Rainbow

Jintai Ding, Ming-Shing Chen, Matthias Kannwischer, Jacques Patarin, Albrecht Petzoldt, Dieter Schmidt, Bo Yin Yang

3rd NIST Post-Quantum Standardization Conference

07.06.2021

Multivariate Cryptography

Public Key: System of multivariate quadratic polynomials

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p^{(1)}(x_1, \ldots, x_n) &= \sum_{i=1}^{n} \sum_{j=i}^{n} p_{ij}^{(1)} \cdot x_i x_j \quad + \quad \sum_{i=1}^{n} p_i^{(1)} \cdot x_i + p_0^{(1)} \\
p^{(2)}(x_1, \ldots, x_n) &= \sum_{i=1}^{n} \sum_{j=i}^{n} p_{ij}^{(2)} \cdot x_i x_j \quad + \quad \sum_{i=1}^{n} p_i^{(2)} \cdot x_i + p_0^{(2)} \\
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Security based on the

Problem MQ: Given m multivariate quadratic polynomials $p^{(1)}(\mathbf{x}), \ldots, p^{(m)}(\mathbf{x})$, find a vector $\bar{\mathbf{x}} = (\bar{x}_1, \ldots, \bar{x}_n)$ such that $p^{(1)}(\bar{\mathbf{x}}) = \ldots = p^{(m)}(\bar{\mathbf{x}}) = 0$.

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Multivariate Signature Schemes

Signature Generation

$$\begin{tikzcd} d \in \{0,1\}^\star \arrow[r, "\mathcal{H}"] & \mathbf{w} \in \mathbb{F}^n \arrow[r, "\mathcal{S}^{-1}"] & \mathbf{x} \in \mathbb{F}^n \arrow[r, "\mathcal{F}^{-1}"] & \mathbf{y} \in \mathbb{F}^m \arrow[r, "\mathcal{T}^{-1}"] & \mathbf{z} \in \mathbb{F}^m \arrow[lll, bend left, "\mathcal{P}"] \end{tikzcd}$$

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Signature Verification: Given: document $d \in \{0,1\}^\star$, signature $\mathbf{z} \in \mathbb{F}^n$, public key $\mathcal{P}$ check if $\mathcal{P}(\mathbf{z}) = \mathcal{H}(d)$

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- central map $\mathcal{F}$ consists of $m := n - v_1$ polynomials $f^{(v_1+1)}, \ldots, f^{(n)}$ of the form

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5. Set $\mathbf{y} = (y_1, \dots, y_n) \in \mathbb{F}^n$.
6. Compute the signature $\mathbf{z} \in \mathbb{F}^n$ by $\mathbf{z} = \mathcal{T}^{-1}(\mathbf{y})$.

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- $\mathbf{w} = \mathcal{H}(d) \in \mathbb{F}^m$.

If $\mathbf{w}' = \mathbf{w}$ holds, the signature is accepted; otherwise it is rejected.

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$$S = \begin{pmatrix} I_{o_1} & S'_{o_1 \times o_2} \\ 0_{o_2 \times o_2} & I_{o_2} \end{pmatrix} \quad T = \begin{pmatrix} I_{v_1} & T^{(1)}_{v_1 \times o_1} & T^{(2)}_{v_1 \times o_2} \\ 0_{o_1 \times v_1} & 1_{v_1} & T^{(3)}_{o_1 \times o_2} \\ 0_{o_2 \times v_1} & 0_{o_2 \times o_1} & I_{o_2} \end{pmatrix}.$$

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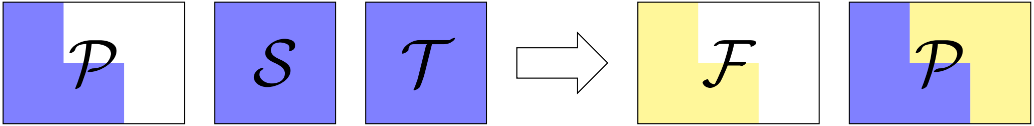
- Our conventions don't weaken the security of the scheme and enable us to speed up the key generation process drastically

CZ-Rainbow



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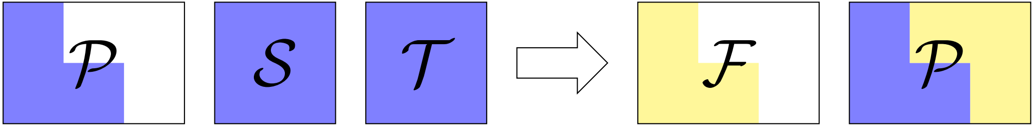

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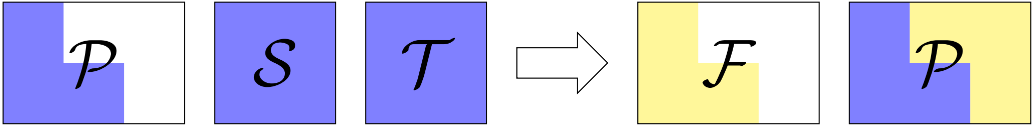

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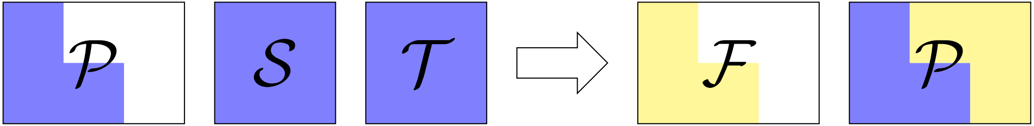

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- We propose also a compressed version in which the central map is generated from a seed, too.

Key and Signature Sizes

NIST security category	parameters (q, v_1, o_2, o_2)	signature size (bit)
I	(16,36,32,32)	528
III	(256,68,32,48)	1,312
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NIST security category	standard Rainbow		CZ-Rainbow		compressed Rainbow	
	$\|pk\|$	$\|sk\|$	$\|pk\|$	$\|sk\|$	$\|pk\|$	$\|sk\|$
I	157.8	101.2	58.8	101.2	58.1	64B
III	861.4	611.3	258.4	611.3	258.4	64B
V	1,885.4	1,375.7	523.6	1,375,7	523.6	64B

Performance

NIST security category	standard Rainbow			CZ-Rainbow			compressed Rainbow		
	keygen	sign	verify	keygen	sign	verify	keygen	sign	verify
I	9.9M	67k	34k	10.7M	67k	3.5M	10.7M	7.0M	3.5M
III	52M	285k	132k	64M	285k	20M	64M	41M	20M
V	192M	739k	392k	235M	739k	47M	235M	118M	47M

Linux / Skylake (using AVX2 instructions)

See also the talk by M. Kannwischer on Implementing Rainbow on Cortex-M4

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- cost of one multiplication
  $\approx (\text{ bit} - \text{length memory transitions}) \times \sqrt{\#\mathit{bits\ non\ sequentially\ accessed}}/2^5$
- For Wiedemann over $(\mathbb{F}_{2^k})^V$ the cost of one multiplication is $\lg V \sqrt{kV}/2^5$ "gates"

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security category	parameters (q, v_1, o_1, o_2)	Intersection		New MinRank		target cost
		mults	cost	mults	cost	
I	(16,36,32,32)	$2^{134.3}$	$2^{162.1}$	$2^{122.4}$	$2^{152.3}$	2^{143}
III	(256,68,32,48)	$2^{105.4}$	$2^{248.3}$	$2^{171.8}$	$2^{216.2}$	2^{207}
V	(256,96,36,64)	$2^{254.5}$	$2^{309.9}$	$2^{219.6}$	$2^{276.2}$	2^{272}

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$\Rightarrow$ Our parameter proposals meet the NIST requirements

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Speeding up the Verification Process

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First Observation: We can stop, as soon as we find an i with $p^{(i)}(\mathbf{z}) \neq w_i$.

Checking a Fixed Subset of Equations

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Security category	parameters (q, v_1, o_1, o_2)	# equations m	# equations to be checked	compl. of sig. forgery classical	quantum
I	(16,36,32,32)	64	55	145	127
III	(256,68,32,48)	80	72	212	185
V	(256,96,36,64)	100	98	280	247

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I	(16,36,32,32)	64	32	143	125
II	(256,68,32,48)	80	48	212	192
III	(256,96,36,64)	100	72	276	245

Checking Randomly Chosen Linear Combinations

- we check k equations of the form $\tilde{p}^{(i)} = \sum_{j=1}^{m} \left(\alpha_j^{(i)} p^{(j)} \right) (\mathbf{z}) = \sum_{j=1}^{m} \alpha_j^{(i)} w_j = \tilde{w}_i$

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Security category	parameters (q, v_1, o_1, o_2)	# equations m	# variables n	# equations to be checked
I	(16,36,32,32)	64	100	32
III	(256,68,32,48)	80	148	24
V	(256,96,36,64)	100	196	32

Rainbow: Advantages and Disadvantages

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- short signatures
- fast key generation
- very fast signature generation
- very fast signature verification

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Disadvantages

- large key sizes
- no security proof

Thank you for your attention

The Team:
Jintai Ding, Ming-Shing Chen, Matthias Kannwischer, Jacques Patarin,
Albrecht Petzoldt, Dieter Schmidt, BoYin Yang

www.pqcrainbow.org