



Tighter proofs of CCA security in the QROM

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Outline

17 different PKE/KEM families in NIST PQC round 2

Core mathematical problem with hashing as glue. Eg:

Start with passively-secure rPKE or dPKE

Turn into KEM by encrypting random m ; then $k \leftarrow H(m, c)$

CCA security requires variant of Fujisaki-Okamoto transform [FO99]:

If rPKE, derandomize by setting $\text{coins} \leftarrow G(m)$

Optional: also use message confirmation $\text{tag} \leftarrow H'(m)$

Recipient checks that m was encrypted properly; if not, reject

Explicit rejection: $k \leftarrow \perp$

Implicit rejection: $k \leftarrow H(\text{prfkey}, c)$

Contributions of this paper

Modular proof that certain KEMs are almost as secure as underlying PKE

Either implicit rejection, or explicit + message confirmation

Consider reaction attacks against PKE with nonzero failure probability

Tightly: adversary must submit a failing ciphertext, **without knowledge of sk**, to gain advantage

Limitations:

QROM proof, not standard model

Some steps aren't tight

Requires dPKE $\text{Encrypt}(pk, \cdot)$ injective whp

Doesn't model multi-key attacks

Doesn't resolve $G(m)$ vs $G(pk, m)$

Related work

[HHK17]: original modular proofs of QROM security

Comprehensive but not very tight

[SXY18]: tighter results using implicit rejection

[JZCWM18, JZM19]: line of improved approaches, mostly using implicit rejection

[HKSU19]: approximately the same overall bound as this work

With/without injectivity requirements depending on version

Uses disjoint simulability (DS) security notion instead of OW-CPA

Classical vs Quantum Random Oracles

Random oracle model: pretend the hash H is a uniformly random function

- Adversary can't run H anymore, has to call an **oracle**

- Simulator can see the calls, choose the outputs

 - (They must still look uniformly, independently random)

Classical ROM

- Simulator can record all oracle queries

- Simulator can reprogram oracle **adaptively**

Quantum ROM

- Queries are quantum superpositions

- Much harder to record oracle queries (see [\[Zha19\]](#))

- Much harder to respond adaptively

Unruh's one-way to hiding (O2H) technique

Suppose simulator changes oracle G to a slightly different oracle H

G, H differ only on a small set S

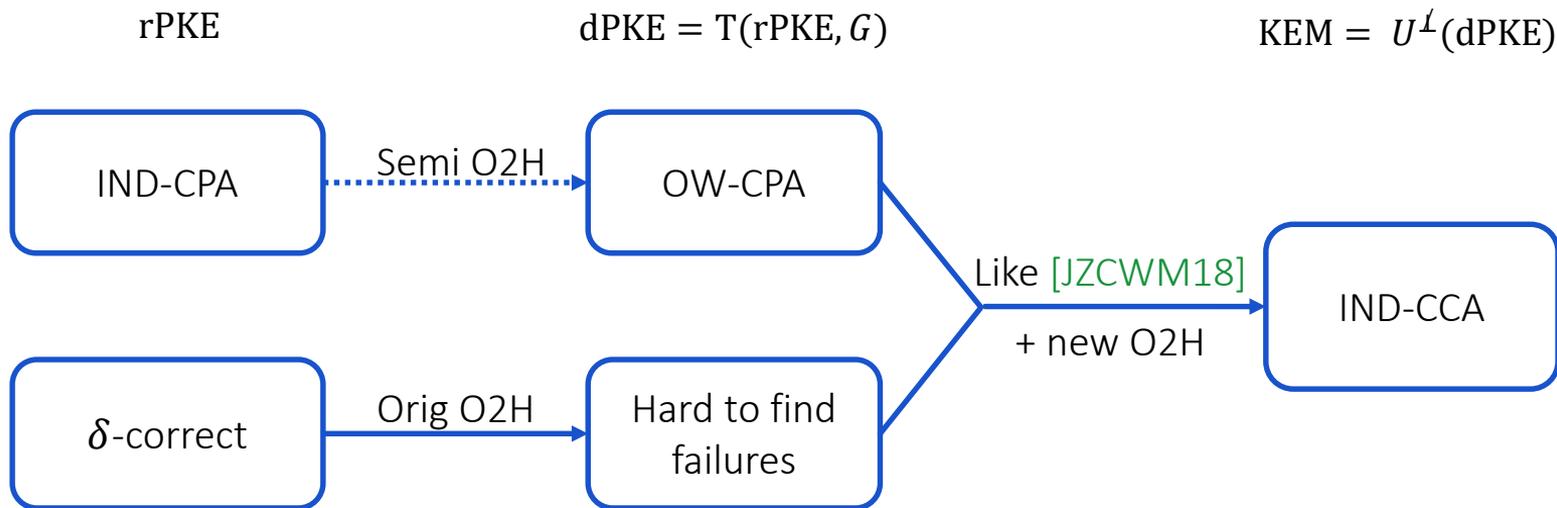
If adversary behaves differently w/p δ , it must be querying some $x \in S$

Simulator can extract x with probability ϵ depending on δ ; depth d

O2H variant	Oracles differ	Sim can simulate	Bound
Original [Unr15]	Arbitrary	G or H	$\delta \leq 2d\sqrt{\epsilon}$
Semi-classical [AHU19]	Arbitrary	$(G$ or $H)$ and S	$\delta \leq 2\sqrt{d\epsilon}$
Double-sided 	One place	G and H	$\delta \leq 2\sqrt{\epsilon}$

Modular reduction outline

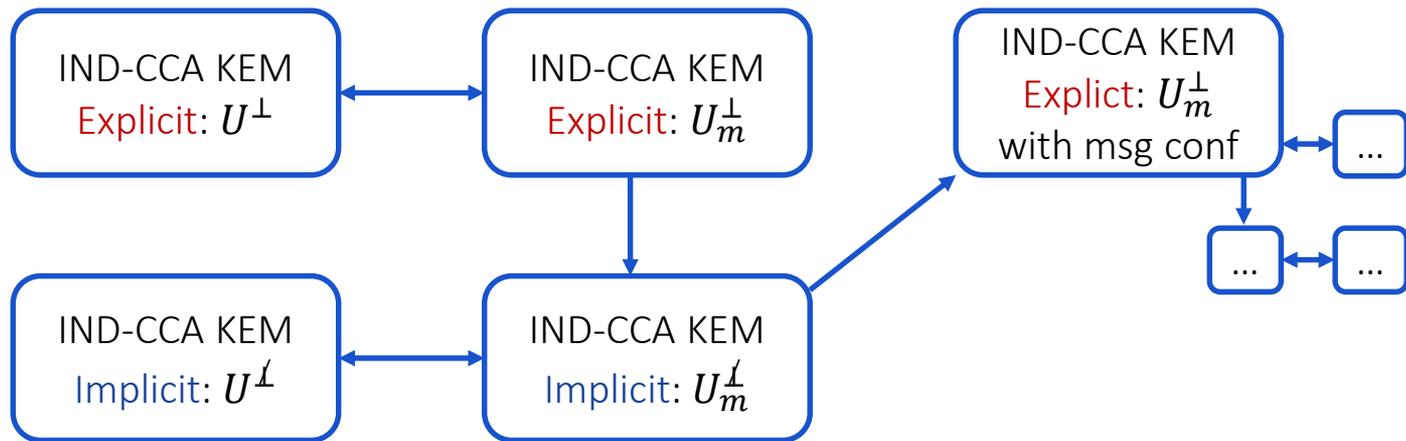
(Assuming $Enc(pk, \cdot)$ injective)



Could start with $OW\text{-}CPA$ instead via orig O2H, at cost of factor of d tightness

Modular reduction outline

(Assuming $Enc(pk, \cdot)$ injective)



$k \leftarrow H(m)$ is as secure as $k \leftarrow H(m, c)$
... in single-target case in QRROM!

Explicit rejection is secure with (short) message confirmation hash

OW-CPA dPKE \rightarrow IND-CCA KEM

Encaps(pk):

$\overset{R}{m} \leftarrow$ message space
 $c \leftarrow \text{Encrypt}(pk, m)$
 $k \leftarrow H(m)$

Decaps($(sk, pk, prfk), c$):

If $c = c^*$: return \perp
 $m' \leftarrow \text{Decrypt}(sk, c)$
If $\text{Encrypt}(pk, m') = c$:
 return $k' \leftarrow H(m)$
Else: return $k' \leftarrow \text{PRF}(prfk, c)$

1. Adv is given $c^* \leftarrow \text{Encrypt}(pk, m^*)$ and either $k^* \leftarrow H(m^*)$ or random

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Else: return $k' \leftarrow R(c)$

1. Adv is given $c^* \leftarrow \text{Encrypt}(pk, m^*)$ and either $k^* \leftarrow H(m^*)$ or random
2. Change $\text{PRF}(prfk, c) \rightarrow R(c)$

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1. Adv is given $c^* \leftarrow \text{Encrypt}(pk, m^*)$ and either $k^* \leftarrow H(m^*)$ or random
2. Change PRF($prfk, c$) $\rightarrow R(c)$
3. Forward $H(m) \rightarrow R(\text{Encrypt}(pk, m))$
 - Requires $\text{Encrypt}(pk, \cdot)$ injective
 - Independent of PRF changes (red $R(c)$) unless decryption failed

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4. Now Decaps oracle is easy

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5. Problem is equivalent to distinguishing $(c^*, k^*, H[m^* \rightarrow k^*]) \leftrightarrow (c^*, k^*, H)$
 - Apply double-sided O2H: can recover m^*

Future goals

Tighter proof

No square roots, possibly using [MW18] notion of IND

No loss of tightness $d \cdot \text{Adv}_A^{\text{IND-CPA}}$

Get rid of injectivity requirements

Find failing message instead of ciphertext

Multi-key security proof with $H(pk, \dots)$

Prove security of explicit rejection without keyconf

Acknowledgments

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Questions?

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