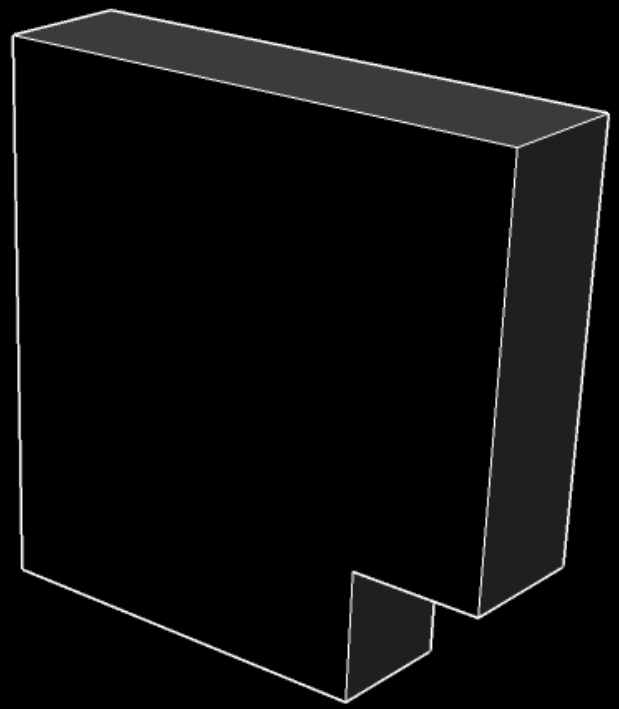
LUOV



Ward Beullens, Bart Preneel, Alan Szepieniec, Frederik Vercauteren

Overview

What is LUOV? (baby don't hurt me)

Unbalanced Oil and Vinegar (UOV) [Patarin 1997]

- Quadratic trapdoor function: $\mathcal{P} : \mathbb{F}_q^n \to \mathbb{F}_q^m$ with $n > m$.
- Trapdoor is a factorization of $\mathcal{P}$ $\quad$ $\mathcal{F} \circ \mathcal{T}$, where $\mathcal{T}$ is linear and $\mathcal{F}$ linear in the last m variables (oil variables).
- Well understood signature scheme, fast, small signatures, but large keys. Used as building block for other MQ schemes (e.g. Rainbow).

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- Generate SK from seed
- Generate most of PK from seed [Petzoldt]
- Field lifting

Given a UOV key pair $(\mathcal{P}, \mathcal{T})$ over $\mathbb{F}_2$, we can use it as a key pair over $\mathbb{F}_{2^r}$.

$$\underbrace{\begin{matrix} x_1^2 + x_1x_2 + x_3 + x_1x_4 + x_4x_5 + x_5 \\ x_2x_3 + x_3^2 + x_2x_6 + x_3x_4 + x_3x_5 + x_6^2 \\ x_1x_2 + x_2x_3 + x_3x_4 + x_2 + x_5x_6 \end{matrix}}_{\mathcal{P}(\mathbf{x})} \quad \underbrace{\begin{matrix} 1 + \alpha^2 + \quad + \alpha^{30} \\ 1 + \alpha + \quad + \alpha^{31} \\ \alpha + \alpha^5 + \quad + \alpha^{31} \end{matrix}}_{\mathcal{H}(M)}$$

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Field Lifting Assumption:

Solving a random system $\mathcal{P}(x) \quad y$ over $\mathbb{F}_{2^r}$ is as hard as solving a random system $\mathcal{P}(x) \quad y$, where $\mathcal{P}$ is defined over $\mathbf{F}_2$,

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Attacks

- Key recovery attacks
  Studied since 1997
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Claimed complexity of the attack:

Parameters	Security lvl	Subfield	Complexity
LUOV-8-58-237	2	$\mathbb{F}_{2^2} \subset \mathbb{F}_{2^8}$	2^{107}
LUOV-48-43-222	2	$\mathbb{F}_{2^8} \subset \mathbb{F}_{2^{48}}$	2^{135}

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We study some generalization of the attack in revised LUOV submission document.

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- Break up PRNG calls into multiple smaller calls.
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- Constant time AVX2 optimized implementation.
- Add option to use Chacha8 instead of SHAKE to expand public randomness. $\Rightarrow$ $\times 2.5$ and $\times 5.2$ faster signing and verification respectively (SL1).

Round 2.1 modifications

- Choose field extension of prime degree.

Original	New
$\mathbb{F}_{2^8}$	$\mathbb{F}_{2^7}$
$\mathbb{F}_{2^{48}}$	$\mathbb{F}_{2^{47}}$
$\mathbb{F}_{2^{64}}$	$\mathbb{F}_{2^{61}}$
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Updated submission package will be online next week.

Some numbers

Key and signature sizes for SL1:

	$\lvert\text{sig}\rvert$	Δ	$\lvert\text{pk}\rvert$	Δ	$\lvert\text{sk}\rvert$
LUOV-7-57-197	239 B	-23%	11.5 KB	-5%	32B
LUOV-47-42-182	1332 B	-17%	4.7 KB	-6%	32B

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[2] Requires 23 KB to store partial signature

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Performance of AVX2 constant-time implementation (SL I):

	PRG	keygen (cycles)	sign (cycles)	verify (cycles)
Standard LUOV	Keccak	1.9 M	1.4 M	1.0 M
	Chacha8	1.1M	515 K	197 K
Precompute Keys[1]	⋆		300 K	90 K
Finish signature [2]	⋆		11 K	

[1] Requires 250 KB to store expanded PK or SK

[2] Requires 23 KB to store partial signature

Conclusion (part 1)

Disadvantages:

- Public key size (11.5 KB)
- Relatively new LUOV assumption

Advantages:

- Small signatures (239 B)
- Small private key (32 B)
- Solid foundation (UOV)
- Simple arithmetic ($\mathbb{F}_{2^7}$)
- Low latency signing (11K cycles)
- No patent claims

Conclusion (part 2)

"All you need is LUOV"

John Lennon

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Questions?