Faster Lattice-based KEMs via Fujisaki-Okamoto Transform in the Multi-User Setting via Prefix-Hashing

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Intro

$$\text{PKE} \xrightarrow[]{\text{Fujisaki-Okamoto (FO) Transform}} \text{KEM}$$

IND-CPA secure PKE → IND-CCA secure KEM

- Standard method of almost all NIST PQC Candidates: start with IND-CPA secure PKE and apply variant of FO [FO99, FO13, HHK17]
- $\mathsf{Encaps}_{pk}(;r) = (\underbrace{\mathsf{Enc}_{pk}(r;\mathsf{G}(r))}_{\text{ciphertext}}, \underbrace{\mathsf{H}(r)}_{key})$

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- Common safeguard against *multi-user attacks:* hash also *public-keys*, notably done by Kyber and Saber
- We *formally* show:
  1. this indeed improves multi-user security.
  2. Too wasteful: hashing a short prefix of pk, gives the same security guarantees

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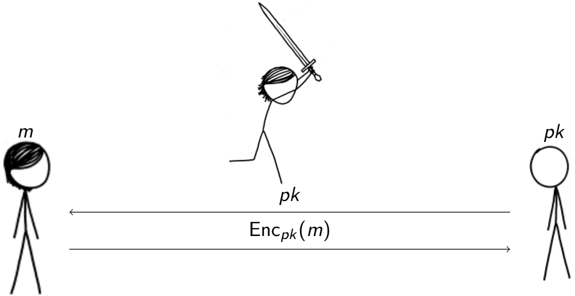
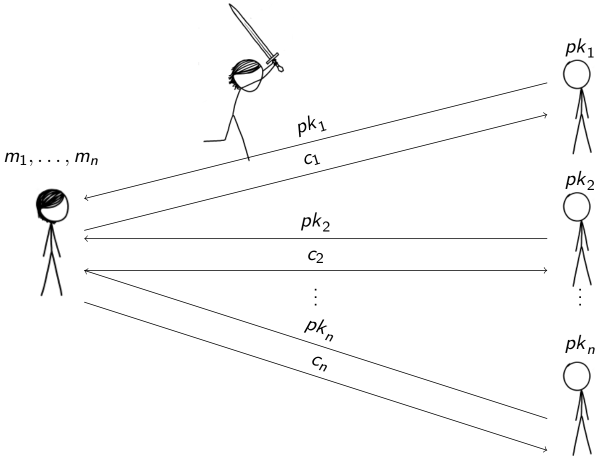
- without weakening multi-user security

Single-User IND-CPA



- Adversary wants to learn some information on plaintext m

Multi-User IND-CPA (n-IND-CPA)



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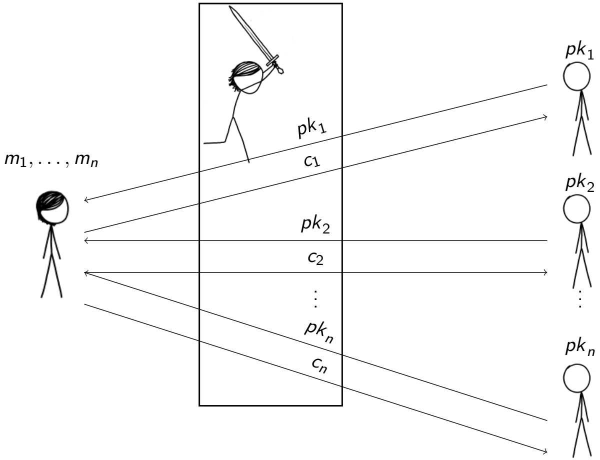

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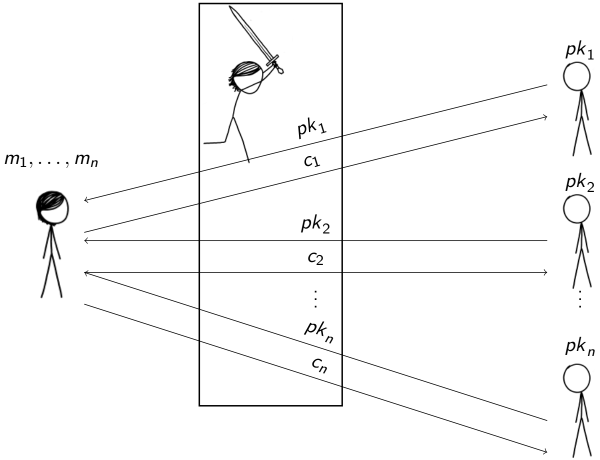

- $\Rightarrow$ Adversary wants to learn some information on the n plaintexts $m_1, \ldots, m_n$

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- $\implies$ PKE needs to be instantiated with worse parameters ☹
- We show: a direct proof of n-IND-CCA yields direct reduction to n-IND-CPA security of PKE
- Beats the hybrid argument if:
  - $\mathsf{Adv}_{\mathsf{PKE}}^{\text{n-IND-CPA}} \ll n \cdot \mathsf{Adv}_{\mathsf{PKE}}^{\text{IND-CPA}}$

FO without *pk* hashing

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- Advantage: more efficient than additionally hashing pk
- Disadvantage: worse multi-user security

FO with Public-Key Hashing

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- $\mathsf{Encaps}_{pk}(;r) = (\underbrace{\mathsf{Enc}_{pk}(r; \mathsf{G}(pk, r))}_{\text{ciphertext}}, \underbrace{\mathsf{H}(pk, r)}_{\text{key}})$
- (Essentially) used by Kyber and Saber to protect against multi-user attacks
- Advantage: improves multi-user security
- Disadvantage: wasteful if e.g. $|pk| \approx 1KB$

FO with Prefix Hashing

- $\mathsf{Encaps}_{pk}(;r) = (\underbrace{\mathsf{Enc}_{pk}(r; \mathsf{G}(\mathsf{id}, r))}_{\text{ciphertext}}, \underbrace{\mathsf{H}(\mathsf{id}, r)}_{\text{key}})$
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- Best of both worlds: improves multi-user security and (almost) as efficient as without any pk hashing

Correctness Errors

- [HHK17] δ-Correctness of PKE informally:
  probability of decryption error for a random key
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- For Kyber and Saber: $\delta < \delta(n) < n \cdot \delta$.

Results (Simplified)

FO variant	$\mathsf{Adv}_{\mathsf{KEM}}^{\text{n-IND-CCA}}$ (ROM)	$\mathsf{Adv}_{\mathsf{KEM}}^{\text{n-IND-CCA}}$ (QROM)
pk hashing	$\mathsf{Adv}_{\mathsf{PKE}}^{\text{n-IND-CPA}} + q_{\mathsf{RO}}\delta(n)$	$\sqrt{q_{\mathsf{RO}}\mathsf{Adv}_{\mathsf{PKE}}^{\text{n-IND-CPA}}} + q_{\mathsf{RO}}^2\delta(n)$
prefix hashing	$\mathsf{Adv}_{\mathsf{PKE}}^{\text{n-IND-CPA}} + q_{\mathsf{RO}}\delta(n) + \frac{n^2}{2^\ell}$	$\sqrt{q_{\mathsf{RO}}\mathsf{Adv}_{\mathsf{PKE}}^{\text{n-IND-CPA}}} + q_{\mathsf{RO}}^2\delta(n) + \frac{n^2}{2^\ell}$
no hashing	$\mathsf{Adv}_{\mathsf{PKE}}^{\text{n-IND-CPA}} + q_{\mathsf{RO}}n\delta(1)$	$n \cdot \sqrt{q_{\mathsf{RO}}\mathsf{Adv}_{\mathsf{PKE}}^{\text{IND-CPA}}} + q_{\mathsf{RO}}^2 n\delta(1)$

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- $\{pk, \text{prefix}\}$-hashing security $>$ no hashing security
- prefix hashing efficiency $\gg$ *pk*-hashing efficiency
- $\Rightarrow$ use prefix hashing

Application to Kyber and Saber

- FO with Prefix Hashing yields significant speed up to Kyber and Saber
- Speedup of Kyber is larger, due to the efficiency of the underlying IND-CPA-secure PKE

NIST Level		Kyber			Saber		
		Original	This Work	Speed-up	Original	This Work	Speed-up
1	**K**	23562	12883	45%	42169	36220	14%
	E	37144	16981	54%	57831	39232	32%
	D	28595	28529	0%	57780	57806	0%
3	**K**	40487	25272	**38%**	74577	64180	**14%**
	E	55726	27624	**50%**	95958	69304	**28%**
	D	43553	43442	0%	95388	95301	0%
5	**K**	55770	38815	30%	116178	102101	12%
	E	77011	40692	47%	142034	109203	23%
	D	61470	61473	0%	142957	143090	0%

Conclusion

- For multi-user security, hashing the prefix of a public-key seems to be the right thing to do in the context of the FO
- Prefix hashing (more than) satisfies the NIST Security requirements
- Significant speedup for Kyber and Saber key-generation and encapsulation using prefix hashing, up to $(56 - 66\ \%)$ and $(30 - 39\ \%)$
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- Thank you for your attention

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