CRYSTALS - Dilithium

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High-Level Scheme Overview

- “Schnorr-like” lattice-based signature scheme
- Based on the hardness of Module-SIS and Module-LWE
- All operations over $R=Z_q[X]/(X^{256}+1)$ for $q=8,380,417$

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KeyGen:
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Sign(pk,sk,μ):
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    $c=\mathrm{Hash}(w,\mu)$
    $z=sc+y$
    $\mathrm{RejectionSample}(pk,sk,z)$
    $\omega = \mathrm{HintVector}(pk,sk,z)$
    $\sigma = (z, \omega, c)$
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```
Verify(μ,σ,pk):
    $w=\mathrm{UseHintVector}(pk,\sigma)$
    check that $c=\mathrm{Hash}(w, \mu)$  and $|z|$ is small
```

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  - Added NIST Level 5 (was not mandatory, but became strongly recommended)
- Sampling in the signing procedure is now uniform within a range with 2^k elements – even simpler than before when the range wasn't a power-of-2
- Slightly simpler and shorter generation of the fixed-weight challenge polynomial

CRYSTALS-Dilithium

AVX2 + AES on Skylake
/ sec is assuming 3GHz freq.

Security Level	Public Key (Bytes)	Signature (Bytes)	pkgen	sign	verify
60	864	1196			
100	992	1843			
128 (NIST II)	1312	2420	50K cyc 60K / sec	150K cyc 20K / sec	65K cyc 45K / sec
2^{159} gates					
2^{98} memory					
192 (NIST III)	1952	3293	80K cyc 35K / sec	200K cyc 15K / sec	95K cyc 30K / sec
2^{217} gates					
2^{139} memory					
256 (NIST V)	2592	4595	125K cyc 24K / cyc	230K cyc 13K / sec	135K cyc 22K / sec
2^{285} gates					
2^{187} memory					
320	2912	5246			
384	3232	5892			

Many Efficiency Trade-Offs Possible

Implementation of Dilithium Signing on Cortex M3 and M4:

[Greconici, Kannwischer, Sprenkels 2020] (Speed numbers extrapolated because the number of repetitions changed)

NIST Level 3	Speed	RAM
Cortex M3	12M cycles	70KB
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[Gonzalez, Hulsing, Kannwischer, Kramer, Lange, Stottinger, Waitz, Wiggers, Yang 2021]

Verification can fit in under 8kB of RAM

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Comparison with Falcon

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Signing a few messages (≈ 100?) shouldn’t leak enough even if the sampling is leaky

Runtime Efficiency Comparison

Dilithium [Greconici, Kannwischer, Sprenkels 2020]

Falcon [Pornin, 2019]

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NIST Level 3	Ver. Speed	Ver. RAM
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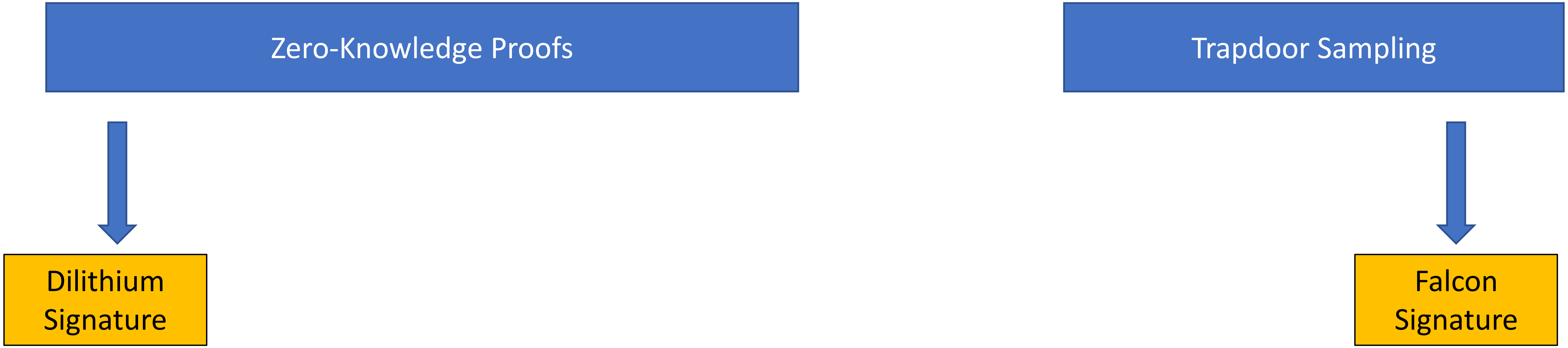
NIST Level 1	Ver. Speed	Ver. RAM
Cortex M4	0.5 M cycles	4KB

- 80% of Dilithium Verification Time is Keccak

Future Uses of Dilithium / Falcon Techniques



Zero-Knowledge Proofs

Dilithium Signature

Trapdoor Sampling

Falcon Signature

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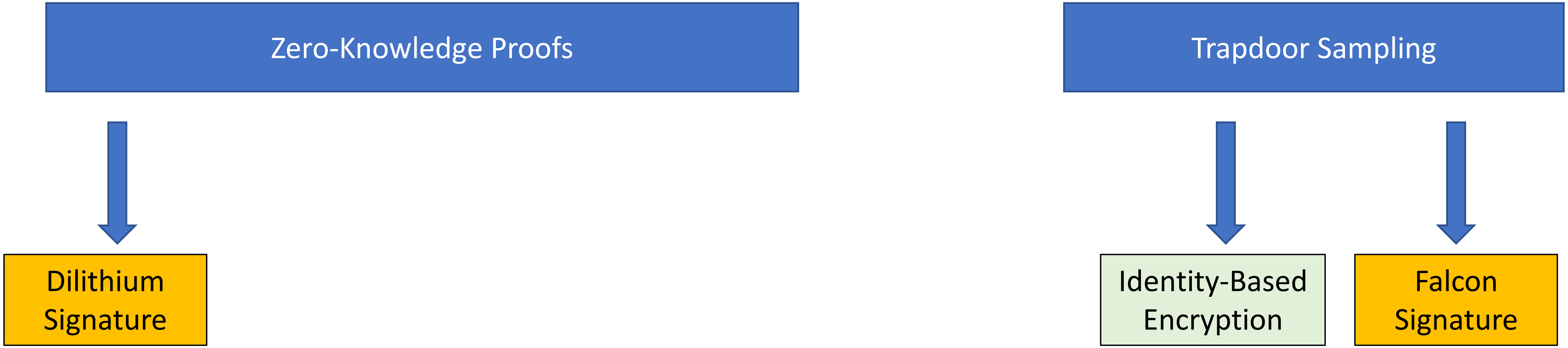

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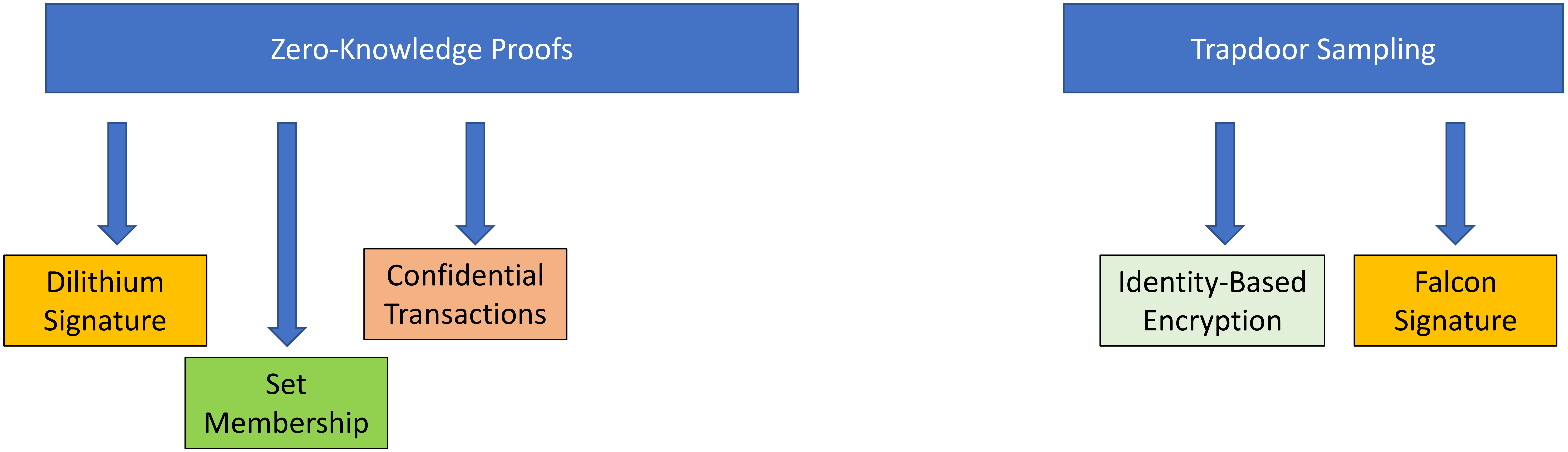

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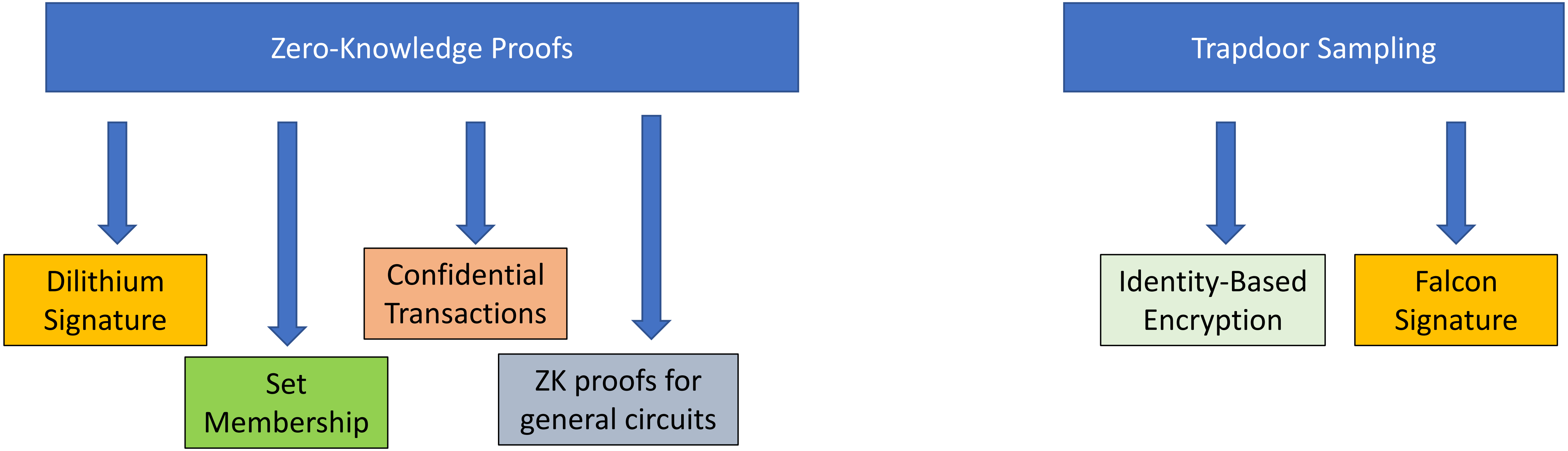

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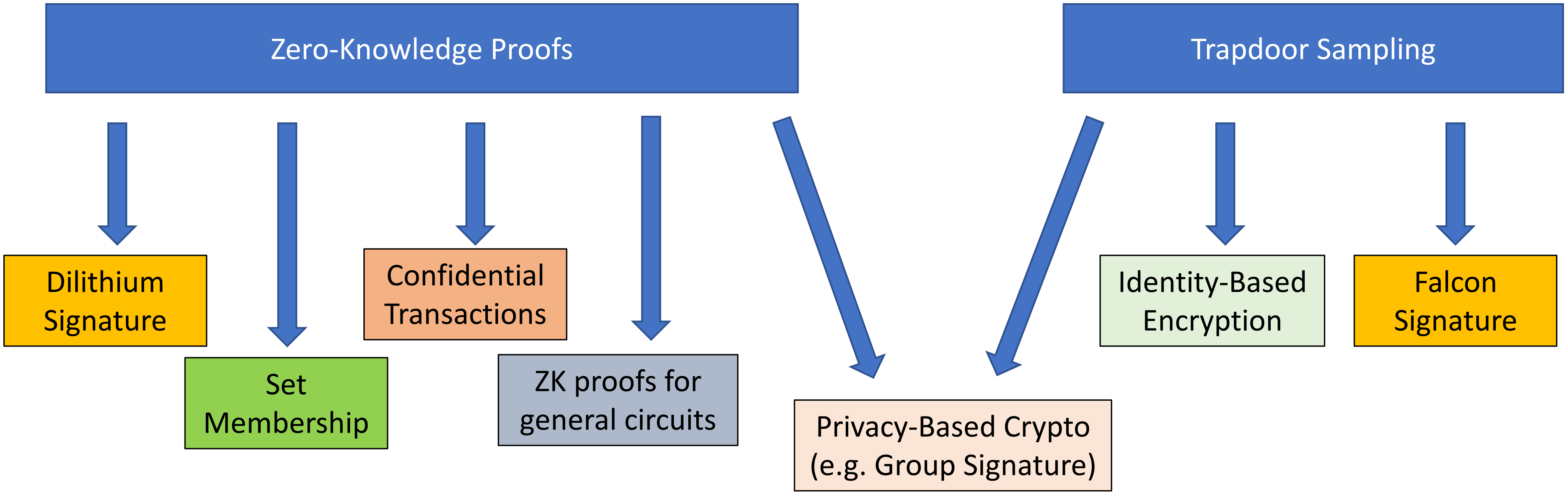

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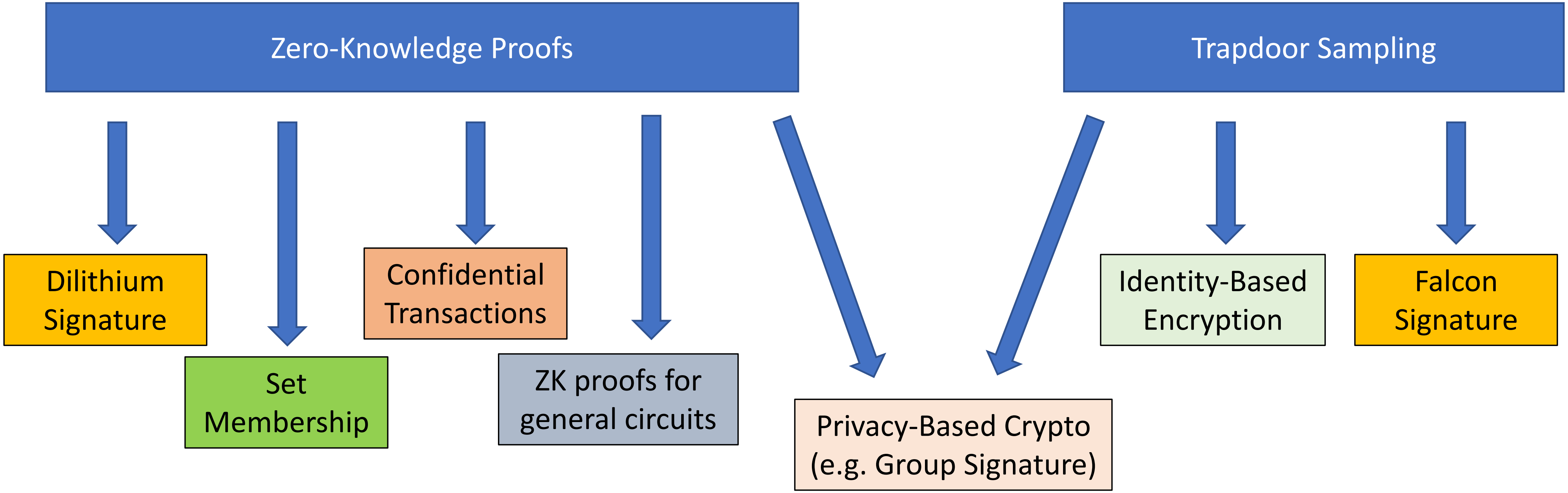

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Lattice-based ZK proofs improved by 3 orders of magnitude in the last 2 years

Lattices are currently the most efficient quantum-safe solution for many of these applications

We should probably get good at the techniques behind them

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  - Could also consider the Falcon ideas with less compact, but easier to use and mask (still Gaussian, though) samplers that don't require floating point ops in the "4th round" :
    - MITAKA [Espitau, Takahashi, Tibouchi, Wallet 2020]
    - Zalcon [Fouque, Gerard, Rossi, Yu 2021]

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https://pq-crystals.org/dilithium/index.shtml

https://github.com/pq-crystals/dilithium

https://github.com/pq-crystals/security-estimates