Lattice-based digital signature scheme qTESLA

(2nd NIST PQC Standardization Conference, 2019)

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Introduction

- qTESLA is a family of post-quantum lattice-based signature schemes
- Based on the decisional R-LWE problem
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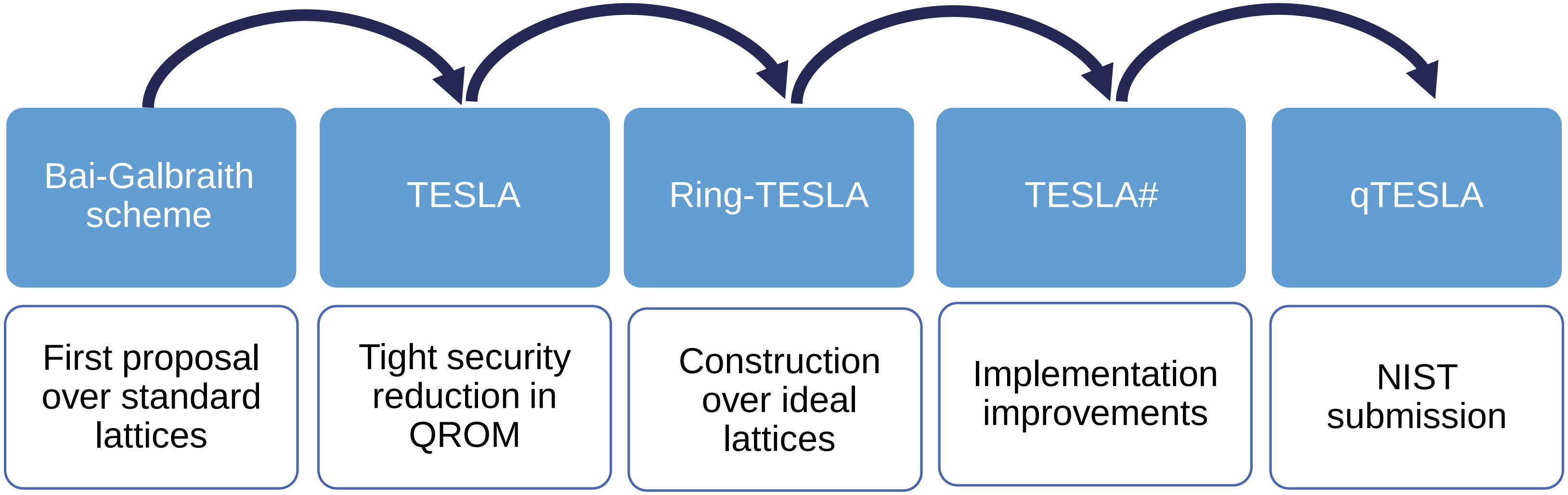


Bai-Galbraith scheme → TESLA → Ring-TESLA → TESLA# → qTESLA

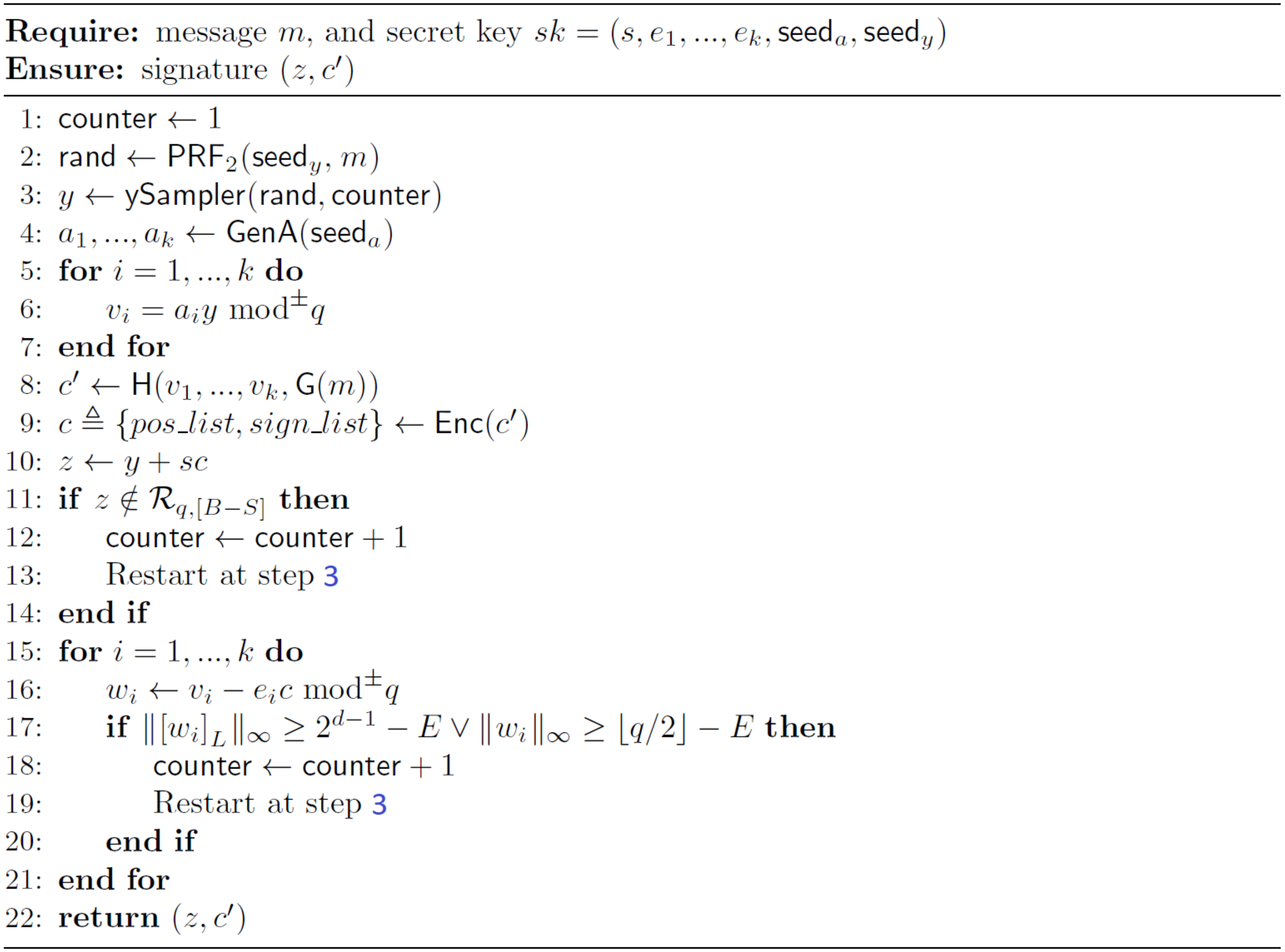
Bai-Galbraith scheme	TESLA	Ring-TESLA	TESLA#	qTESLA
First proposal over standard lattices	Tight security reduction in QROM	Construction over ideal lattices	Implementation improvements	NIST submission

qTESLA – Key generation

- Secret key:
  - $s, e_1, \dots, e_k \overset{\sigma}{\longleftarrow} \mathcal{R} = \mathbb{Z}[x]/\langle x^n + 1 \rangle$, “small enough”
  - $seed_a, seed_y$

- Public key:
  - $t_1 \leftarrow a_1 s + e_1 \bmod q, \dots, t_k \leftarrow a_k s + e_k \bmod q$ with $a_1, \dots, a_k \leftarrow GenA(seed_a)$
  - $seed_a$

qTESLA – Signing



Require: message m, and secret key $sk = (s, e_1, ..., e_k, \mathsf{seed}_a, \mathsf{seed}_y)$
Ensure: signature (z, c')

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 1: counter ← 1
 2: rand ← $\mathsf{PRF}_2(\mathsf{seed}_y, m)$
 3: $y$ ← ySampler(rand, counter)
 4: $a_1, ..., a_k \leftarrow \mathsf{GenA}(\mathsf{seed}_a)$
 5: for $i = 1, ..., k$ do
 6:     $v_i = a_i y \bmod^{\pm} q$
 7: end for
 8: $c' \leftarrow \mathsf{H}(v_1, ..., v_k, \mathsf{G}(m))$
 9: $c \triangleq \{pos\_list, sign\_list\} \leftarrow \mathsf{Enc}(c')$
10: $z \leftarrow y + sc$
11: if $z \notin \mathcal{R}_{q,[B-S]}$ then
12:     counter ← counter + 1
13:     Restart at step 3
14: end if
15: for $i = 1, ..., k$ do
16:     $w_i \leftarrow v_i - e_i c \bmod^{\pm} q$
17:     if $\|[w_i]_L\|_\infty \geq 2^{d-1} - E \vee \|w_i\|_\infty \geq \lfloor q/2 \rfloor - E$ then
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Pseudo-randomness expansion

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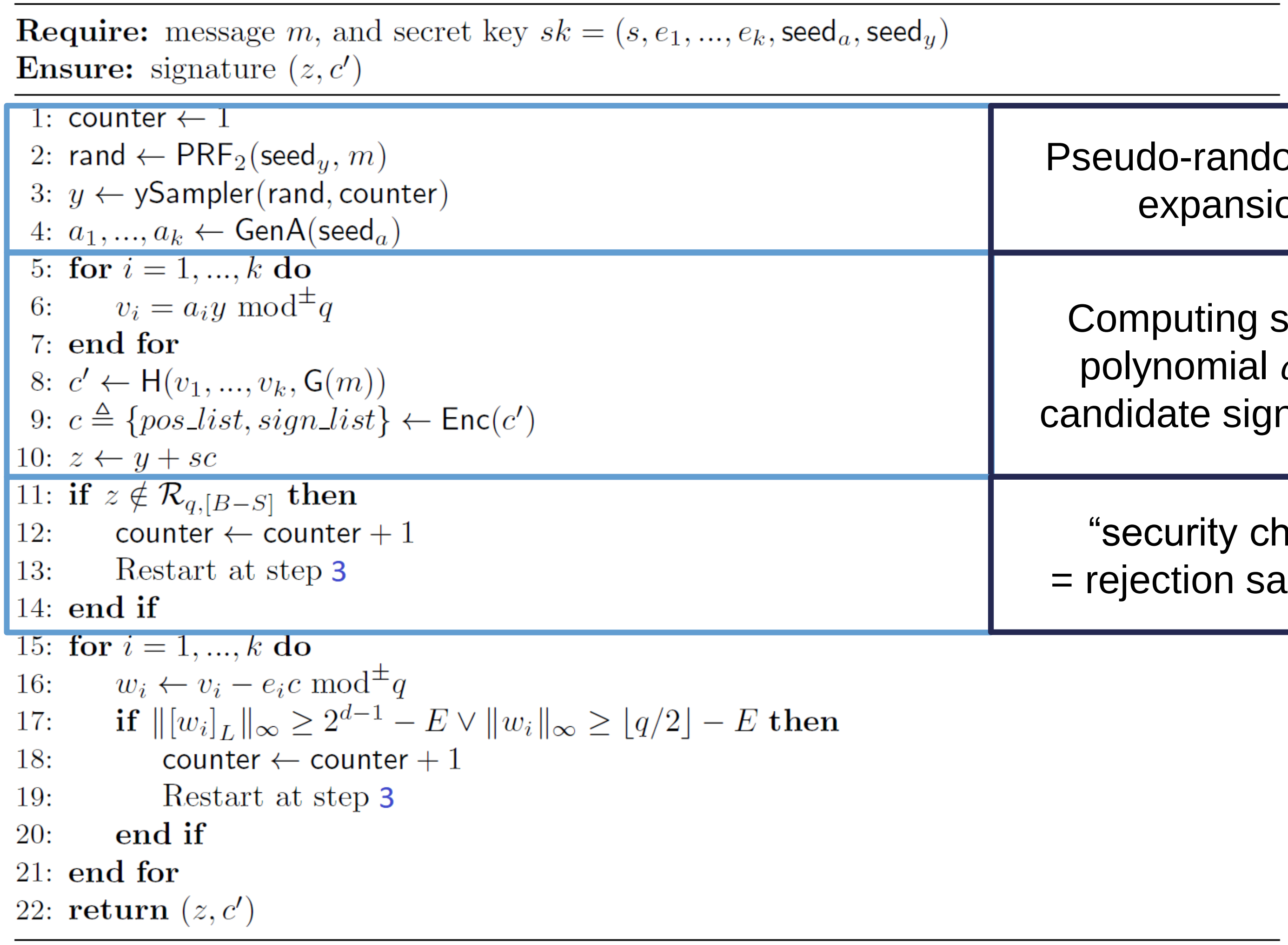

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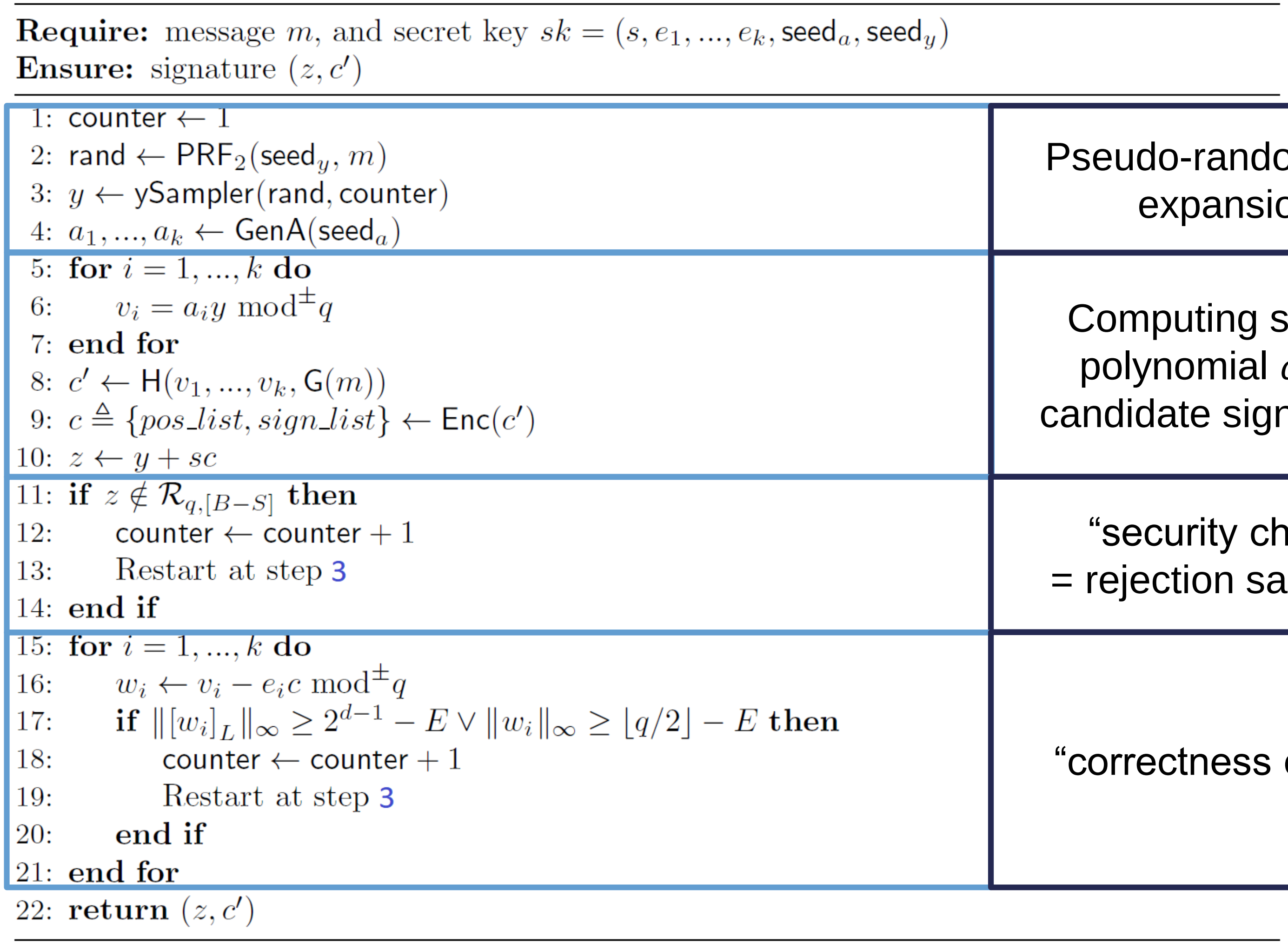

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“correctness check“

Round 2 modifications

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Support for power-of-two cyclotomic ring $\mathcal{R}_q = \mathbb{Z}_q[x]/\langle \phi_{2^\ell}(x)\rangle$.	**Added** support for non-power-of-two cyclotomic ring $\mathcal{R}_q = \mathbb{Z}_q[x]/\langle \phi_{2^\ell 9}(x)\rangle$.

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Simplified Bernoulli sampler: • Portability issues • Hard to make fully constant-time.	**Replaced by** simpler, faster, portable, constant-time CDT-based Gaussian sampler.

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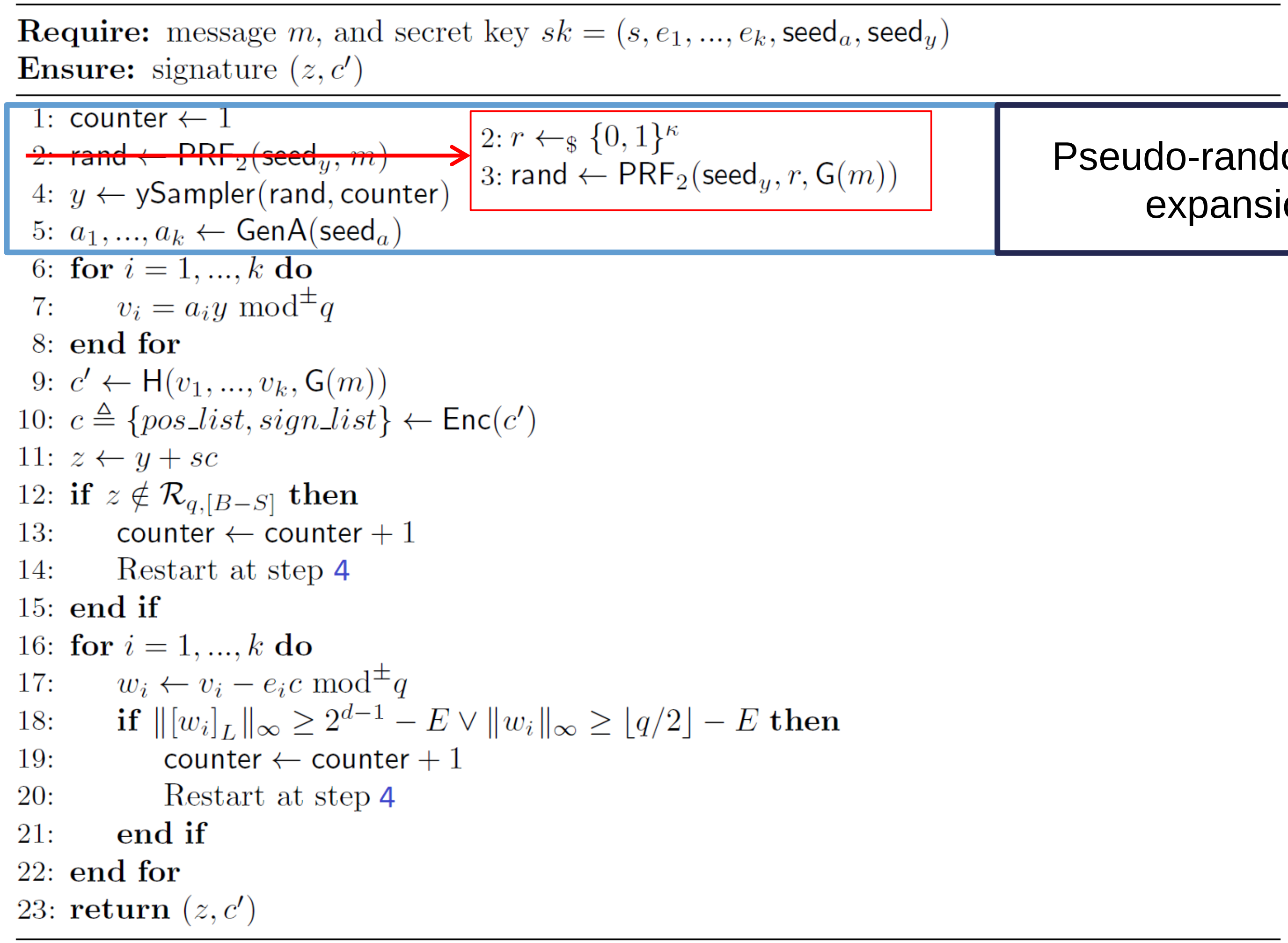

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    2: $r \leftarrow_{\$} \{0,1\}^{\kappa}$
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Deterministic signatures.	**Converted to** probabilistic.
Security reduction in the QROM using conjecture.	**Refined** conjecture and **backed it up** experimentally.

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  - A security proof following [KLS18] that reduces hardness of ST-R-SIS and R-LWE to the security of heuristic qTESLA allows generation of secure parameters
    - Main change involves increasing number of R-LWE samples from 1 to 2

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  - However, we decided to **drop the heuristic parameters**

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Parameter sets

Parameter set	Heuristic			Provable	
	qTESLA-I	qTESLA-II	qTESLA-III	qTESLA-p-I	qTESLA-p-III
NIST category	1	2	3	1	3
R-LWE hardness	111	138	188	140	279
SIS hardness	50	71	95	-	-
Targeted hardness	95	128	160	95	160
pk size [bytes]	1,504	2,336	3,104	14,880	38,432
sig size [bytes]	1,376	2,144	2,848	2,592	5,664

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Fixed parameter sets

Parameter set	Heuristic			Provable	
	qTESLA-I	qTESLA-II	qTESLA-III	qTESLA-p-I	qTESLA-p-III
NIST category	1	2	3	1	3
R-LWE hardness	97	130	178	140	279
SIS hardness	100	143	197	-	-
Targeted hardness	95	128	160	95	160
pk size [bytes]	2,976	4,832	6,432	14,880	38,432
sig size [bytes]	1,400	2,336	3,104	2,592	5,664

Updated parameter sets (round 2+)

Parameter set	Provable	
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NIST category	1	3
R-LWE hardness	140	279
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pk size [bytes]	14,880	38,432
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Performance (round 2+)

Performance (in kilocycles) of the constant-time **reference implementation** on a 3.40GHz Intel Core i7-6700 (Skylake) processor

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Total (sign + verify)	**2,996**	**8,115**

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- E.g., qTESLA-p-I produces signatures in **0.68 msec.** or **1,470 signs/sec**.

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  - Reduces {theoretical, practical} attack surface
- **By default** built-in protection against some side-channel and fault attacks
- Very conservative security
  - qTESLA instantiations are **provably-secure** in the QROM

Potential avenues of improvement

- Further optimization of implementation (e.g., using assembly).
- Use of Dilithium’s pk compression technique.

Thanks!

qTESLA website: https://qtesla.org/

Updated specs: https://qtesla.org/wp-content/uploads/2019/08/qTESLA_round2_08.19.2019.pdf

Updated package: https://qtesla.org/wp-content/uploads/2019/08/qTESLA_NIST_update_08.19.2019.zip

Code: https://github.com/qtesla/qTesla