Security Analysis of KNOT-AEAD and KNOT-Hash

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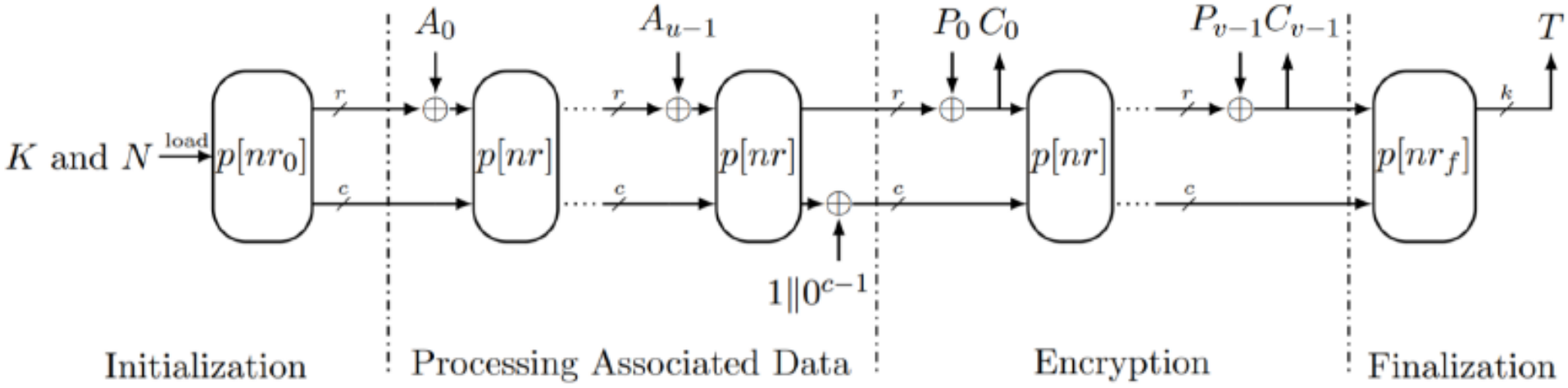
1.The KNOT Family

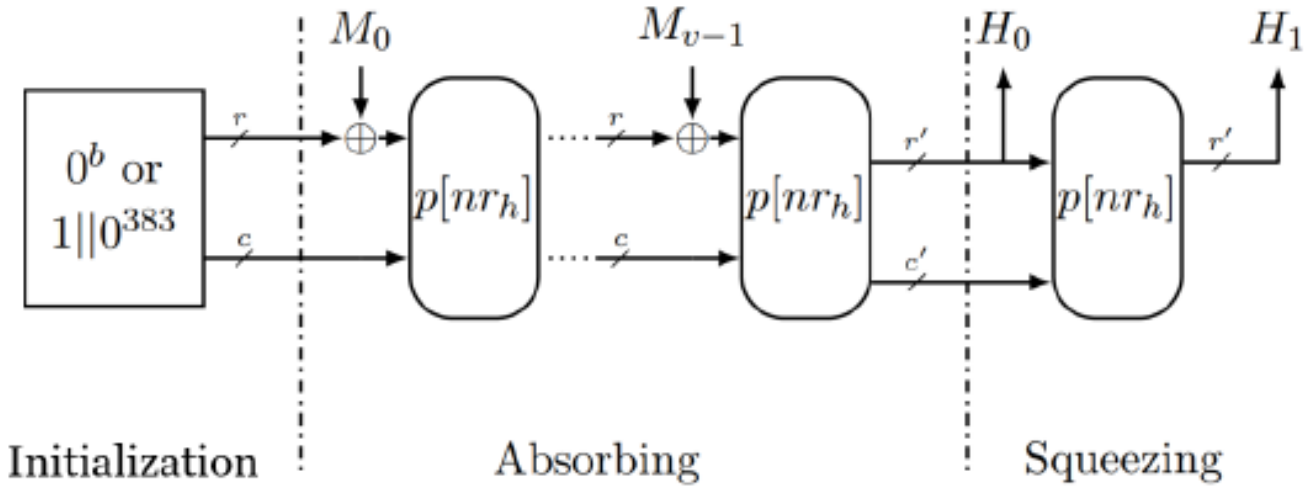
- KNOT is a family of bit-slice lightweight AEAD and hashing algorithms:
  - **KNOT-AEAD**
  - **KNOT-Hash**

- KNOT uses permutation-based modes.



Encryption of KNOT-AEAD

- KNOT-AEAD: **MonkeyDuplex,** with a reduced number of rounds in the data processing and finalization phases



KNOT-Hash

- KNOT-Hash: **an extended Sponge**, with a different bit rate in the squeezing phase

- **KNOT-AEAD has 4 members**. They are different in state size, bit rate and key size.

- Accordingly, **KNOT-Hash also has 4 members.**

- For **each KNOT-pair**, the corresponding KNOT-AEAD and KNOT-Hash have **the same state size** and use **an identical round transformation**:
  - the primary pair, 256-bit state

The KNOT permutations

- **State width b**: 256/384/512 bits

$$\begin{bmatrix} a_{0,\frac{b}{4}-1} & \cdots & a_{0,1} & a_{0,0} \\ a_{1,\frac{b}{4}-1} & \cdots & a_{1,1} & a_{1,0} \\ a_{2,\frac{b}{4}-1} & \cdots & a_{2,1} & a_{2,0} \\ a_{3,\frac{b}{4}-1} & \cdots & a_{3,1} & a_{3,0} \end{bmatrix}$$

A b-bit state

The KNOT permutations

- **Structure:** SP-network, each of the rounds has 3 steps

$$
\begin{aligned}
\{ \quad & AddRoundConstant_b(STATE, RC) \\
& SubColumn_b(STATE) \\
& ShiftRow_b(STATE) \\
\}
\end{aligned}
$$

where RC denotes a round constant.

SubColumn: parallel application of sboxes to the 4 bits in the same column, $S : \mathrm{F}_2{}^4 \longrightarrow \mathrm{F}_2{}^4$

$$\begin{pmatrix} a_{0,\frac{b}{4}-1} \\ a_{1,\frac{b}{4}-1} \\ a_{2,\frac{b}{4}-1} \\ a_{3,\frac{b}{4}-1} \end{pmatrix} \cdots \begin{pmatrix} a_{0,1} \\ a_{1,1} \\ a_{2,1} \\ a_{3,1} \end{pmatrix} \begin{pmatrix} a_{0,0} \\ a_{1,0} \\ a_{2,0} \\ a_{3,0} \end{pmatrix}$$

$$\downarrow S \qquad \cdots \qquad \downarrow S \qquad \downarrow S$$

$$\begin{pmatrix} b_{0,\frac{b}{4}-1} \\ b_{1,\frac{b}{4}-1} \\ b_{2,\frac{b}{4}-1} \\ b_{3,\frac{b}{4}-1} \end{pmatrix} \cdots \begin{pmatrix} b_{0,1} \\ b_{1,1} \\ b_{2,1} \\ b_{3,1} \end{pmatrix} \begin{pmatrix} b_{0,0} \\ b_{1,0} \\ b_{2,0} \\ b_{3,0} \end{pmatrix}$$

ShiftRow: a left rotation to each row over different offsets

$$
\begin{aligned}
&\left(a_{0,\frac{b}{4}-1} \quad \cdots \quad a_{0,1} \quad a_{0,0}\right) \xrightarrow{\lll 0} \left(a_{0,\frac{b}{4}-1} \quad \cdots \quad a_{0,1} \quad a_{0,0}\right) \\
&\left(a_{1,\frac{b}{4}-1} \quad \cdots \quad a_{1,1} \quad a_{1,0}\right) \xrightarrow{\lll c_1} \left(a_{1,\frac{b}{4}-c_1-1} \quad \cdots \quad a_{1,\frac{b}{4}-c_1+1} \quad a_{1,\frac{b}{4}-c_1}\right) \\
&\left(a_{2,\frac{b}{4}-1} \quad \cdots \quad a_{2,1} \quad a_{2,0}\right) \xrightarrow{\lll c_2} \left(a_{2,\frac{b}{4}-c_2-1} \quad \cdots \quad a_{2,\frac{b}{4}-c_2+1} \quad a_{2,\frac{b}{4}-c_2}\right) \\
&\left(a_{3,\frac{b}{4}-1} \quad \cdots \quad a_{3,1} \quad a_{3,0}\right) \xrightarrow{\lll c_3} \left(a_{3,\frac{b}{4}-c_3-1} \quad \cdots \quad a_{3,\frac{b}{4}-c_3+1} \quad a_{3,\frac{b}{4}-c_3}\right)
\end{aligned}
$$

2. Motivation

- In the KNOT design document, the **designers evaluated in detail the security of the underlying permutations** against various cryptanalytic approaches:
  - among which DC and LC are the most powerful approaches

- Although **distinguishers of the permutations** can give insights in the resistance of the AEAD and hash primitives against various cryptanalytic attacks, they **usually can not be directly used in an attack.**

- To have a better understanding of the security of KNOT, we furthermore study **the best differential/linear distinguishers with constraints**, which can be **directly used to mount attacks on KNOT-AEAD and KNOT-Hash**.

- The evaluation of the KNOT permutations against DC and LC is achieved by using **Matsui's search algorithm and its improvements**. However, **when constraints are added, the efficiency is not satisfied,** e.g., the 256-bit KNOT permutation, the best linear trail:
  - constraints: all active bits of both the input and output mask are only allowed in the rate part.
  - search time: 98 hours for 11 rounds; additional 49 hours for 12 rounds.

- The design of the KNOT permutations inherits the design of RECTANGLE.

- **An observation on RECTANGLE** is that **the best long-round differential and linear trail always contains iterative sub-trails**, which motivates us to study differential/linear distinguishers containing iterative sub-trails for KNOT.

3. Our Method

- Main idea:
  1). By using the algorithm of finding elementary circuits (a circuit is elementary if no vertex but the first and last appears twice), we can **find all elementary iterative differential/linear trails** for each of the KNOT permutations.
  2). Then, by checking all the differential/linear iterative trails within the scope of consideration and **connecting them repeatedly**, we can efficiently derive the difference/linear propagations for a given number of rounds and given input/output difference (or input/output mask).
  3). Finally, by using Matsui's search algorithm, we can **extend** the difference (or linear) propagations obtained in the previous step **both forward and backward** for several rounds.

- The **accuracy of our calculation** on differential probability (linear correlation) is closely related with **the number of trails collected:**
  - The more trails we collect, the more accurate the estimation is, yet the more time and memory the program costs.
  - In our experiments **for each KNOT member**, we try to make an **appropriate tradeoff between the accuracy and efficiency.**

- For more details, please refer to our paper:

  Ding Tianyou, Zhang Wentao, Ji Fulei, Zhou Chunning: *An Automatic Search Tool for Iterative Trails and its Application to KNOT, PRESENT, GIFT-64 and RECTANGLE*, https://eprint.iacr.org/2020/1152.

4.Attack Models

- KNOT-AEAD: 6 attack models
- KNOT-Hash: 2 attack models

6 attack models for KNOT-AEAD

Note: all the attacks are in the nonce-respecting setting; for forgery attacks , nonce can be reused in decryption oracle.

	Target phase	Distinguisher Type	Constraints	Attack Goal
Diff-Init-D	Initialization	truncated difference propagation	**input difference:** active bits only allowed in nonce part **output difference**: the rate part is some fixed value	distinguishing
Linear-Ini-KR		linear propagation	**input mask**: the key part is active **output mask**: active bits only allowed in rate part	key recovery
Linear-Ini-D		linear propagation	**input mask**: the key part is zero **output mask:** active bits only allowed in rate part	distinguishing
Diff-Enc-F	Encryption	difference propagation	**both input and output difference**: active bits only allowed in rate part	forgery
Linear-Enc-D		linear propagation	**both input and output mask**: active bits only allowed in rate part	distinguishing
Diff-Final-F	Finalization	truncated difference propagation	**input difference**: active bits only allowed in rate part **output difference**: the tag part is some fixed value	Forgery

2 attack models for KNOT-Hash

	Distinguisher Type	Constraints	Attack Goal
Diff-Col-I	difference propagation	**both input and output difference**: active bits only allowed in rate part	collsion
Diff-Col-II	truncated difference Propagation	**input difference**: active bits only allowed in rate part **output difference**: the hash-value part is zero	near collision

5.Experimental Results for KNOT

- According to the claimed security strength and data limit of KNOT, we study:
  - the security margin of **each KNOT-AEAD** member **against the 6 attack models** in the **single-key scenario**
  - the security margin of **each KNOT-Hash** member against **the 2 attack models**.

	Plaintext Confidentiality	Ciphertext Integrity	Data Limit
KNOT-AEAD(128, 256, 64)	125	125	2^{64}
KNOT-AEAD(128, 384, 192)	128	128	2^{64}
KNOT-AEAD(192, 384, 96)	189	189	2^{96}
KNOT-AEAD(256, 512, 128)	253	253	2^{128}

KNOT-AEAD: claimed security strength and data limit

Name	Security (bit)			Data limit
	pre.	2nd pre.	col.	
KNOT-Hash(256, 256, 32, 128)	128	112	112	2^{64}
KNOT-Hash(256, 384, 128, 128)	128	128	128	2^{64}
KNOT-Hash(384, 384, 48, 192)	192	168	168	2^{96}
KNOT-Hash(512, 512, 64, 256)	256	224	224	2^{128}

KNOT-Hash: claimed security strength and data limit

- **data complexity** of a **differential/linear attack**:
  - For a difference propagation, *P*: the **probability**, $-\log_2 p$: **weight.**
    The **data complexity** is proportional to p^{-1}
  - For a linear propagation, *cor*: the **correlation,** $-\log_2 cor$: **weight.**
    The **data complexity** is proportional to cor^{-2}.

- The following 5 tables present our experimental results for KNOT:
  - **the numbers in red color** denote **the highest weight of an effective distinguisher** considering **the data limit**, and the corresponding number of rounds is **the highest number of rounds of an effective distinguisher**

KNOT-AEAD v1 (primary version): weights of the best r-round distinguisher

r	Diff-Init-D	Linear-Init-KR	Linear-Init-D	Diff-Enc-F	Linear-Enc-D	Diff-Final-F
10	43	25	26	52.4	27	47.1
11	47.9	28	28	57.4	30	51.8
12	52.6	30	31	62.4	32	56.6
13	57.4	33	33	67.7	35	61.4
14	62.2	35	36	72.4	37	66.2
15	67	38	38	77.1	40	71.0

- For the primary version of KNOT-AEAD, **the highest number of rounds** of an effective distinguisher are **14, 12, and 13 rounds** respectively for the **initialization, encryption** and **finalization phase** :
  - note that the full number of rounds are 52 , 28 and 32 rounds respectively, the highest number of rounds of an effective distinguisher is less than 43% of the full number of rounds.

KNOT-AEAD v2: weights of the best r-round distinguisher

r	Diff-Init-D	Linear-Init-KR	Linear-Init-D	Diff-Enc-F	Linear-Enc-D	Diff-Final-F
10	46.4	25	26	51.4	25.4	44.0
11	51.2	28	29	56.7	28	48.6
12	56	30	31	61.4	30.4	53.2
13	60.8	33	34	66.1	33	57.9
14	65.6	35	36	71.1	35.4	62.6
15	70.4	38	39	76.1	38	67.4

KNOT-AEAD v3: weights of the best r-round distinguisher

r	Diff-Init-D	Linear-Init-KR	Linear-Init-D	Diff-Enc-F	Linear-Enc-D	Diff-Final-F
10	43	25	26	52.4	27	47.1
11	47.9	28	28	57.4	30	51.8
12	52.6	30	31	62.4	32	56.6
13	57.4	33	33	67.7	35	61.4
14	62.2	35	36	72.4	37	66.2
15	67	38	38	77.1	40	71.0
16	71.8	40	41	82.1	42	75.8
17	76.6	43	43	87.1	45	80.6
18	81.4	45	46	92.1	47	85.4
19	86.2	48	48	96.8	50	90.1
20	91	50	51	101.5	52	94.9
21	95.8	53	53	106.5	55	99.7
22	100.5	55	56	111.5	57	104.5

KNOT-AEAD v4: weights of the best r-round distinguisher

r	Diff-Init-D	Linear-Init-KR	Linear-Init-D	Diff-Enc-F	Linear-Enc-D	Diff-Final-F
10	43	25	26	52.4	27	47.1
11	47.9	28	28	57.4	30	51.8
12	52.6	30	31	62.4	32	56.6
13	57.4	33	33	67.7	35	61.4
14	62.2	35	36	72.4	37	66.2
15	67	38	38	77.1	40	71.0
16	71.8	40	41	82.1	42	75.8
17	76.6	43	43	87.1	45	80.6
18	81.4	45	46	92.1	47	85.4
19	86.2	48	48	96.8	50	90.1
20	91	50	51	101.5	52	94.9
21	95.8	53	53	106.5	55	99.7
22	100.6	55	56	111.5	57	104.5
23	105.3	58	58	116.3	60	109.3
24	110.1	60	61	121.1	62	114.1
25	114.9	63	63	125.9	65	118.9
26	119.7	65	66	130.9	67	123.7
27	124.5	68	68	135.8	70	128.5
28	129.3	70	71	140.5	72	133.3

KNOT hash: weights of the best r-round distinguisher

	KNOT-Hash v1		KNOT-Hash v2		KNOT-Hash v3		KNOT-Hash v4	
r	Diff-Col-I	Diff-Col-II	Diff-Col-I	Diff-Col-II	Diff-Col-I	Diff-Col-II	Diff-Col-I	Diff-Col-II
10	53.7	47.1	51.4	44.1	54.0	47.1	53.7	47.1
11	59.4	51.8	56.7	48.7	57.4	51.8	57.4	51.8
12	62.7	56.6	61.4	53.4	62.4	56.6	62.4	56.6
13	67.7	61.4	66.1	58.1	67.8	61.4	67.7	61.4
14	72.7	66.2	71.1	62.8	72.4	66.2	72.4	66.2
15	-	-	76.1	67.5	77.4	71.0	77.1	71.0
16	-	-	-	-	82.7	75.8	82.1	75.8
17	-	-	-	-	88.7	80.6	87.1	80.6
18	-	-	-	-	95.2	85.4	92.1	85.4
19	-	-	-	-	100.8	90.1	96.8	90.1
20	-	-	-	-	106.5	94.9	101.5	94.9
21	-	-	-	-	112.2	99.7	106.5	99.7
22	-	-	-	-	-	-	111.5	104.5
23	-	-	-	-	-	-	116.3	109.3
24	-	-	-	-	-	-	121.1	114.1
25	-	-	-	-	-	-	126.1	118.9
26	-	-	-	-	-	-	131.2	123.7
27	-	-	-	-	-	-	136.2	128.5

To sum up

- The **security margin** of **KNOT-AEAD** against **the 6 attack models**:
  - for all the 4 members, the highest number of rounds of an effective distinguisher is less than 50% of the full number of rounds, which means a **50% security margin**; especially, for **initialization phase**, the security margin **is higher than 72%.**

- The **security margin** of **KNOT-Hash** against **the 2 attack models**:
  - for all the 4 members, the highest number of rounds of an effective distinguisher is less than 20% of the full number of rounds, which means a **80% security margin**.

6.Validity of Our New Method

- we investigate **the accuracy of our new method** in **two different ways**:
  - Firstly, we apply our new method to RECTANGLE, which is an ancestor of the KNOT permutations, and **compare the results obtained by our new method with those in the specification document of RECTANGLE**.
  - Secondly, **we use MILP method** to compute the **differential and linear clustering effect** of the **256-bit KNOT permutation** and **compare the results** obtained by MILP method with those obtained by our new method.

- **Differential clustering of 14-round RECTANGLE:**
  - The designers of RECTANGLE claim that: *" For 14-round RECTANGLE, the probability of the best differential trail is 2^{-61} . We have searched for all 14-round differential trails with probability between 2^{-61} and 2^{-71}, and examined all the difference propagations made up of these investigated trails."* Then they present a 14-round difference propagation with probability $2^{-60.66}$.
  - Fixing the input and output difference and **applying our method**, we obtain **the same probability $2^{-60.66}$**.

- **Linear clustering of 14-round RECTANGLE:**
  - The designers of RECTANGLE claim that: **"For 14-round RECTANGLE, we have searched for all the linear trails with a correlation (absolute value) between 2^{-34} and 2^{-37} and examined all the linear propagations made up of the investigated trails."** Then they present a linear propagation with correlation $2^{-31.61}$.
  - Fixing the input and output mask and **applying our method**, we obtain a correlation $2^{-32.32}$, which is very close to the designers' result $2^{-31.61}$.

- **10-round difference propagation of 256-bit KNOT permutation:**
  - for Diff-Col-I type distinguishers, a 10-round difference propagation is given by

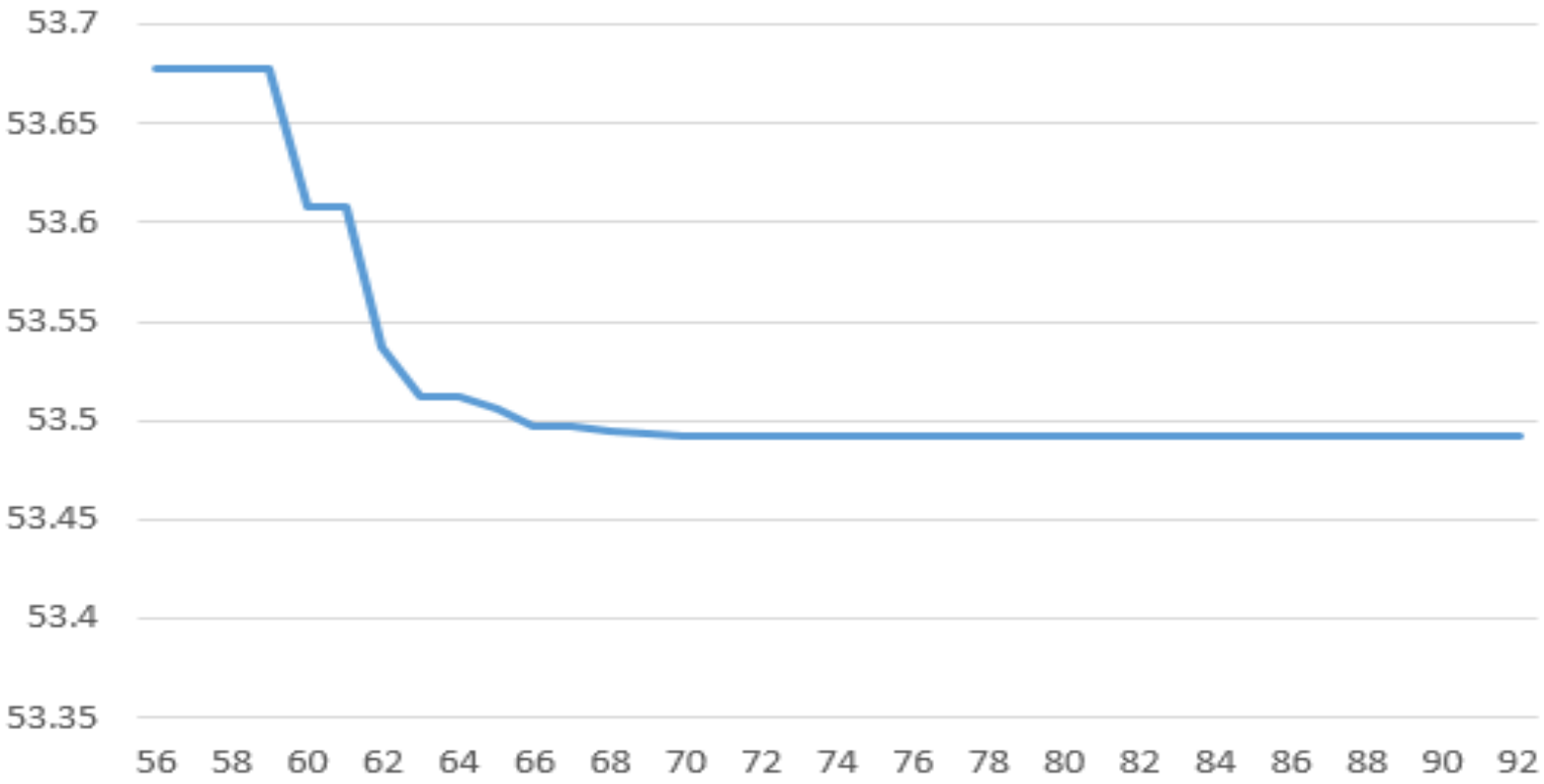
0000000000000000000000000000100000000000000010000000000000

↓

0000000000000000000000000000100000000000000000000000000001

with probability $2^{-53.7}$.

  - Fixing the input and output difference and **using MILP method**, we have searched for all the differential trails with probability between 2^{-56} and 2^{-92}, and get **a total probability $2^{-53.5}$**.



For the 10-round difference propagation, y = f(x) is the accumulated weight by considering all the differential trails with weight less than or equal to x.

- **10-round linear propagation of the 256-bit KNOT permutation:**
  - for Linear-Enc-D type distinguisher, a 10-round linear propagation is given by

1000000000000000000000010000000000000000000000000000000000000000

↓

0000000000000000000000010000000000000000100000000000000000000000

with correlation amplitude 2^{-27}.

  - Fixing the input and output mask and **using MILP method**, we have searched for all the linear trails with probability between 2^{-30} and 2^{-40}, and get **a correlation amplitude $2^{-25.4}$**.

- we use an **11-round linear propagation** (Linear-Enc-D type) for further verification

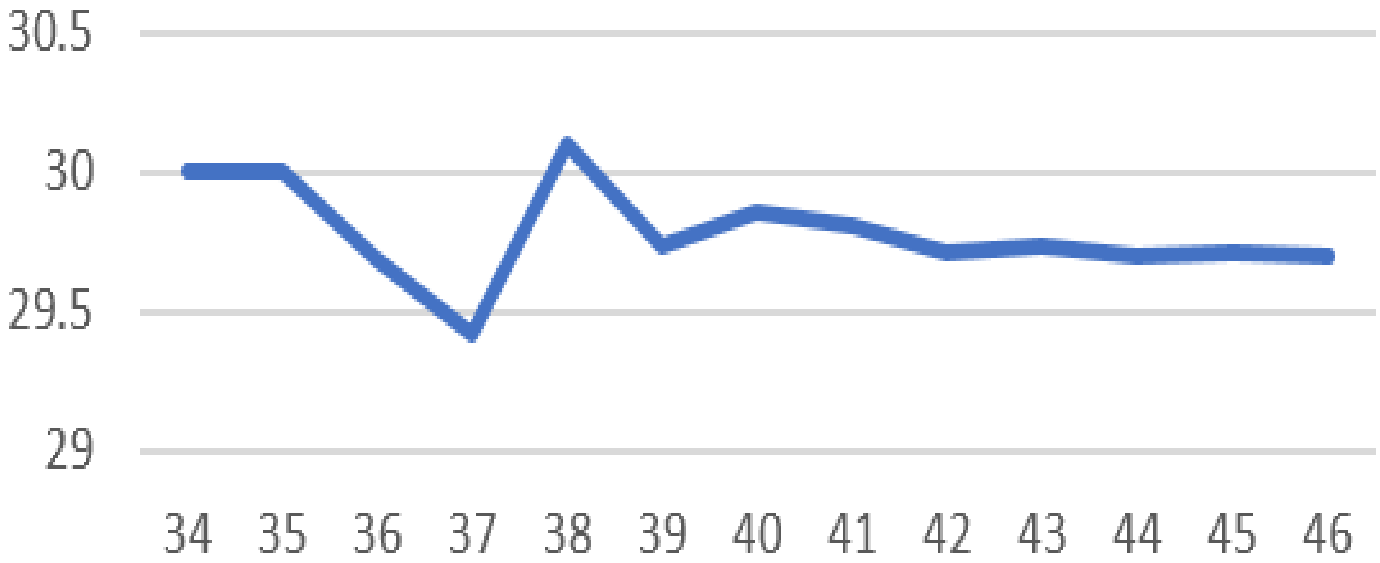
  1000000000000000000000010000000000000000000000000000000000

  ↓

  0000000000000000000000010000000000000010000000000000000000

  with correlation amplitude 2^{-30} using our method.

  - Fixing the input and output mask and **using MILP method**, we have searched for all the linear trails with a probability between 2^{-34} and 2^{-46}, and get **a correlation amplitude $2^{-29.7}$**, which is very close to the value 2^{-30}.



For the 11-round linear propagation, $y = f(x)$ is the accumulated weight by considering all the linear trails with weight less than or equal to x.

7.Summary and Discussion

- By studying **the class of differential/linear trails which contain iterative sub-trails**, we have **efficiently obtained effective differential/linear distinguishers** for each KNOT-AEAD member w.r.t. 6 attack models and for each KNOT-Hash member w.r.t. 2 attack models.

- Furthermore, we investigated the accuracy of our new method in two different ways. Based on these comparative results, we can reasonably infer that the results using our new approach provide **a quite accurate security evaluation** of KNOT-AEAD and KNOT-Hash:
  - **each KNOT-AEAD member** has **at least 50% security margin** against the 6 attack models. Especially, **72% security margin of the initialization phase**, *possibility of reducing the number of rounds in initialization phase.*
  - **each KNOT-Hash member** has **at least 80% security margin** against the 2 attack models, *possibility of reducing the number of rounds.*

- Security decreases in a multi-key scenario:
  - If taking multi-key attacks into account, the number of rounds of KNOT need to be increased.

- However, in practice, the data limit 2^{64} is sufficient for lightweight applications.

Thanks for your attention!

Questions?