On the Security of COMET Authenticated Encryption Scheme

NIST Lightweight Workshop '19

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Motivation

Lightweight Authenticated Encryption Design

- Block cipher based.
- Rate-1.
- Small state size (close to $(n + \kappa)$-bit).
- Simple design (simple operations like XOR, shifts and rotations).

COunter Mode Encryption with authentication Tag

Design Summary

- Rate-1 and Feedback-based authenticated encryption mode.
- Combined feedback function:

  input is a function of current output and next plaintext block.

- Nonce and block counter-based rekeying.
- Parametrized by the block size, $n \in \{64, 128\}$. Tag size $t = n$.
- Two variants:
  - COMET-128: Here $n = 128$, key size $\kappa = 128$, nonce size $r = 128$.
  - COMET-64: Here $n = 64$, key size $\kappa = 128$, nonce size $r = 120$.

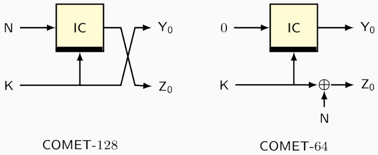
Nonce-based Initial State Derivation

- For COMET-128:

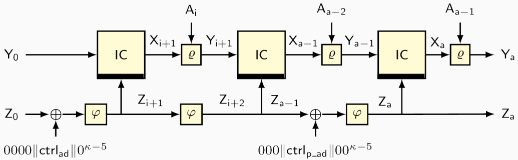
$$(Y_0, Z_0) := (K, IC_K(N))$$

- For COMET-64:

$$(Y_0, Z_0) := (IC_K(0), K \oplus N \| 0^{32})$$



Associated Data Processing



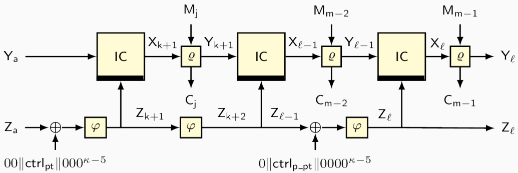
Here,

$$0 \le i \le a - 3$$

$$\mathsf{ctrl}_{\mathsf{ad}} = \begin{cases} 1 & \text{if } |A| > 0, \\ 0 & \text{o.w.} \end{cases}$$

$$\mathsf{ctrl}_{\mathsf{p\_ad}} = \begin{cases} 1 & \text{if } |A_{a-1}| < n, \\ 0 & \text{o.w.} \end{cases}$$

Plaintext Processing



Here,

$0 \leq j \leq m-3,$

$k = a + j$, and

$\ell = a + m$

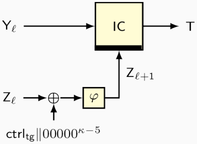
$$\mathsf{ctrl_{pt}} = \begin{cases} 1 & \text{if } |M| > 0, \\ 0 & \text{o.w.} \end{cases}$$

$$\mathsf{ctrl_{p\_pt}} = \begin{cases} 1 & \text{if } |M_{m-1}| < n, \\ 0 & \text{o.w.} \end{cases}$$

Ciphertext processing is symmetrically defined.

COMET : High-level Overview

Tag Generation



Here,

$$\ell = a + m$$

$$\mathsf{ctrl}_{\mathsf{tg}} = \begin{cases} 1 & \text{for tag generation,} \\ 0 & \text{o.w.} \end{cases}$$

Design Features

- Design simplicity: Only requires shift and XOR operations apart from block cipher calls.
- Small state size: Possibility of close to $(n + \kappa)$-bit state size in area optimized implementation.
- Inverse free: No need for block cipher decryption.
- Dynamic key updation: No two blocks share the same key non-trivially.
- Efficiency: Single-pass scheme.

COMET : Recommendations

Submissions to NIST LwC Standardization Project

- COMET-128_AES-128/128 instantiated with AES-128/128. [Primary]
- COMET-128_CHAM-128/128 instantiated with CHAM-128/128.
- COMET-64_Speck-64/128 instantiated with Speck-64/128.
- COMET-64_CHAM-64/128 instantiated with CHAM-64/128.

COMET : Security Claims

Submissions	Confidentiality		Integrity	
	Time	Data (in bytes)	Time	Data (in bytes)
COMET-128_AES-128/128	2^{119}	2^{64}	2^{119}	2^{64}
COMET-128_CHAM-128/128	2^{119}	2^{64}	2^{119}	2^{64}
COMET-64_Speck-64/128	2^{119}	2^{64}	2^{112}	2^{45}
COMET-64_CHAM-64/128	2^{119}	2^{64}	2^{112}	2^{45}

We focus on the security of COMET-128 .

COMET-128 : Security Model

AEAD Security Game

- Indistinguishability game between the ideal system $\mathcal{O}_0$ and real system $\mathcal{O}_1$, where

$$\mathcal{O}_0 := (\$, \perp, \mathsf{IC}^{\pm}) \quad \mathcal{O}_1 := (\text{COMET-128}.\mathsf{E}_\mathsf{K}, \text{COMET-128}.\mathsf{E}_\mathsf{K}, \mathsf{IC}^{\pm}).$$

- Advantage of any adversary $\mathscr{A}$ against COMET-128 is defined as:

$$\mathbf{Adv}^{\text{aead}}_{\text{COMET-128}}(\mathscr{A}) := \left| \Pr\left[\mathscr{A}^{\mathcal{O}_1} = 1\right] - \Pr\left[\mathscr{A}^{\mathcal{O}_0} = 1\right] \right|.$$

- $\mathscr{A}$ is computationally unbounded, but bounded in number of queries to its oracle.
- $\mathscr{A}$ operates under two restrictions:
  - Nonce-respecting: No two encryption query share the same nonce.
  - Non-trivial forger: An encryption query (N, A, M) yields (C, T), a decryption query (N, A, C, T) is not allowed.

COMET-128 : Security Result

Theorem

For $\sigma_e, \sigma_d < 2^{127}$, $q_p < 2^{127}$, and $(q_e, q_d, \sigma_e, \sigma_d, q_p)$-adversary $\mathscr{A}$ we have

$$\mathbf{Adv}^{\mathrm{aead}}_{\mathrm{COMET\text{-}128}}(\mathscr{A}) \leq \frac{4\sigma_c^2}{2^{256}} + \frac{14\sigma_c q_p}{2^{249}} + \frac{3\sigma_c^2}{2^{128}} + \frac{3.01 q_p}{2^{121}} + \frac{4\sigma_c}{2^{128}} + \frac{q_c}{2^{64}} + \frac{6 q_p \sigma_d}{2^{188.5}}$$

- q_e and q_d denote the number of queries to COMET-128.E_K and COMET-128.D_K, respectively.
- σ_e and σ_d denote the sum of input (associated data and message) lengths across all encryption and decryption queries, respectively;

  $q_c = q_e + q_d$ and $\sigma_c = \sigma_e + \sigma_d$.
- q_p denotes the number of direct queries to the block cipher.

Proof tool: Coefficient-H Technique

- Concentrates on the query-response tuple, called the transcript, generated by $\mathscr{A}$'s interaction with the oracle at hand.
- Let Θ_1: transcript random variable corresponding to $\mathcal{O}_1$.
- Let Θ_0: transcript random variable corresponding to $\mathcal{O}_0$.
- Identify a set of bad transcripts, Ω_{bad}.
- Compute $\Pr\left[\Theta_0 \in \Omega_{\mathsf{bad}}\right] \leq \epsilon_{\mathsf{bad}}$.
- Show that $\dfrac{\Pr\left[\Theta_1 = \omega\right]}{\Pr\left[\Theta_0 = \omega\right]} \geq (1 - \epsilon_{\mathsf{ratio}})$ for all $\omega \notin \Omega_{\mathsf{bad}}$.
- Then, $\mathbf{Adv}^{\mathsf{aead}}_{\mathsf{COMET\text{-}128}}(\mathscr{A}) \leq \epsilon_{\mathsf{bad}} + \epsilon_{\mathsf{ratio}}$.

COMET-128 : Security Proof Sketch

Notational Conventions

- Variables in encryption queries are defined as per the figures.
- Variables in decryption queries are defined analogously, topped with a bar.
- Variables in primitive queries are defined analogously, topped with a hat.

Oracle description

- Real oracle: Faithfully responds to encryption, decryption and primitive queries.
- Ideal oracle:

  For the encryption query: samples $X_1, \ldots, X_\ell, T \leftarrow_\$ \{0,1\}^n$, and sets $(Y_j, C_j) = \varrho(X_{a+j+1}, M_j)$ for all $0 \le j \le m$. Sets $Y_j = X_j \oplus A_j$ for $1 \le j \le a$. Returns (C, T).
  For decryption query: Returns $\perp$ symbol.
  For primitive query: Responds faithfully using $IC^{\pm}$.
- After the query phase, both the oracles release all encryption query internal variables and the secret key.

Identifying bad events

- `Kcoll` (key guessing/recovery):

  B1: $\exists i \in [q_e], j \in [m^i]$, such that $\mathsf{Z}_j^i = \mathsf{K}$.
  B2: $\exists i \in [q_d], j \in [\bar{m}^i]$, such that $\bar{\mathsf{Z}}_j^i = \mathsf{K}$.
  B3: $\exists i \in [q_p]$, such that $\widehat{\mathsf{Z}}^i = \mathsf{K}$.
  B4: $\exists i \in [q_e]$, such that $\mathsf{Z}_0^i = *\|0^{n/2}$.
  B5: $\exists i \in [q_d]$, such that $\bar{\mathsf{Z}}_0^i = *\|0^{n/2}$.
  B6: $\exists (i,j) \in [q_e] \times [m^i], (i',j') \in [q_d] \times [\bar{m}^{i'}]$, such that $\mathsf{N}^i \neq \bar{\mathsf{N}}^{i'}$ and $\mathsf{Z}_j^i = \bar{\mathsf{Z}}_{j'}^{i'}$.

- `EEmatch` (encryption-encryption state matching):

  B7: $\exists (i,j) \in [q_e] \times [m^i], (i',j') \in [q_e] \times [m^{i'}]$, such that $(\mathsf{Z}_j^i, \mathsf{Y}_j^i) = (\mathsf{Z}_{j'}^{i'}, \mathsf{Y}_{j'}^{i'})$.
  B7: $\exists (i,j) \in [q_e] \times [m^i], (i',j') \in [q_e] \times [m^{i'}]$, such that $(\mathsf{Z}_j^i, \mathsf{X}_j^i) = (\mathsf{Z}_{j'}^{i'}, \mathsf{X}_{j'}^{i'})$.

Identifying bad events

- `EPmatch` (encryption-primitive state matching):

  B9: $\exists(i,j) \in [q_e] \times [m^i]$ and $i' \in [q_p]$, such that $(\mathsf{Z}^i_j, \mathsf{Y}^i_j) = (\widehat{\mathsf{Z}}^{i'}, \widehat{\mathsf{Y}}^{i'})$.

  B10: $\exists(i,j) \in [q_e] \times [m^i]$ and $i' \in [q_p]$, such that $(\mathsf{Z}^i_j, \mathsf{X}^i_j) = (\widehat{\mathsf{Z}}^{i'}, \widehat{\mathsf{X}}^{i'})$.

- `EPKcoll` (technical requirement: key exhaustion via primitive query):

  B11: $\exists(i,j) \in [q_e] \times [m^i]$ such that $|\{j \in [q_p] : \widehat{\mathsf{Z}}^j = \mathsf{Z}^i\}| \geq 2^{n-1}$.

COMET-128 : Security Proof Sketch

Identifying bad events

- Chain (valid forgery via primitive (and encryption) queries):
  Let domain$(\omega_p) := \{(\hat{Z}_i, \hat{Y}_i)\}_{i \in [q_p]}$ and range$(\omega_p) := \{(\hat{Z}_i, \hat{X}_i)\}_{i \in [q_p]}$.

$$\delta_i := \begin{cases} \max_{\bar{C}^i_{0\ldots k-1} = C^j_{0\ldots k-1}} (\bar{a}^i + k) & \text{if } \bar{A}^i = A^j \wedge (\bar{A}^i, \bar{C}^i) \neq (A^j, C^j) \\ \max_{\bar{A}^i_{0\ldots k-1} = A^j_{0\ldots k-1}} (k) & \text{otherwise.} \end{cases}$$

$$\delta'_i := \begin{cases} \max_{\bar{X}^i_{\delta_i+1}, \ldots, \bar{X}^i_j \in \text{range}(\omega_p)} (j) & \text{if } \bar{X}^i_{\delta_i+1} \in \text{range}(\omega_p) \\ \delta_i & \text{otherwise.} \end{cases}$$

B12: chain using primitive queries
$\exists i \in [q_d]$ such that $\delta_i \geq 0$, $\delta'_i = \bar{\ell}^i$ and $\bar{X}^i_{\bar{\ell}^i+1} = \bar{T}^i$.

B13: partial chain using primitive queries followed by encryption query
$\exists i \in [q_d], (i', j') \in [q_e] \times [m^{i'}]$ such that $0 \leq \delta_i < \delta'_i < \bar{\ell}^i$ and $(\bar{Z}^i_{\delta'_i}, \bar{Y}^{i'}_{\delta'_i}) = (Z^{i'}_{j'}, Y^{i'}_{j'})$.

Bounding $\Pr[\Theta_0 \in \Omega_{\text{bad}}]$

- $\Pr[\texttt{Kcoll}]$: using the fact that $\mathsf{K} \leftarrow_{\$} \{0,1\}^{\kappa}$

$$\Pr[\texttt{B1}] \leq \frac{\sigma_e}{2^{\kappa}}; \;\; \Pr[\texttt{B2}] \leq \frac{\sigma_d}{2^{\kappa}}; \;\; \Pr[\texttt{B3}] \leq \frac{q_p}{2^{\kappa}}.$$

$$\Pr[\texttt{B4}|\neg\texttt{B3}] \leq \frac{q_e}{2^{n/2}}; \;\; \Pr[\texttt{B5}|\neg\texttt{B3}] \leq \frac{q_d}{2^{n/2}}; \;\; \Pr[\texttt{B6}] \leq \frac{\sigma_e\sigma_d}{2^{\kappa}}.$$

- $\Pr[\texttt{EEmatch}|\neg\texttt{Kcoll}]$: using the fact that $\mathsf{K} \leftarrow_{\$} \{0,1\}^{\kappa}$ and $\mathsf{X}^{i}_{j}, \mathsf{X}^{i'}_{j'} \leftarrow_{\$} \{0,1\}^{n}$.

$$\Pr[\texttt{B7}] \leq \frac{\sigma_e^2}{2^{n+\kappa}}; \;\; \Pr[\texttt{B8}] \leq \frac{\sigma_e^2}{2^{n+\kappa}}.$$

COMET-128 : Security Proof Sketch

Bounding $\Pr[\Theta_0 \in \Omega_{\text{bad}}]$

- $\Pr[\texttt{EPmatch} | \neg\texttt{Kcoll}]$:
  - Primitive query occurs before encryption query:

$$\Pr[\texttt{EPmatch} | \neg\texttt{Kcoll}] \leq 2q_p\sigma_e/2^{n+\kappa}.$$

  - Primitive query after encryption query:
    Let, $\text{mcoll}(x) := \left|\{\mathsf{X}_j^i = x : (i,j) \in [q_e] \times [m^i]\}\right|$ and $\texttt{Mcoll}$ denote the event $\max_x \text{mcoll}(x) \geq n$. Then,

$$\begin{aligned}\Pr[\texttt{EPmatch} | \neg\texttt{Kcoll}] &\leq \Pr[\texttt{Mcoll}] + \Pr[\texttt{EPmatch} | \neg(\texttt{Kcoll} \vee \texttt{Mcoll})] \\ &\leq \frac{\sigma_e}{2^{n-1}} + \frac{2nq_p}{2^\kappa}.\end{aligned}$$

- $\Pr[\texttt{EPKcoll}]$: using the fact that the number of keys which are repeated in primitive queries at least 2^{n-1} times is at most $q_p/2^{n-1}$.

$$\Pr[\texttt{EPKcoll}] \leq \frac{2\sigma_e q_p}{2^{n+\kappa}}.$$

COMET-128 : Security Proof Sketch

Bounding $\Pr[\Theta_0 \in \Omega_{\text{bad}}]$

- $\Pr[\texttt{Chain} | \neg(\texttt{Kcoll} \vee \texttt{EEmatch} \vee \texttt{EPmatch})]$:

  Using graph-based analysis (similar to Beetle).
  Let $\mathcal{G}_{\omega_p} = (\mathcal{V}, \mathcal{E})$ be an edge-labeled graph where $\mathcal{V} = \text{domain}(\omega_p)$ and $((\hat{Z}_j, \hat{Y}_i), (\hat{Z}_j, \hat{Y}_j), C^*) \in \mathcal{E}$ if and only if

  $$(\hat{Z}_j, \hat{Y}_j) = (\mathsf{IC}_{\hat{Z}_i}(\hat{Y}_i), \mathsf{IC}_{\hat{Z}_i}(\hat{Y}_i) \oplus C^*)$$

  A walk $\mathcal{W}$ from vertex W_0 to W_k with label $C = (C_1, \ldots, C_k)$, denoted $W_0 \xrightarrow{C} W_k$, is

  $$W_0 \xrightarrow{C_1} W_1 \cdots W_{k-1} \xrightarrow{C_k} W_k.$$

COMET-128 : Security Proof Sketch

Bounding $\Pr[\Theta_0 \in \Omega_{\text{bad}}]$

- $\Pr[\texttt{Chain} | \neg(\texttt{Kcoll} \vee \texttt{EEmatch} \vee \texttt{EPmatch})]$:

  A multi-chain with label $\mathsf{C} = (\mathsf{C}_1, \ldots, \mathsf{C}_k)$, denoted $\mathcal{C}_{\mathsf{C}}$, is a set of labeled walks $\{\mathcal{W}_1, \ldots, \mathcal{W}_s\}$ such that for all $1 \leq i \leq s$,

$$\mathcal{W}_i : (\hat{\mathsf{Z}}^i_0, \hat{\mathsf{Y}}^i_0) \xrightarrow{\mathsf{C}} (\hat{\mathsf{Z}}^i_k, \hat{\mathsf{Y}}^i_k) \ \wedge \ \hat{\mathsf{Y}}^1_0 = \cdots = \hat{\mathsf{Y}}^s_0 \ \wedge \ \hat{\mathsf{X}}^1_{k+1} = \cdots = \hat{\mathsf{X}}^s_{k+1}.$$

$$\begin{array}{ll}
\mathcal{W}_1 : & (\hat{\mathsf{Z}}^1_0, \hat{\mathsf{Y}}^1_0) \xrightarrow{\mathsf{C}_1} (\hat{\mathsf{Z}}^1_1, \hat{\mathsf{Y}}^1_1) \xrightarrow{\mathsf{C}_2} (\hat{\mathsf{Z}}^1_2, \hat{\mathsf{Y}}^1_2) \xrightarrow{\mathsf{C}_3} (\hat{\mathsf{Z}}^1_3, \hat{\mathsf{Y}}^1_3) \xrightarrow{\mathsf{C}_4} (\hat{\mathsf{Z}}^1_3, \hat{\mathsf{Y}}^1_4) \overset{\mathsf{IC}}{\dashrightarrow} \hat{\mathsf{X}}^1_5 \\
\mathcal{W}_2 : & (\hat{\mathsf{Z}}^2_0, \hat{\mathsf{Y}}^2_0) \xrightarrow{\mathsf{C}_1} (\hat{\mathsf{Z}}^2_1, \hat{\mathsf{Y}}^2_1) \xrightarrow{\mathsf{C}_2} (\hat{\mathsf{Z}}^2_2, \hat{\mathsf{Y}}^2_2) \xrightarrow{\mathsf{C}_3} (\hat{\mathsf{Z}}^2_3, \hat{\mathsf{Y}}^2_3) \xrightarrow{\mathsf{C}_4} (\hat{\mathsf{Z}}^2_3, \hat{\mathsf{Y}}^2_4) \overset{\mathsf{IC}}{\dashrightarrow} \hat{\mathsf{X}}^2_5 \\
\vdots & \\
\mathcal{W}_s : & (\hat{\mathsf{Z}}^s_0, \hat{\mathsf{Y}}^s_0) \xrightarrow{\mathsf{C}_1} (\hat{\mathsf{Z}}^s_1, \hat{\mathsf{Y}}^s_1) \xrightarrow{\mathsf{C}_2} (\hat{\mathsf{Z}}^s_2, \hat{\mathsf{Y}}^s_2) \xrightarrow{\mathsf{C}_3} (\hat{\mathsf{Z}}^s_3, \hat{\mathsf{Y}}^s_3) \xrightarrow{\mathsf{C}_4} (\hat{\mathsf{Z}}^s_3, \hat{\mathsf{Y}}^s_4) \overset{\mathsf{IC}}{\dashrightarrow} \hat{\mathsf{X}}^s_5
\end{array}$$

$$\boxed{\Pr[\texttt{B11} | \neg(\texttt{Kcoll} \vee \texttt{EEmatch} \vee \texttt{EPmatch})] \leq \sum_{i \in [q]_d} \Pr\left[\left|\mathcal{C}_{\bar{\mathsf{C}}_{\delta_i \ldots \bar{m}^i}}\right| \geq \lambda_i\right] + \frac{\lambda_i}{2^\kappa}.}$$

COMET-128 : Security Proof Sketch

Bound on $\Pr\left[|\mathcal{C}_{\bar{\mathsf{C}}_{\delta_i\ldots\bar{m}^i}}| \geq \lambda_i\right]$ and λ_i

- Three ways to construct a multi-chain structure:
  - Forward-only: all queries of the form $(\hat{\mathsf{Z}}_i, \hat{\mathsf{Y}}_i)$.

$$\Pr\left[\mathcal{C}_{\mathrm{fwd}} \geq n\left\lceil\frac{q_p}{2^n}\right\rceil\right] \leq \frac{1}{2^n},$$

    (by bounding the multicollisions on $\hat{\mathsf{X}}_j$)
  - Backward-only: all queries of the form $(\hat{\mathsf{Z}}_i, \hat{\mathsf{X}}_i)$.

$$\Pr\left[\mathcal{C}_{\mathrm{bck}} \geq n\left\lceil\frac{q_p}{2^n}\right\rceil\right] \leq \frac{1}{2^n}.$$

    (by bounding the multicollisions on $\hat{\mathsf{Y}}_j$)
  - Both forward and backward type queries:
    reduced to multicollision event at some index $1 \leq i \leq \bar{\ell}^i$ (using Pigeonhole-principle).

$$\Pr\left[\mathcal{C}_{\mathrm{fwd\text{-}bck}} \geq \bar{\ell}^i\,\frac{2\sqrt{n}q_p}{2^{n/2}} + \frac{2q_p}{2^n}\right] \leq \frac{1}{2^n}.$$

- $\Pr\left[|\mathcal{C}_{\bar{\mathsf{C}}_{\delta_i\ldots\bar{m}^i}}| \geq \bar{\ell}^i\,\frac{2\sqrt{n}q_p}{2^{n/2}} + 2n\left\lceil\frac{q_p}{2^n}\right\rceil + \frac{2q_p}{2^n}\right] \leq \frac{3}{2^n}.$

COMET-128 : Security Proof Sketch

- $\Pr\left[\mathtt{B11}|\neg(\mathtt{Kcoll} \vee \mathtt{EEmatch} \vee \mathtt{EPmatch})\right] \leq \frac{2\sqrt{n}\sigma_d q_p}{2^{\kappa+n/2}} + \frac{2nq_d}{2^\kappa}\left\lceil\frac{q_p}{2^n}\right\rceil + \frac{2q_d q_p}{2^{n+\kappa}} + \frac{3q_d}{2^n}$.
- $\Pr\left[\mathtt{B12}|\neg(\mathtt{Kcoll} \vee \mathtt{EEmatch} \vee \mathtt{EPmatch})\right]$ can be bounded in a similar fashion.

$$\Pr\left[\mathtt{B12}|\neg(\mathtt{Kcoll} \vee \mathtt{EEmatch} \vee \mathtt{EPmatch} \vee \mathtt{B11})\right] \leq \frac{2\sqrt{n}\sigma_d q_p}{2^{\kappa+n/2}} + \frac{2nq_d}{2^\kappa}\left\lceil\frac{q_p}{2^n}\right\rceil + \frac{2q_d q_p}{2^{n+\kappa}} + \frac{3q_d}{2^n}.$$

Finally, $\Pr\left[\mathtt{Chain}|\neg(\mathtt{Kcoll} \vee \mathtt{EEmatch} \vee \mathtt{EPmatch})\right] \leq \frac{6\sqrt{n}\sigma_d q_p}{2^{\kappa+n/2}} + \frac{6nq_d}{2^\kappa}\left\lceil\frac{q_p}{2^n}\right\rceil + \frac{4q_d q_p}{2^{n+\kappa}} + \frac{6q_d}{2^n}$.

COMET-128 : Security Proof Sketch

Good transcript analysis

Given any good transcript ω:

$$\frac{\Pr[\Theta_1 = \omega]}{\Pr[\Theta_0 = \omega]} \geq \left(1 - \frac{2\sigma_d(\sigma_e + q_p)}{2^{\kappa+n}} - \frac{2q_d}{2^n}\right).$$

- First term bounds the probability that for some decryption query i an intermediate input $(\hat{Z}^i_j, \hat{Y}^i_j)$ collides with some encryption/primitive input, for $j > \delta_i$.
- The second term bounds the probability that som decryption forgery succeeds given that all intermediate inputs are fresh.

This completes the proof.

Thank you. Questions...

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