NTRU

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Second round update
2019-08-24

NTRU-HRSS-KEM (NIST Round 1)

- Perfect correctness
- Arbitrary-weight trinary vectors
- One nice parameter set
- Probabilistic encryption
- CCA2 KEM via Dent “Table 5” / Targhi–Unruh

NTRUEncrypt (NIST Round 1)

- Imperfect correctness
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- CCA2 PKE via NAEP

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Saito–Xagawa–Yamakawa (Eurocrypt 2018)

- Deterministic encryption
- CCA2 KEM via re-encryption and implicit rejection

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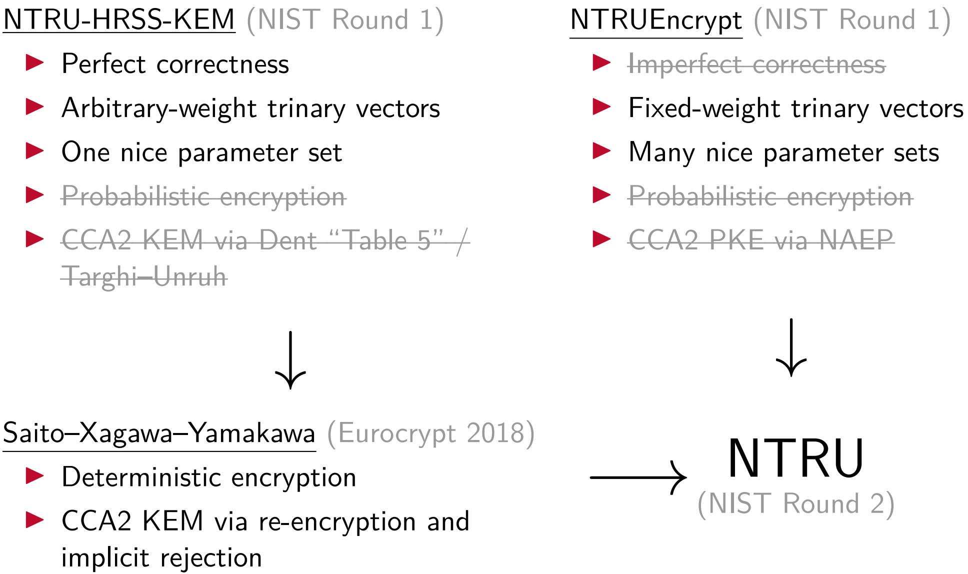
(NIST Round 2)



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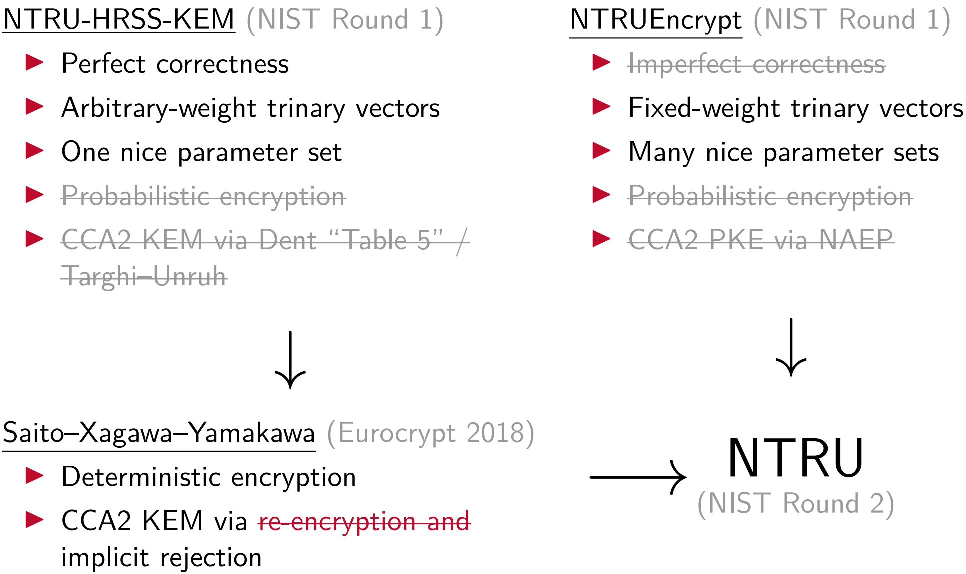

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(NIST Round 2)

Eliminating re-encryption: rigidity

Bernstein–Persichetti (ePrint 2019/256):

$$\text{ROM CCA2 KEM} \;\leq\; \text{correct rigid deterministic PKE} + \text{implicit rejection}$$

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$$\text{Rigid: } (\text{Encrypt}(m) = c) \Leftarrow (\text{Decrypt}(c) = m)$$

Rigidity is often enforced through re-encryption...
some schemes can avoid it.

NTRU

- Integer parameters n and q.
- Polynomial arithmetic modulo $\mathbf{x}^n - 1 = \mathbf{\Phi}_1\mathbf{\Phi}_n$.
- **Private key:** A pair of polynomials $(\mathbf{f}, \mathbf{g})$.
- **Public key:** A polynomial $\mathbf{h}$ that satisfies
  - $\mathbf{hf} \equiv 3\mathbf{g} \pmod{(q, \mathbf{\Phi}_1\mathbf{\Phi}_n)}$, and
  - $\mathbf{h} \equiv 0 \pmod{(q, \mathbf{\Phi}_1)}$.
- **Plaintext:** A pair of polynomials $(\mathbf{r}, \mathbf{m})$, with
  - $\mathbf{m} \equiv 0 \pmod{(q, \mathbf{\Phi}_1)}$.
- **Ciphertext:** $\mathbf{c} = \mathbf{rh} + \mathbf{m} \bmod (q, \mathbf{\Phi}_1\mathbf{\Phi}_n)$.
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- Check: (Decrypt$(c) = m$) $\Rightarrow$ (Encrypt$(m) = c$).

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Decrypt((f, h), c)
1: $\mathbf{a} = (\mathbf{cf}) \bmod (q, \Phi_1\Phi_n)$
2: $\mathbf{m} = (\mathbf{a}/\mathbf{f}) \bmod (3, \Phi_n)$
3: $\mathbf{r} = ((\mathbf{c} - \mathbf{m})/\mathbf{h}) \bmod (q, \Phi_n)$
4: if $\mathbf{c} \equiv 0 \pmod{(q, \Phi_1)}$ and
   $(\mathbf{r}, \mathbf{m})$ is in the message space
   then
5:     return $(\mathbf{r}, \mathbf{m})$
6: end if
7: return $\perp$
```

Suppose Decrypt$((\mathbf{f}, \mathbf{h}), \mathbf{c}) = (\mathbf{r}, \mathbf{m})$. Then, by Line 3,

$$\begin{aligned} \text{Encrypt}(\mathbf{h}, (\mathbf{r}, \mathbf{m})) &= \mathbf{rh} + \mathbf{m} \bmod (q, \Phi_1\Phi_n) \\ &\equiv \mathbf{c} \pmod{(q, \Phi_n)} \end{aligned}$$

Lines 4-7 provide rigidity because

1. $\mathbf{h} \equiv 0 \bmod (q, \Phi_1)$, and
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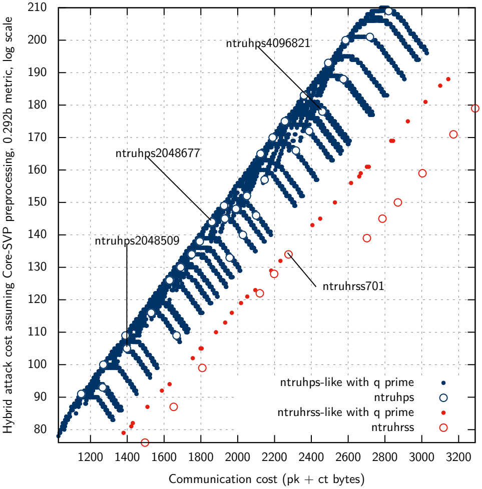
Eliminating re-encryption: implicit rejection

- The user stores an additional 256 bit secret, s.

```
Encaps($\mathbf{h}$):
    Sample $\mathbf{r}$ and $\mathbf{m}$.
    return $\mathbf{rh} + \mathbf{m} \bmod (q, \mathbf{\Phi}_1\mathbf{\Phi}_n)$.
```

```
Decaps($(\mathbf{f}, \mathbf{h}, s), \mathbf{c}$):
    $result$ = Decrypt($(\mathbf{f}, \mathbf{h}), \mathbf{c}$)
    if $result = \perp$ then
        return $\mathrm{SHA3\text{-}256}(s \mid \mathbf{c})$
    else
        return $\mathrm{SHA3\text{-}256}(result)$
    end if
```

Parameter selection process



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Recommended parameters

	pk bytes	ct bytes	Core-SVP dim.
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ntruhps2048509	171**k**	38**k**	49**k**
ntruhps2048677	292**k**	53**k**	73**k**
ntruhrss701	283**k**	52**k**	76**k**
ntruhps4096821	-	-	-

$\mathbf{k} = 1000$ Haswell cycles.

one second $= 3\,100\,000\mathbf{k}$

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ntruhps2048677	277**k**	35**k**	69**k**
ntruhrss701	255**k**	27**k**	71**k**
ntruhps4096821	-	-	-

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Faster key generation?

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Optimize! Most expensive component is inversion mod $(3, \mathbf{\Phi}_n)$:

- Original ntruhrss701 software:
  150**k** Haswell cycles
- New software from Dan Bernstein and Bo-Yin Yang, ePrint 2019/266:
  90**k** Haswell cycles

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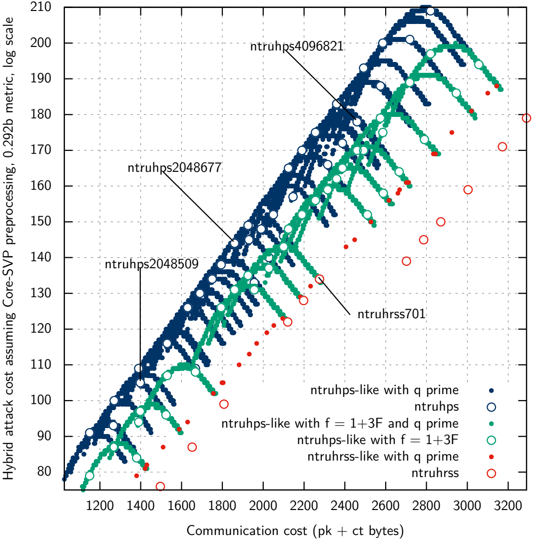
Other avenues to explore:

- Use $\mathbf{f} = 1 + 3\mathbf{F}$ in an ephemeral-only setting.
- Choose perfectly correct parameters compatible with $\mathbf{f} = 1 + 3\mathbf{F}$.

Neither option is currently recommended.

Correct parameters with faster key gen



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