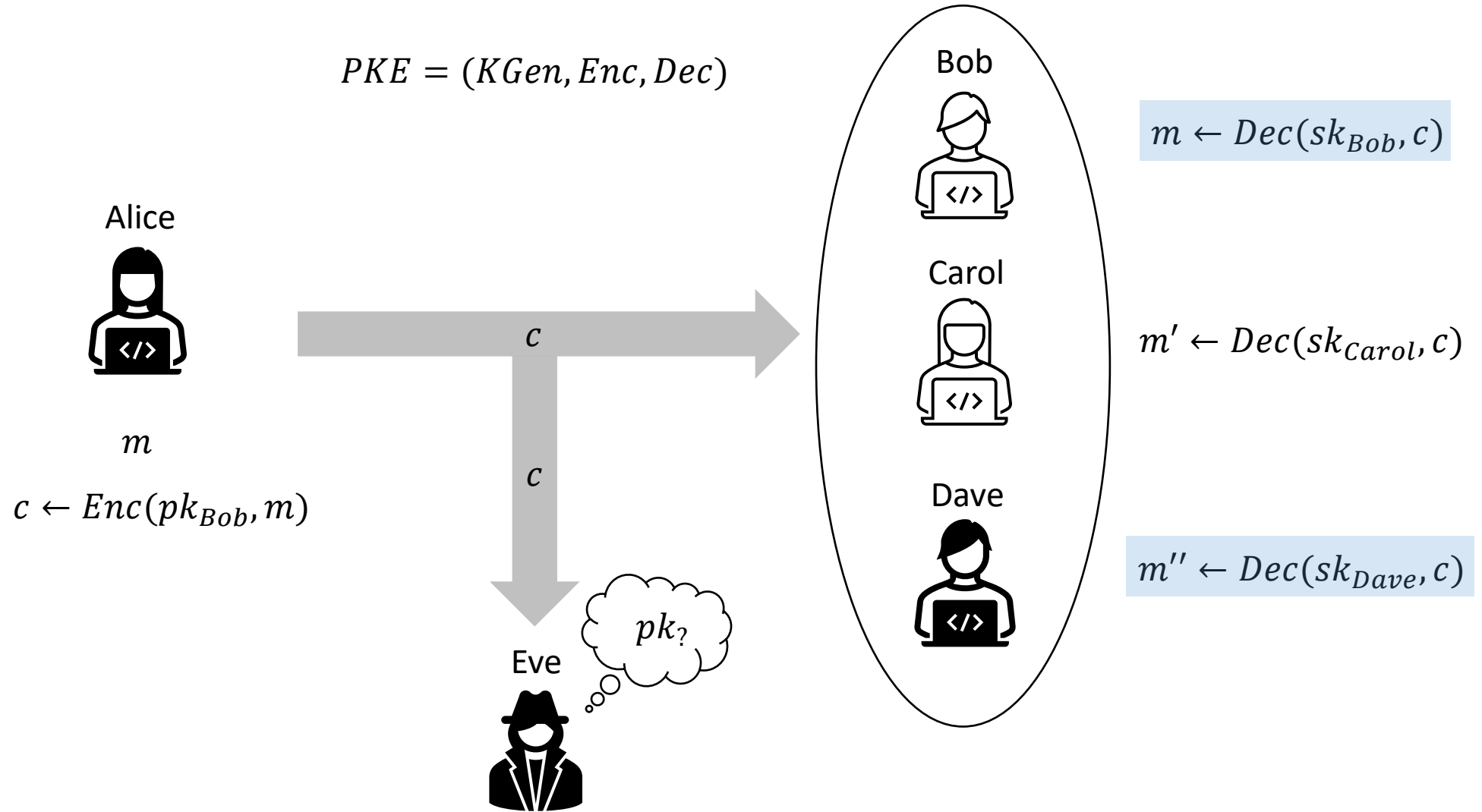
$pk?$



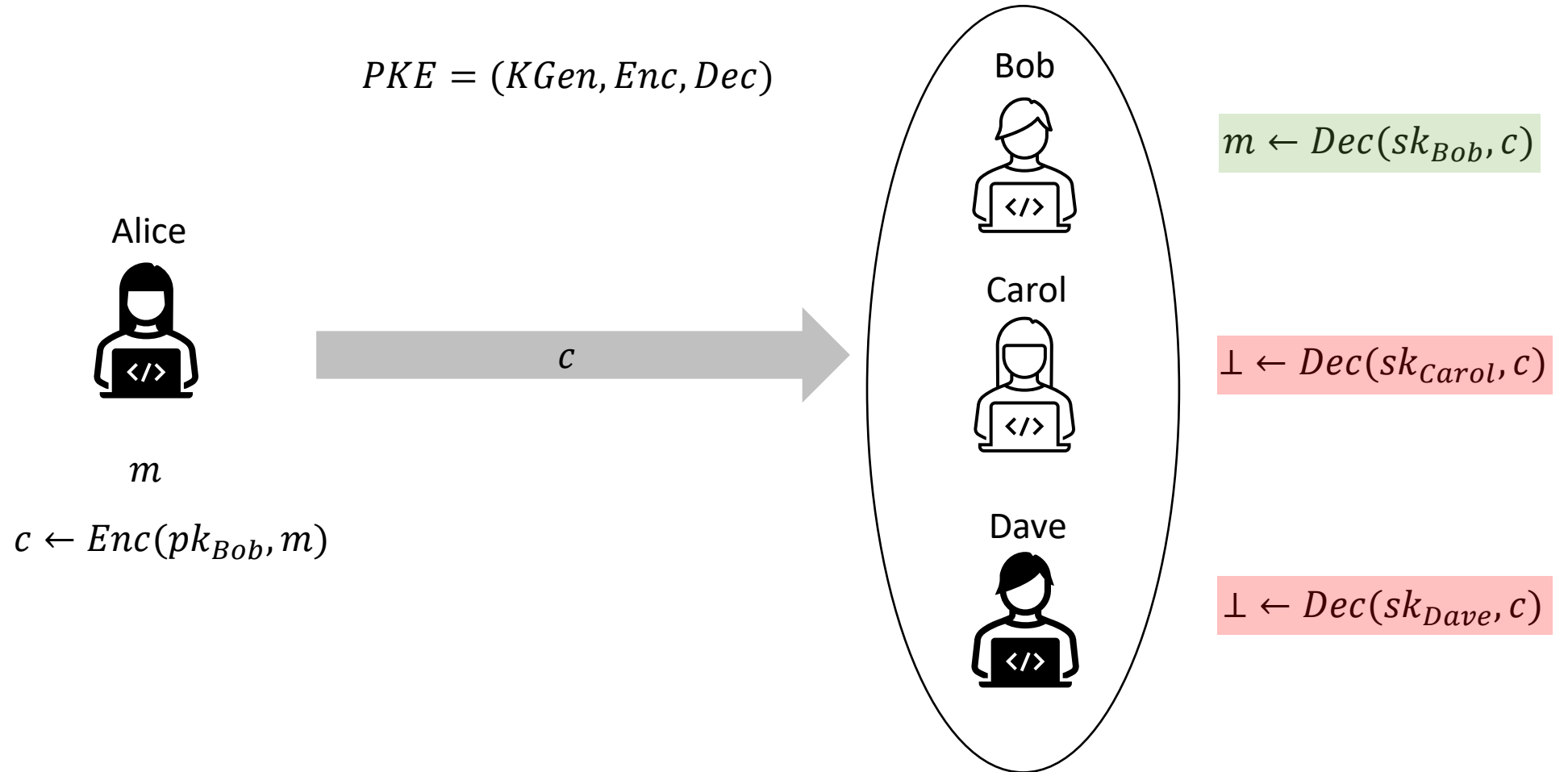
(Image taken from <https://z.cash>)

(Image taken from <https://digiday.com/marketing/ad-buyers-programmatic-auction/>)

Anonymity (ANO-CCA security)

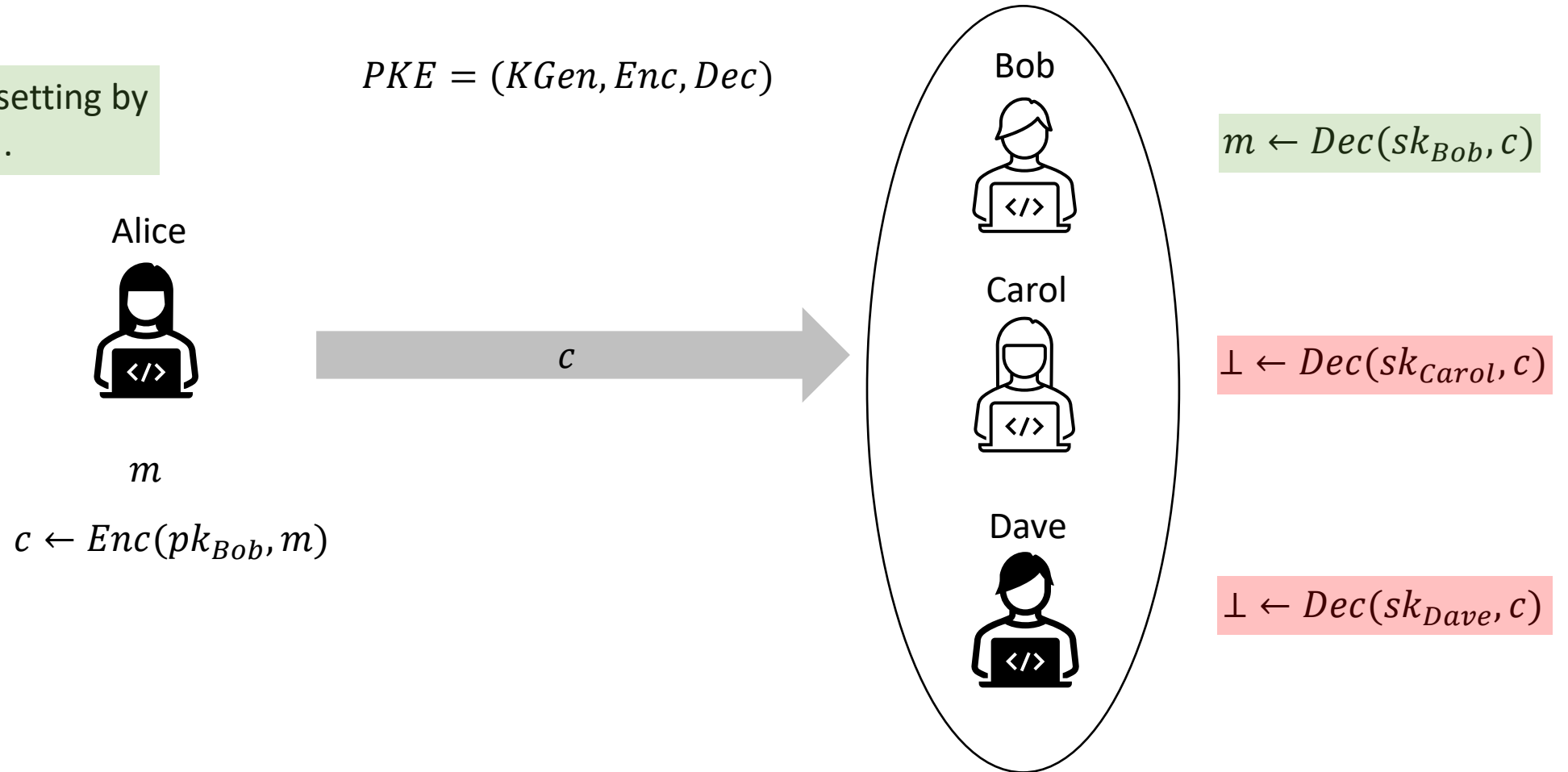


Robustness (SROB-CCA security)



Robustness (SROB-CCA security)

Formalized in a public-key setting by [Abdalla-Bellare-Neven'10].



KEM-DEM Paradigm

Public-Key Encryption/KEMs

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CRYSTALS-KYBER

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IND-CCA secure

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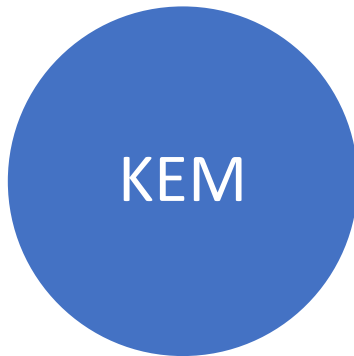
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IND-CCA secure

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IND-CCA secure

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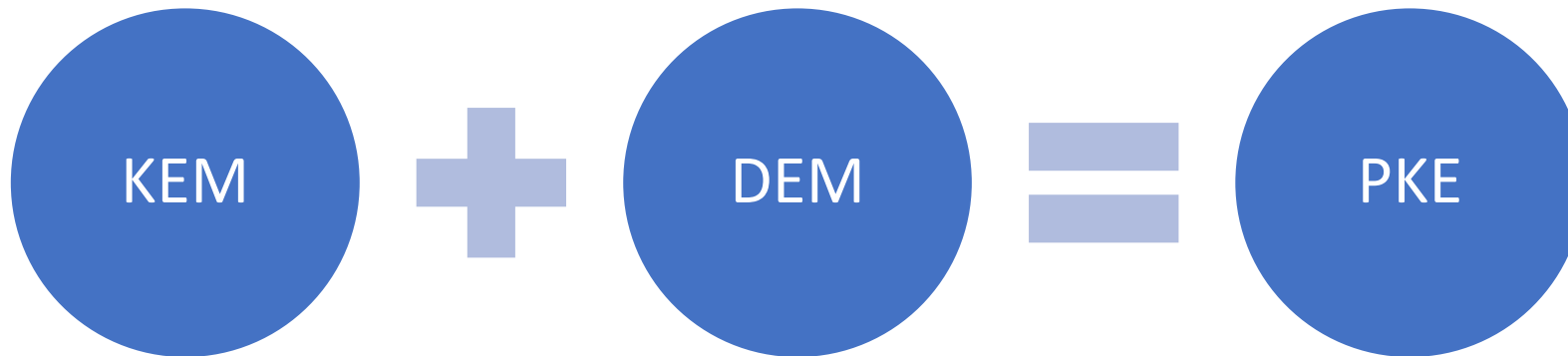
Classic McEliece

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$KEM = (KGen, Encap, Decap)$ $DEM = (Enc^{sym}, Dec^{sym})$ $PKE = (KGen, Enc, Dec)$



IND-CCA secure

(one-time) IND-CCA secure

IND-CCA secure

KEM-DEM Paradigm

Public-Key Encryption/KEMs

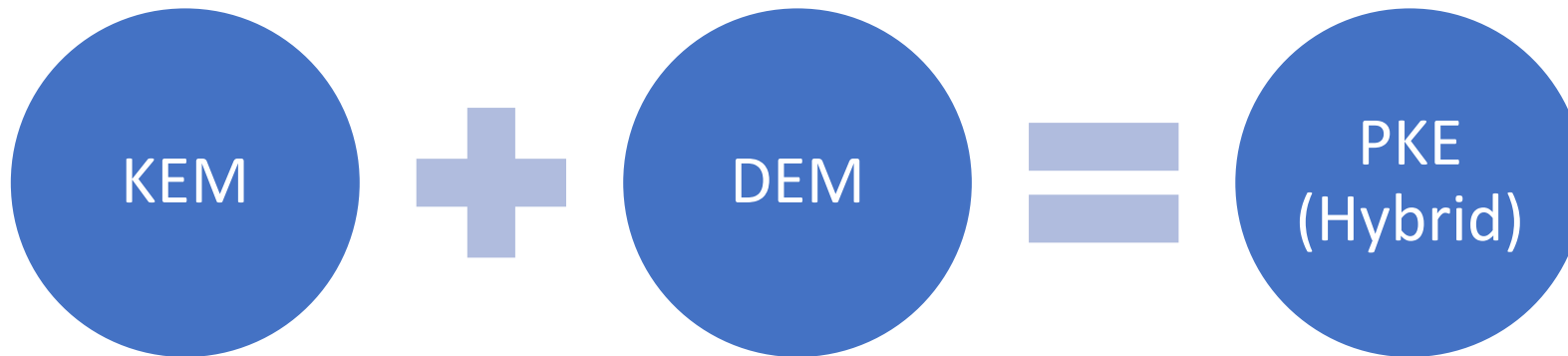
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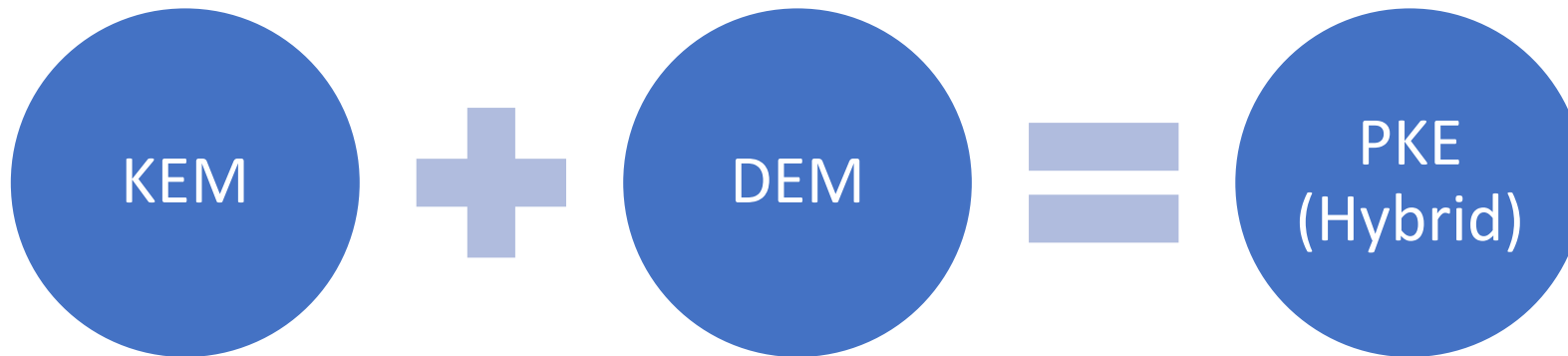
Classic McEliece

CRYSTALS-KYBER

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$$KEM = (KGen, Encap, Decap) \quad DEM = (Enc^{sym}, Dec^{sym}) \quad PKE = (KGen, Enc, Dec)$$



$$(c_{KEM}, k) \leftarrow Encap(pk_{Bob})$$

IND-CCA secure

$$c_{DEM} \leftarrow Enc^{sym}(k, m)$$

(one-time) IND-CCA secure

$$(c_{KEM}, c_{DEM}) \leftarrow Enc(pk_{Bob}, m)$$

IND-CCA secure

KEM-DEM Paradigm

Public-Key Encryption/KEMs

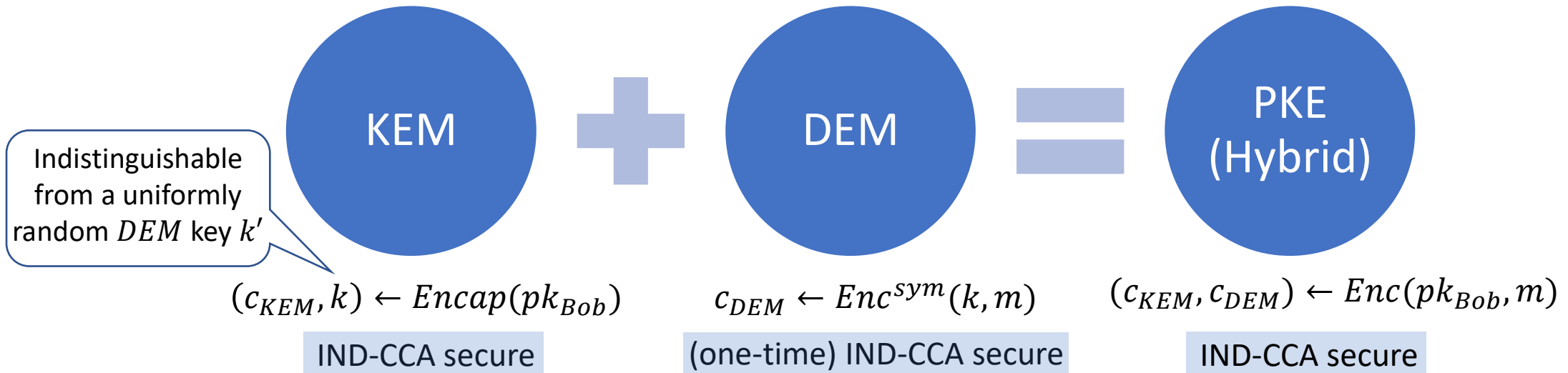
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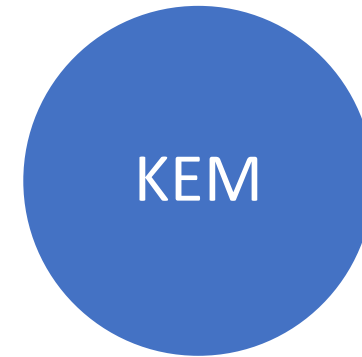
NTRU

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Fujisaki-Okamoto Transformation

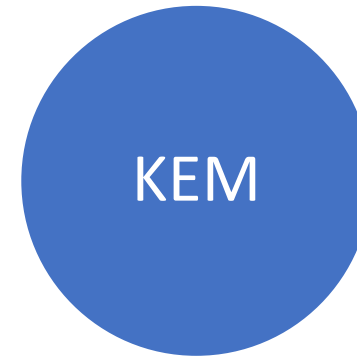


IND-CCA secure

Fujisaki-Okamoto Transformation

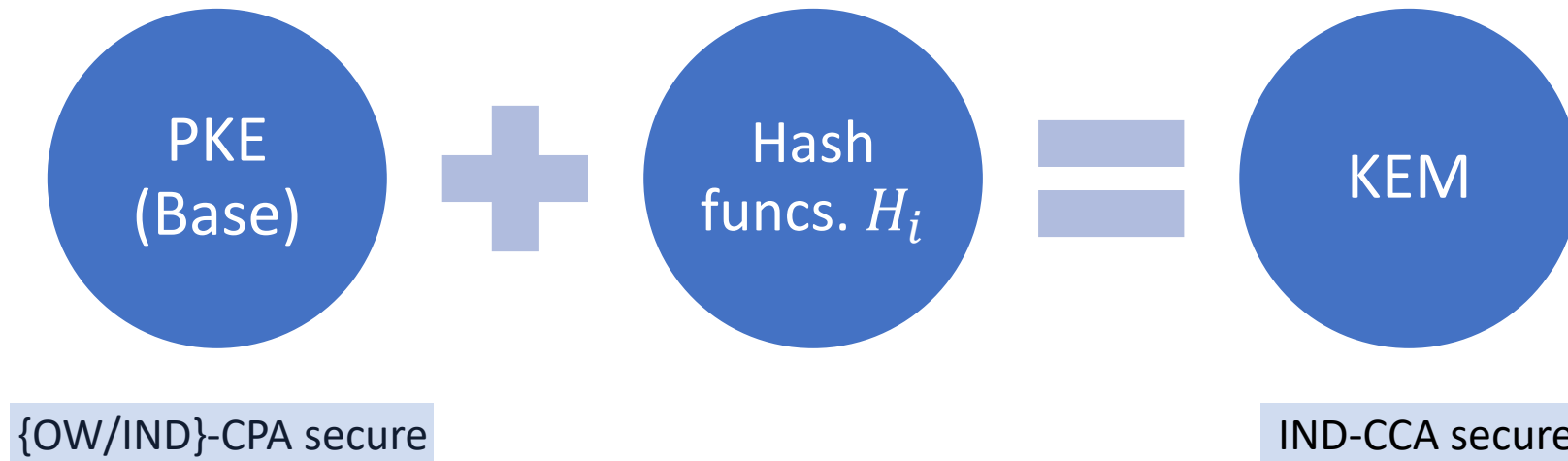


{OW/IND}-CPA secure

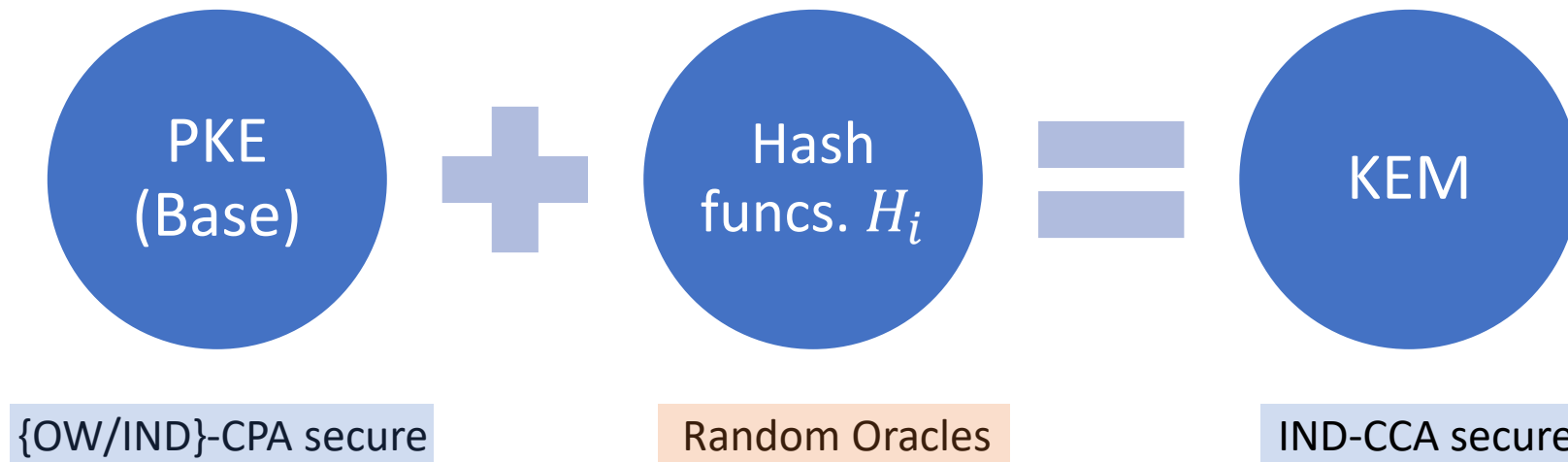


IND-CCA secure

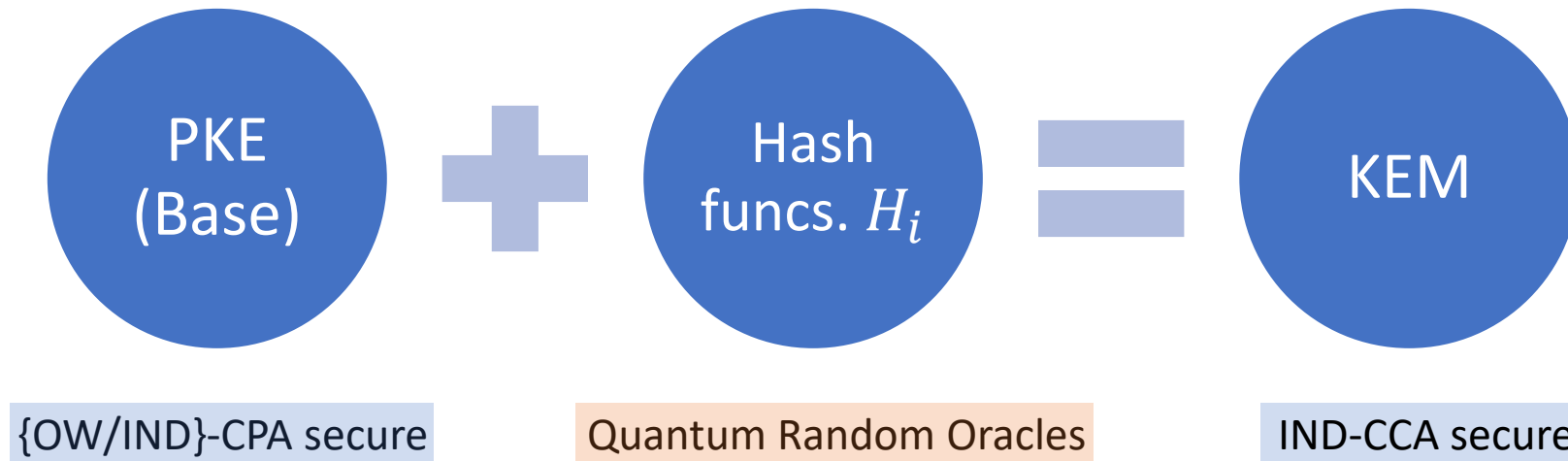
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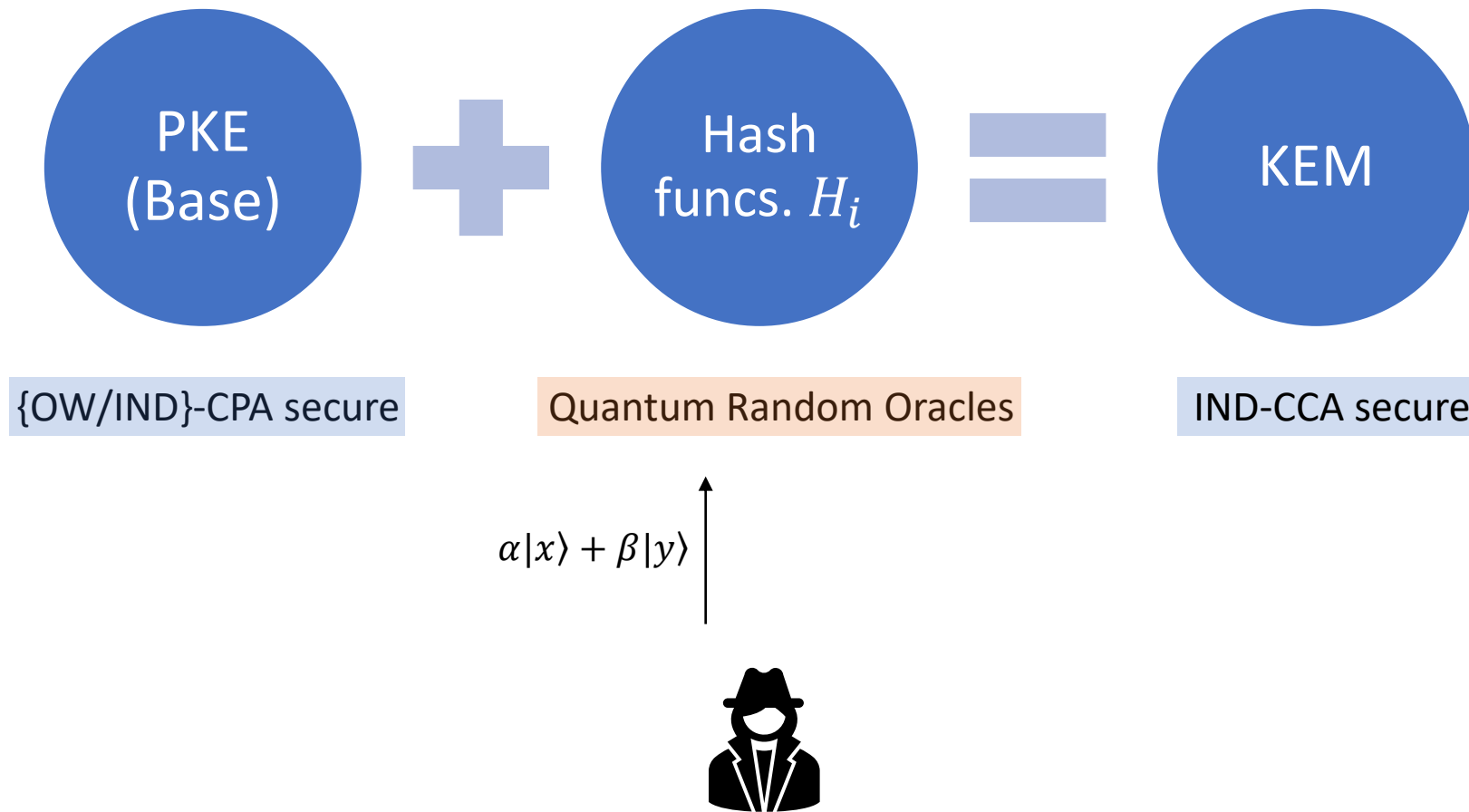
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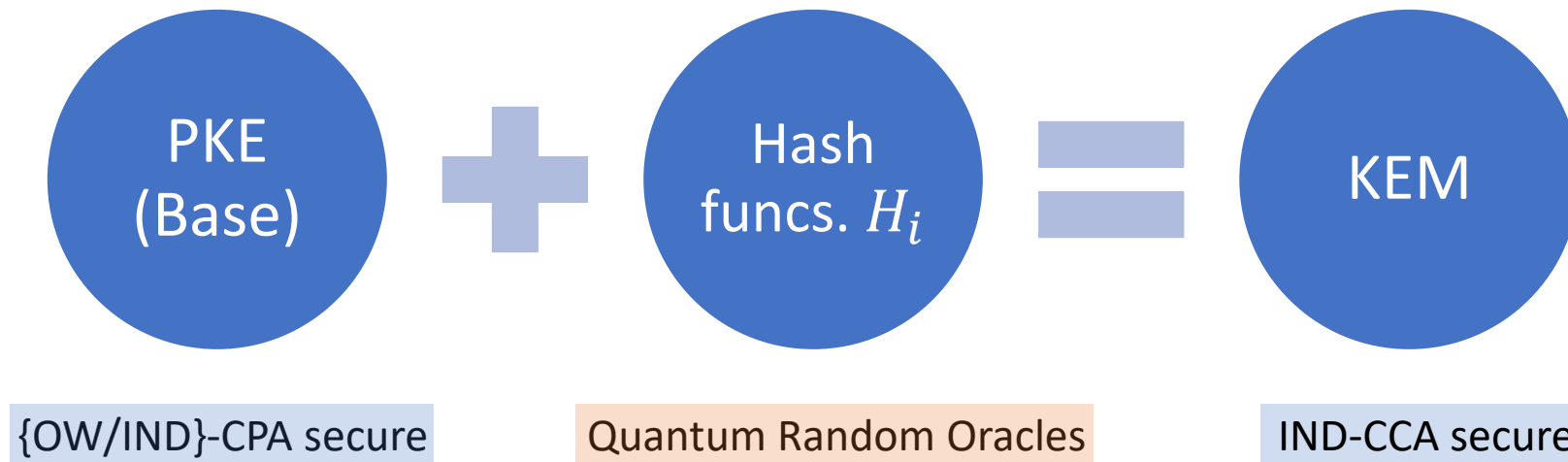
Fujisaki-Okamoto Transformation



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Fujisaki-Okamoto Transformation



$$\alpha|x\rangle + \beta|y\rangle \begin{array}{c} \uparrow \\ \downarrow \end{array} \alpha|H_i(x)\rangle + \beta|H_i(y)\rangle$$



Fujisaki-Okamoto Transformation

Classic McEliece

CRYSTALS-KYBER

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Fujisaki-Okamoto Transformation

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Classic McEliece

CRYSTALS-KYBER

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KGen'	Encap(pk)	Decap(sk', c)
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2: $s \leftarrow_{\$} \mathcal{M}$	2: $c \leftarrow \text{Enc}(pk, m; G(m))$	2: $m' \leftarrow \text{Dec}(sk, c)$
3: $sk' = (sk, s)$	3: $k \leftarrow H(m, c)$	3: $c' \leftarrow \text{Enc}(pk, m'; G(m'))$
4: return (pk, sk')	4: return (c, k)	4: if $c' = c$ then
		5: return $H(m', c)$
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The KEM $\text{FO}^{\perp}[\text{PKE}, G, H]$.

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[Jiang-Zhang-Chen-Wang-Ma'18] showed the IND-CCA security of KEMs obtained from these two “standard” FO transforms in the QROM.

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The KEM $\text{FO}_m^{\neq}[\text{PKE}, G, H]$.

6.1.2 Security in the Quantum Random Oracle Model

Jiang et al. [24] provide a security reduction against a quantum adversary in the quantum random oracle model from IND-CCA security to OW-CPA security. IND-CPA with a

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[Jiang-Zhang-Chen-Wang-Ma'18] showed the IND-CCA security of KEMs obtained from these two "standard" FO transforms in the QROM.

Anonymity from FO transforms

(Informal) **Theorem** [Grubbs-Maram-Paterson'21]:

*Hybrid PKE schemes obtained from FO[⊥] KEMs via the generic KEM-DEM composition are also ANO-CCA secure in the QROM.**

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*(*Provided the base PKE scheme satisfies some additional mild security properties.)*

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(Informal) **Theorem** [Grubbs-Maram-Paterson'21]:

*Hybrid PKE schemes obtained from $\text{FO}^{\cancel{\chi}}$ KEMs via the generic KEM-DEM composition are also ANO-CCA secure in the QRROM.**

*(*Provided the base PKE scheme satisfies some additional mild security properties.)*

Status of NTRU (which uses a close variant of $\text{FO}_m^{\cancel{\chi}}$) with respect to anonymity and robustness properties is open.

Classic McEliece (CM)

(Informal) **Theorem** [Grubbs-Maram-Paterson'21]:

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The base PKE scheme needs to *randomized* (specifically, γ -spread).

CM uses a *deterministic* base PKE scheme.

Classic McEliece (CM)

2.2.3 Encoding subroutine

The following algorithm ENCODE takes two inputs: a weight- t column vector $e \in \mathbb{F}_2^n$; and a public key T , i.e., an $(n - k) \times k$ matrix over \mathbb{F}_2 . The algorithm output ENCODE(e, T) is a vector $C_0 \in \mathbb{F}_2^{n-k}$. Here is the algorithm:

1. Define $H = (I_{n-k} \mid T)$.
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Fix any “message” $e = \begin{pmatrix} e_{n-k} \\ 0^k \end{pmatrix}$:

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- $(n - k \geq t$ in all CM parameters)

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- $(n - k \geq t$ in all CM parameters)
- $C_0 = (I_{n-k} \mid T) \begin{pmatrix} e_{n-k} \\ 0^k \end{pmatrix} = e_{n-k}$ – i.e., independent of public-key T .

Classic McEliece (CM)

2.2.3 Encoding subroutine

The following algorithm ENCODE takes two inputs: a weight- t column vector $e \in \mathbb{F}_2^n$; and a public key T , i.e., an $(n - k) \times k$ matrix over \mathbb{F}_2 . The algorithm output ENCODE(e, T) is a vector $C_0 \in \mathbb{F}_2^{n-k}$. Here is the algorithm:

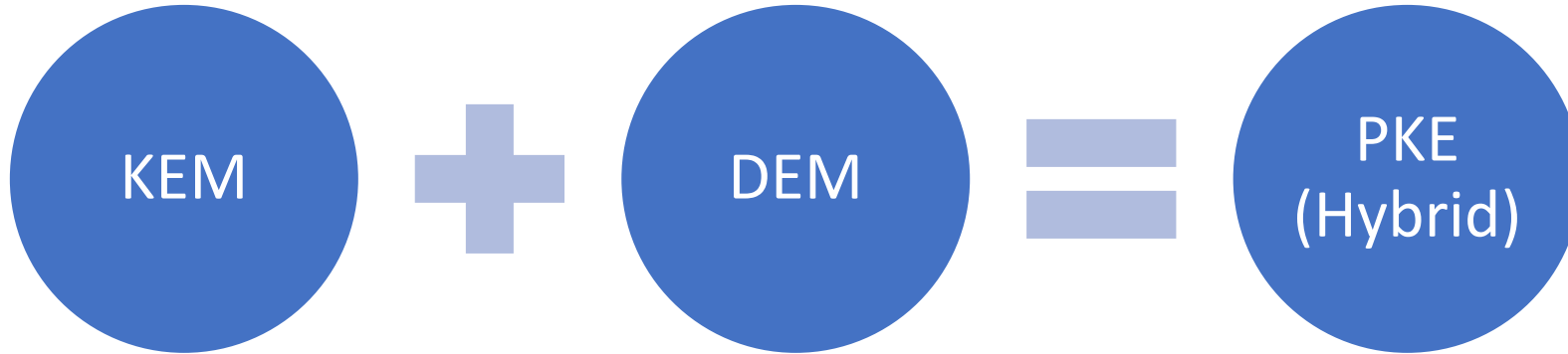
1. Define $H = (I_{n-k} \mid T)$.
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- $C_0 = (I_{n-k} \mid T) \begin{pmatrix} e_{n-k} \\ 0^k \end{pmatrix} = e_{n-k}$ – i.e., independent of public-key T .
- Because of perfect correctness, C_0 must decrypt to fixed e under *any private key* of CM’s base PKE scheme.

Classic McEliece (CM)

$KEM = (KGen, Encap, Decap)$ $DEM = (Enc^{sym}, Dec^{sym})$ $PKE = (KGen, Enc, Dec)$



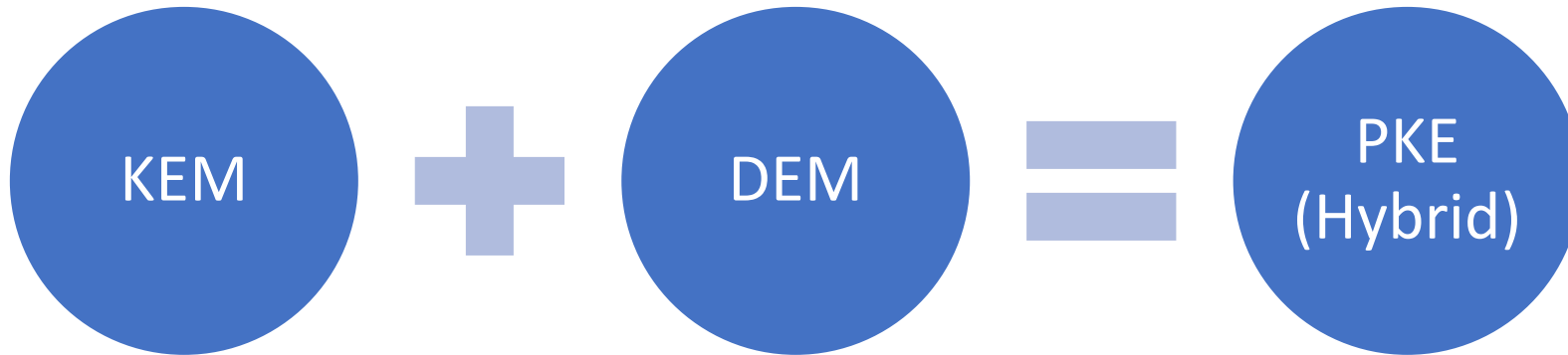
$(c_{KEM}, k) \leftarrow Encap(pk_{Bob})$

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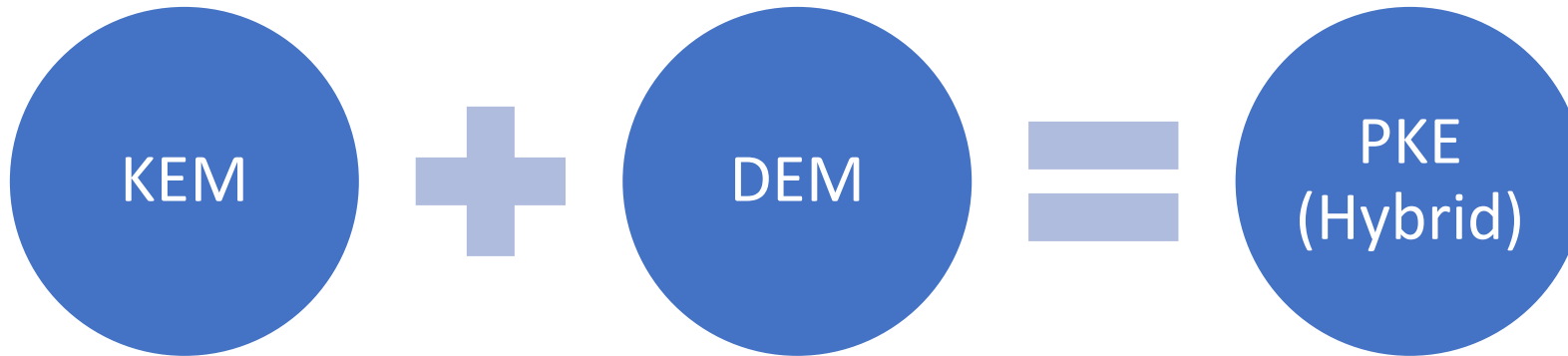
2.4.5 Encapsulation

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2. Compute $C_0 = \text{ENCODE}(e, T)$.
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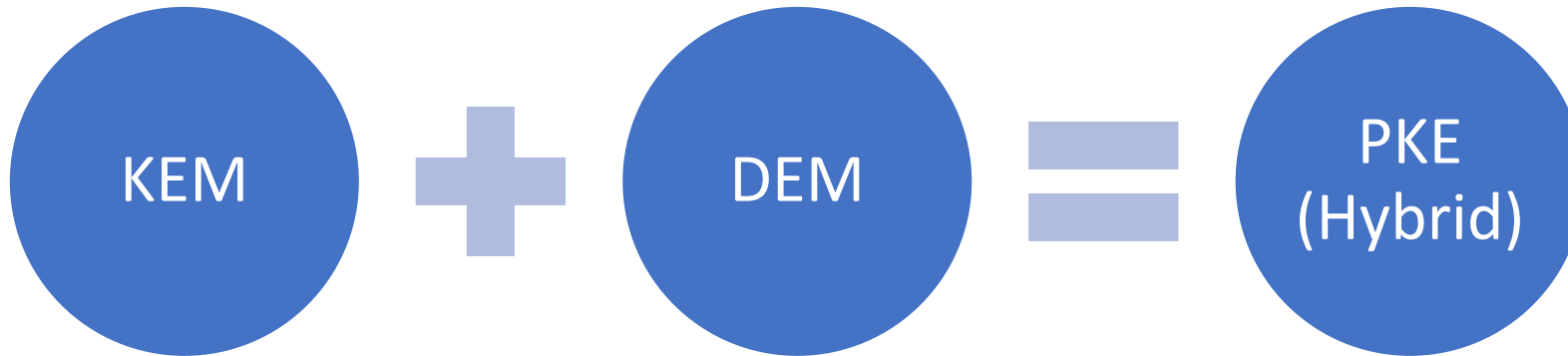
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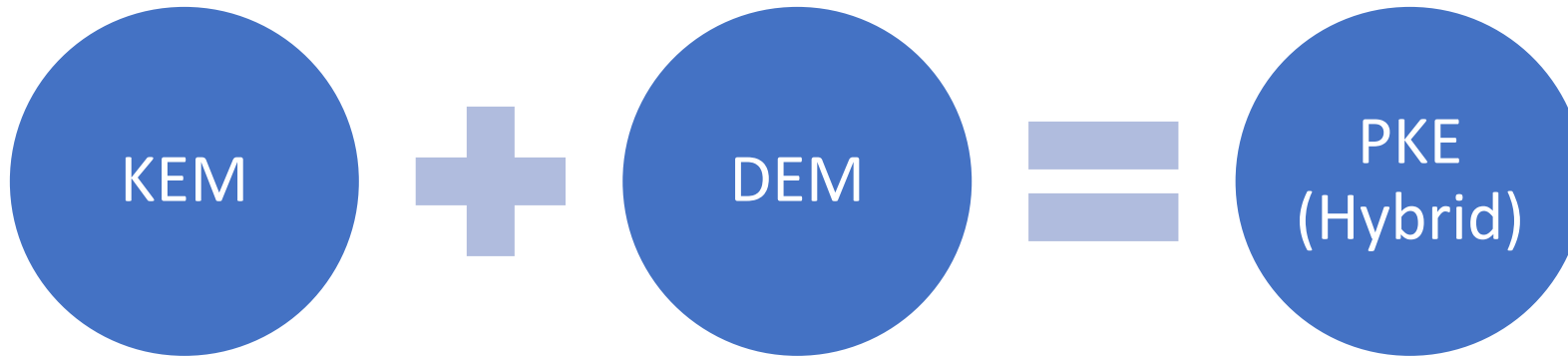
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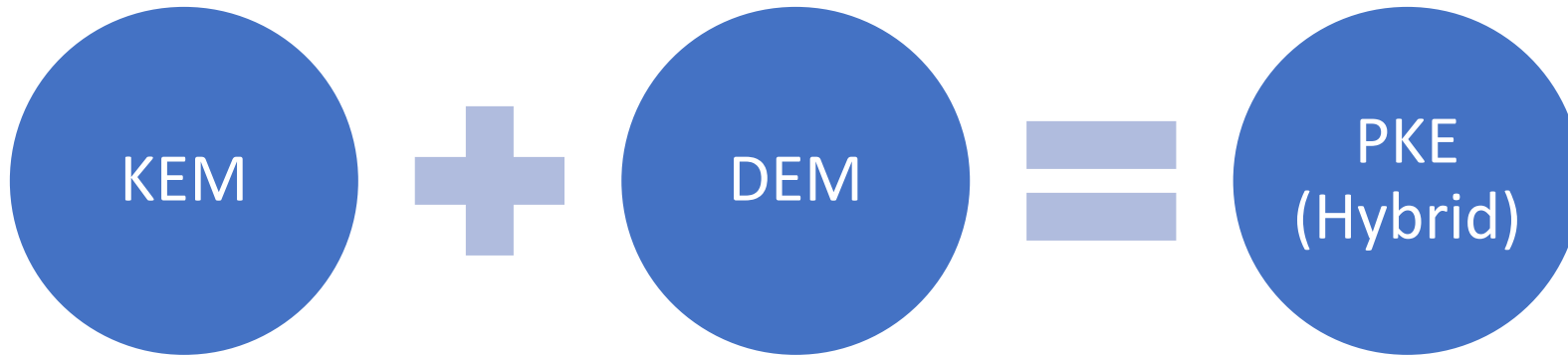
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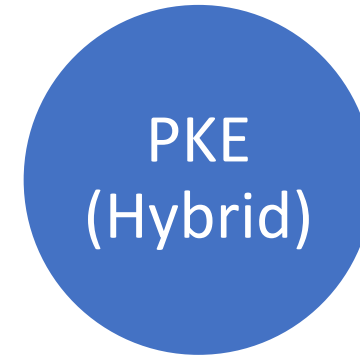
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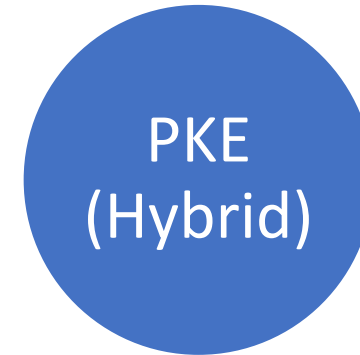
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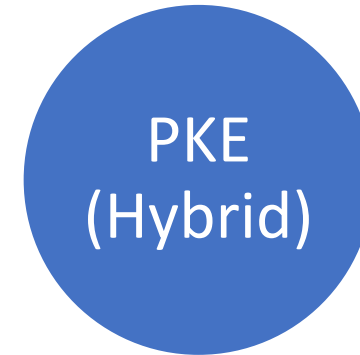
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$Dec(sk_*, c) = m (\neq \perp)$.

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Since CM closely follows the FO^{\neq} transform to construct its KEM, the analysis of [Jiang-Zhang-Chen-Wang-Ma'18] can be extended to CM to obtain (relatively) tight security bounds in the QRROM.

SaberCore vs Saber

SaberCore: the “core” scheme

Saber: the “actual” implemented scheme

SaberCore vs Saber

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2.5.2 Saber.KEM Key Encapsulation

The Saber key encapsulation is specified by the following algorithm and makes use of Saber.PKE.Enc as specified in Algorithm 2.

Algorithm 5: Saber.KEM.Encaps($pk := (seed_A, \mathbf{b})$)

- 1 $m \leftarrow \mathcal{U}(\{0, 1\}^{256})$
- 2 $(\hat{K}, r) = \mathcal{G}(\mathcal{F}(pk), m)$
- 3 $c = \text{Saber.PKE.Enc}(pk, m; r)$
- 4 $K = \mathcal{H}(\hat{K}, c)$
- 5 **return** (c, K)

8.5.2 Saber.KEM.Encaps

This function generates a session key and the ciphertext corresponding to the key. The algorithm is described in Alg 21.

Algorithm 21: Algorithm Saber.KEM.Encaps for generating session key and ciphertext.

Input: $PublicKey_{cca}$: public key generated by Saber.KEM.KeyGen

Output: $SessionKey_{cca}$: session key,

$CipherText_{cca}$: cipher text corresponding to the session key

- 1 $\text{randombytes}(m, \text{SABER_KEYBYTES})$
- 2 $\text{SHA3-256}(m, m, \text{SABER_KEYBYTES})$
- 3 $\text{SHA3-256}(\text{hash_pk}, PublicKey_{cca}, \text{SABER_INDCPA_PUBKEYBYTES})$
- 4 $buf = (\text{hash_pk} \parallel m)$
- 5 $\text{SHA3-512}(kr, buf, 2 \times \text{SABER_KEYBYTES})$
- 6 Split kr in two equal chunks of length SABER_KEYBYTES and obtain $(r \parallel k) = kr$
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- 11 **return** $(SessionKey_{cca}, CipherText_{cca})$

“ $k \leftarrow H(\hat{k}, H(c))$ ”

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SaberCore

**Saber: Module-LWR based key exchange,
CPA-secure encryption and CCA-secure KEM**

Jan-Pieter D’Anvers, Anshuman Karmakar
Sujoy Sinha Roy, and Frederik Vercauteren

imec-COSIC, KU Leuven
Kasteelpark Arenberg 10, Bus 2452, B-3001 Leuven-Heverlee, Belgium
firstname.lastname@esat.kuleuven.be

Theorem 6 (QROM, Jiang et al. [32]). *For any IND-CCA quantum adversary B , making at most $q_{\mathcal{H}}$ and $q_{\mathcal{G}}$ queries to respectively the random quantum oracle \mathcal{G} and \mathcal{H} , and q_D many (classical) queries to the decryption oracle, there exists an adversary A such that:*

$$Adv_{Saber.KEM}^{ind-cca}(B) \leq 2q_{\mathcal{H}} \frac{1}{\sqrt{2^{256}}} + 4q_{\mathcal{G}}\sqrt{\delta} + 2(q_{\mathcal{G}} + q_{\mathcal{H}})\sqrt{Adv_{Saber.PKE}^{ind-cpa}(A)}$$

[D’Anvers-Karmakar-Roy-Vercauteren’18]

SaberCore

Theorem 6.5. *In the quantum random oracle model, where \mathcal{G} and \mathcal{H} are assumed to be random oracles, for any IND-CCA quantum adversary B , making at most $q_{\mathcal{H}}$ and $q_{\mathcal{G}}$ queries to respectively the random quantum oracle \mathcal{G} and \mathcal{H} , and q_D many (classical) queries to the decryption oracle, there exists an adversary A , with approximately the same running time as B , such that:*

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[Saber's specification document]

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(Informal) **Theorem** [Grubbs-Maram-Paterson'21]:

*Hybrid PKE schemes obtained from $\text{FO}^{\not\perp}$ KEMs via the generic KEM-DEM composition are also ANO-CCA secure in the QROM.**

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Jiang et al. [24] provide a security reduction against a quantum adversary in the quantum random oracle model from IND-CCA security to OW-CPA security. IND-CPA with a

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<u>KGen'</u>	<u>Encap(pk)</u>	<u>Decap(sk', c)</u>
1 : (pk, sk) \leftarrow KGen	1 : $m \leftarrow_{\$} \mathcal{M}$	1 : Parse $\text{sk}' = (\text{sk}, s)$
2 : $s \leftarrow_{\$} \mathcal{M}$	2 : $r \leftarrow G(m)$	2 : $m' \leftarrow \text{Dec}(\text{sk}, c)$
3 : $\text{sk}' = (\text{sk}, s)$	3 : $c \leftarrow \text{Enc}(\text{pk}, m; r)$	3 : $r' \leftarrow G(m')$
4 : return (pk, sk')	4 : $k \leftarrow H(m, c)$	4 : $c' \leftarrow \text{Enc}(\text{pk}, m'; r')$
	5 : return (c, k)	5 : if $c' = c$ then
		6 : return $H(m', c)$
		7 : else return $H(s, c)$

FO^q

(Image taken from <https://eprint.iacr.org/2021/708.pdf> [Grubbs-Maram-Paterson'21])

(Image taken from <https://www.esat.kuleuven.be/cosic/pqcrypto/saber/files/saberspecround3.pdf>)

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FO[✗]

KGen'	Encap(pk)	Decap(sk', c)
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SaberCore

(Image taken from <https://eprint.iacr.org/2021/708.pdf> [Grubbs-Maram-Paterson'21])

(Image taken from <https://www.esat.kuleuven.be/cosic/pqcrypto/saber/files/saberspecround3.pdf>)

SaberCore

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 - Effectively, $\text{Decap}(sk, c)$ can be simulated by returning $H'(c)$.

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We were able to adapt the simulation trick to SaberCore, by observing that the nested hashing of m is *length-preserving*, i.e., $m \in \{0,1\}^{256}$ and $\hat{k} \in \{0,1\}^{256}$.

SaberCore

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(Informal) **Theorem** [Grubbs-Maram-Paterson’21]:

Hybrid PKE schemes obtained from SaberCore via the generic KEM-DEM composition in the QROM are:

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FrodoKEM

In addition, the following eight candidate algorithms will advance to the third round:

Alternate Candidates

Public-Key Encryption/KEMs

BIKE;

FrodoKEM

HQC

NTRU Prime

SIKE

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We expect our results on SaberCore to be applicable to the “actual” FrodoKEM.

CRYSTALS-KYBER, Saber

(Informal) **Theorem** [Grubbs-Maram-Paterson'21]:

*Hybrid PKE schemes obtained from $\text{FO}^{\not\sim}$ KEMs via the generic KEM-DEM composition are also ANO-CCA secure in the QRROM.**

*(*Provided the base PKE scheme satisfies some additional mild security properties.)*

CRYSTALS-KYBER and Saber use a transform that deviates *even further* from $\text{FO}^{\not\sim}$ when compared to SaberCore/FrodoKEM.

CRYSTALS-KYBER, Saber

KGen'	Encap(pk)	Decap(sk', c)
1: (pk, sk) \leftarrow KGen	1: $m \leftarrow_s \mathcal{M}$	1: Parse $sk' = (sk, pk, F(pk), s)$
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4: return (pk, sk')	4: $c \leftarrow \text{Enc}(pk, m; r)$	4: $c' \leftarrow \text{Enc}(pk, m'; r')$
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CRYSTALS-KYBER, Saber

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- The “nested” (compressing) hash of ciphertext, namely " $F(c)$ ", in the key-derivation " $k \leftarrow KDF(\hat{k}, F(c))$ " acts as a barrier w.r.t. establishing the IND-CCA security of CRYSTALS-KYBER and Saber in the QRROM with the claimed tightness.

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- *Because we recover a corresponding tight IND-CCA security proof in the QRROM,*
- *and obtain similarly tight security proofs of anonymity and robustness.*

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Need to re-evaluate the IND-CCA security claims of the finalists in the QROM.