

**Visualizing  
size-security tradeoffs  
for lattice-based encryption**

**Daniel J. Bernstein**

# Horizontal axis: ciphertext size

Why focus on size instead of CPU time?

— Fitting into existing frameworks and protocols.

Data from Google. Long term: Hardware trends.

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e.g. `ntrup` beats `sntrup` in key size,

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— Google's 2016 experiment used key+ciphertext.

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e.g. `ntrup` beats `sntrup` in key size,  
but `sntrup` beats `ntrup` in ciphertext size.

— Google's 2016 experiment used `key+ciphertext`.  
But long term: Use IND-CCA2 to multicast+cache  
public keys (2015 McGrew). Lattice traffic is then  
much closer to ciphertext than to `key+ciphertext`.

# Vertical axis: Core-SVP security estimate

Beware (potential/actual) oversimplifications inside lattice security estimates. Can lead to:

- Overstating security.
- Understating security—damaging deployment.
- Damaging comparisons: e.g. omitting “hybrid attacks”; e.g. overstating `sntrup` “rotations”.

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Security estimate where (claimed) data points are easiest to find: “Core-SVP” pre-quantum estimate.

# Vertical axis: Core-SVP security estimate

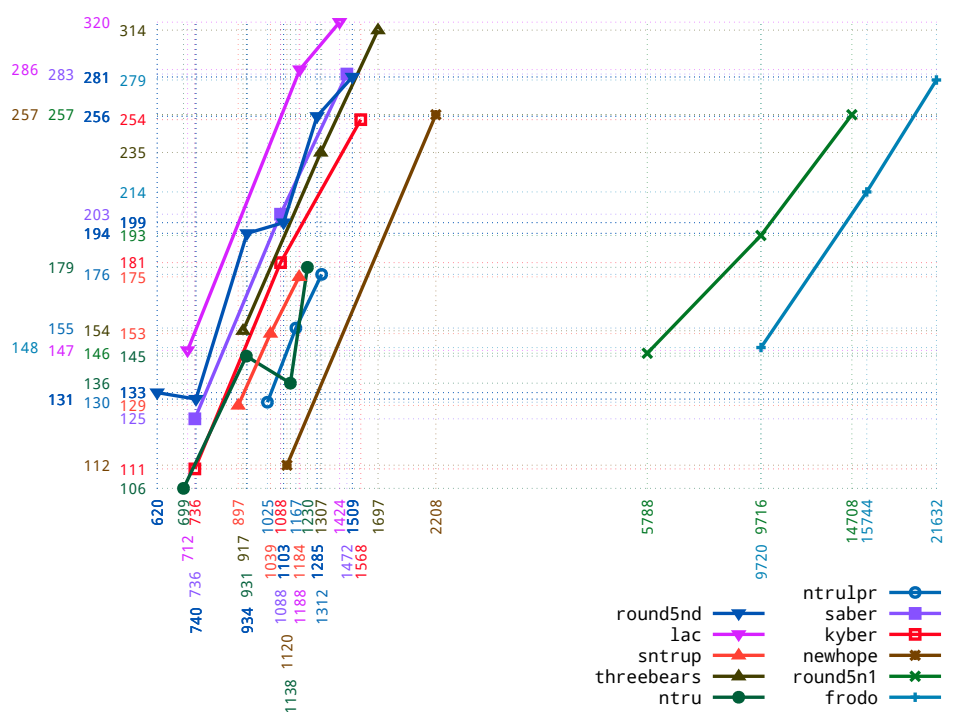
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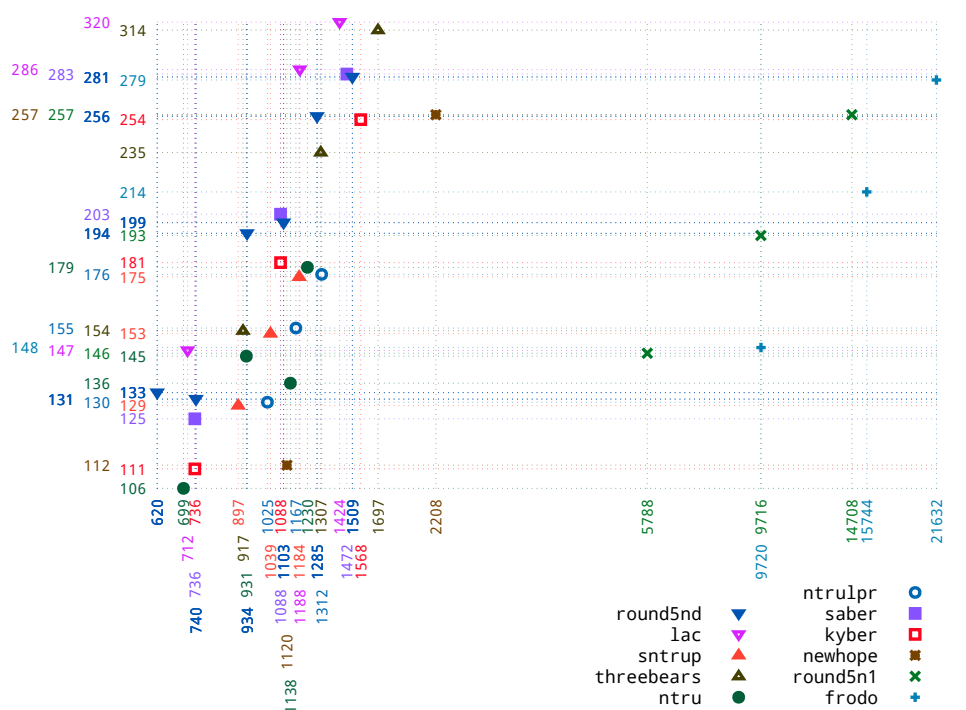
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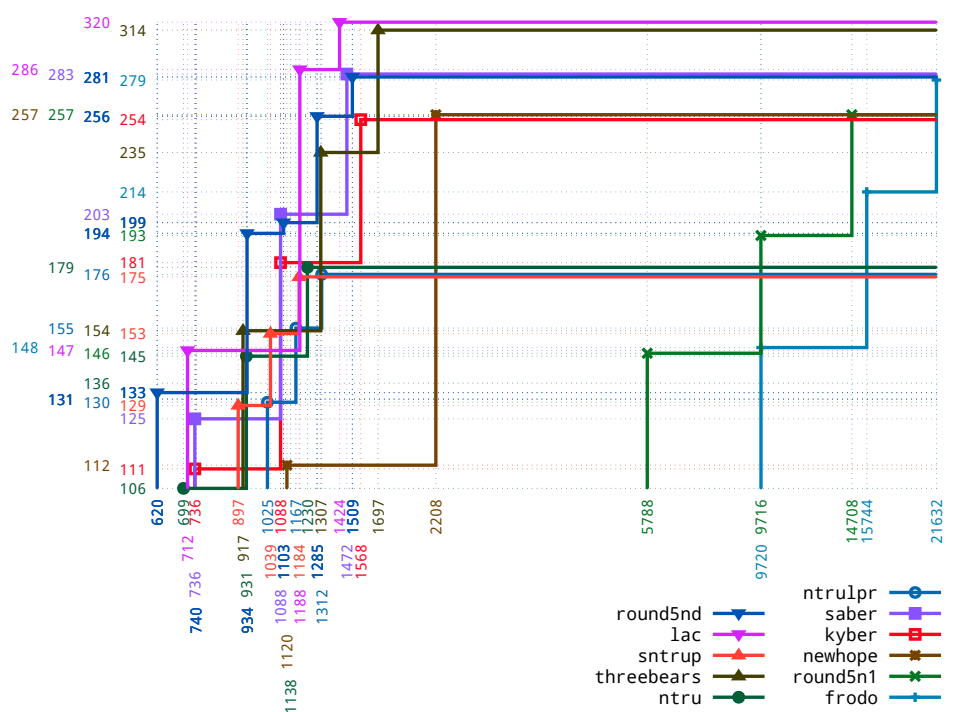
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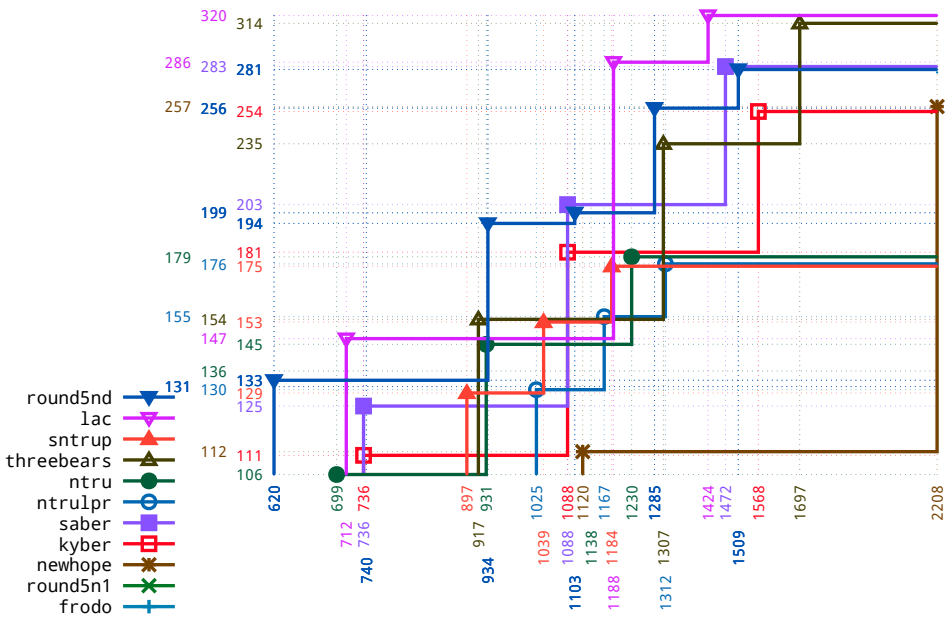
Some work on better estimates; should continue this work, re-estimate all the schemes, draw new graphs.





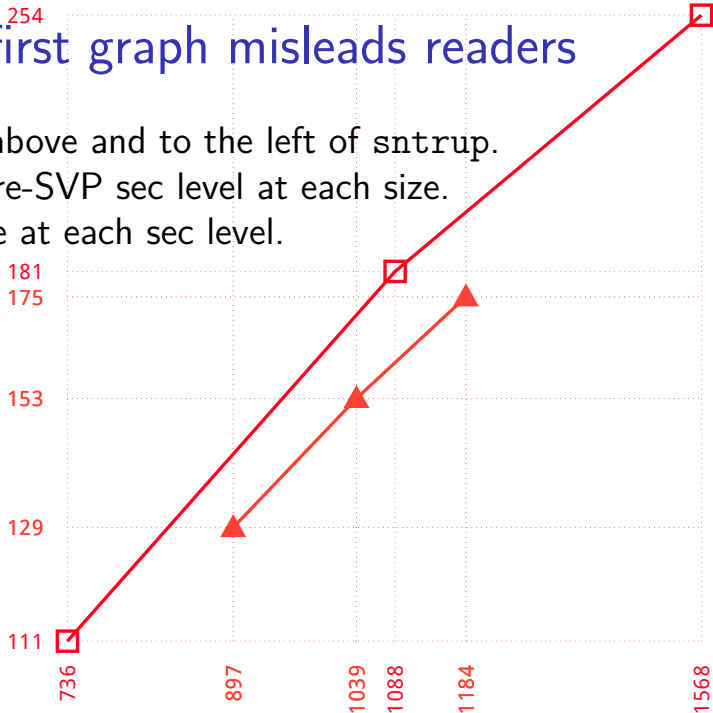






# How the first graph misleads readers

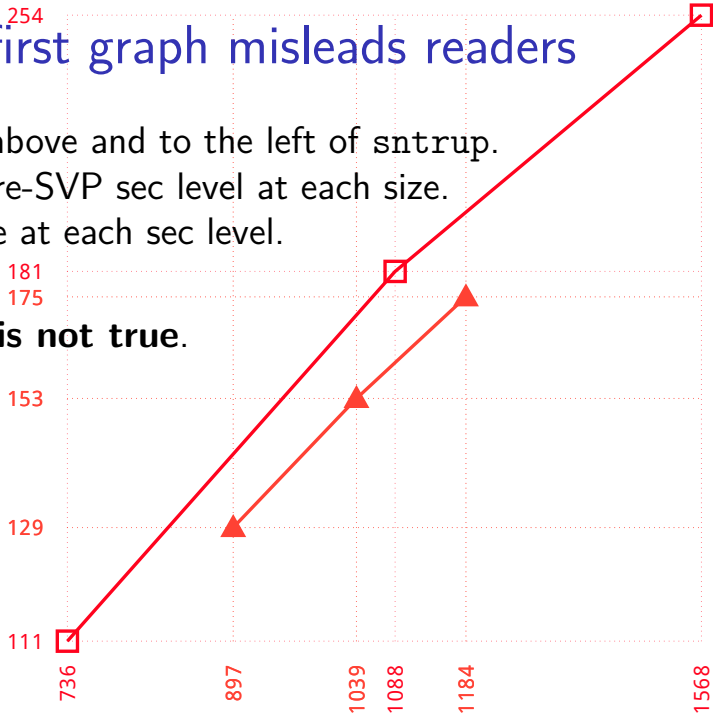
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Better Core-SVP sec level at each size.  
Better size at each sec level.



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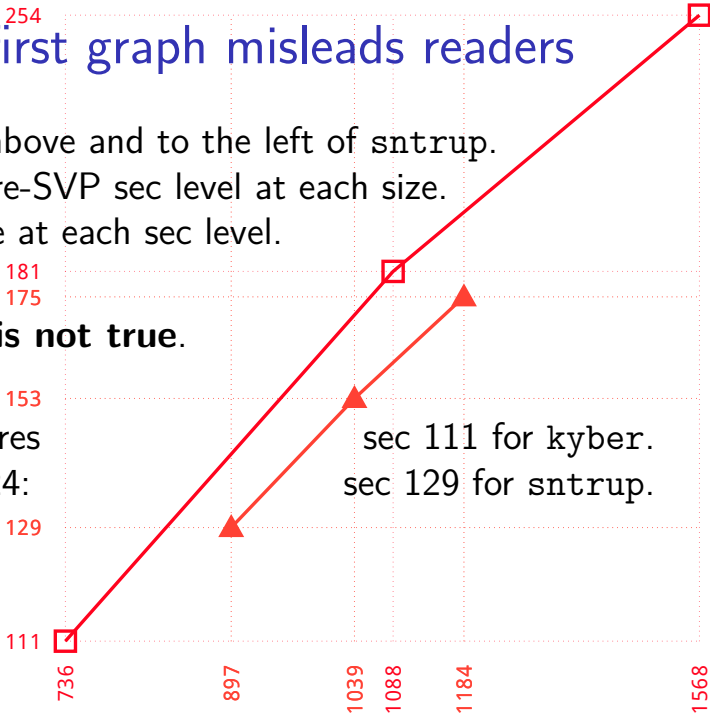
# How the first graph misleads readers

kyber is above and to the left of sntrup.  
Better Core-SVP sec level at each size.  
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**But this is not true.**

User requires  
size  $\leq 1024$ :

sec 111 for kyber.  
sec 129 for sntrup.



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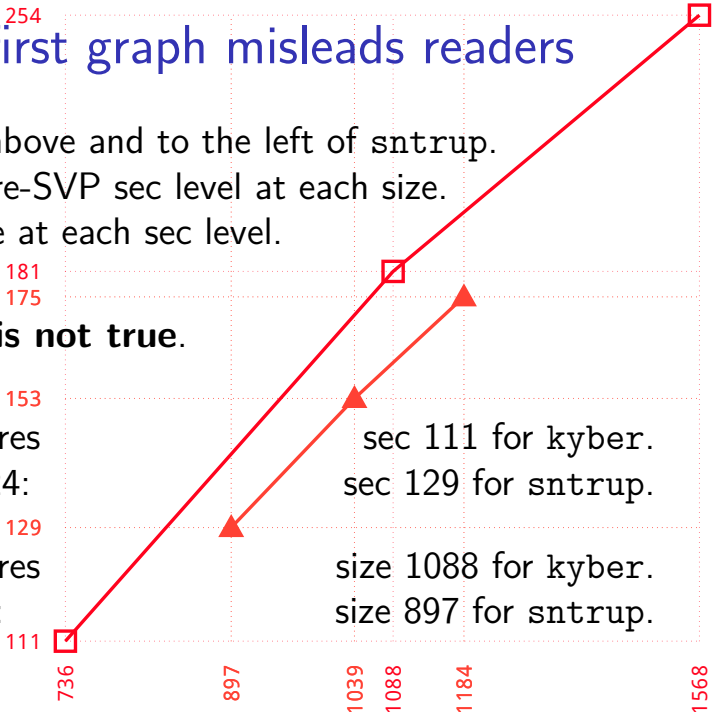
**But this is not true.**

User requires  
size  $\leq 1024$ :

User requires  
sec  $\geq 128$ :

sec 111 for kyber.  
sec 129 for sntrup.

size 1088 for kyber.  
size 897 for sntrup.





# Ciphertext-size comparison examples

Core-SVP for sntrup options: 129, 153, 175.

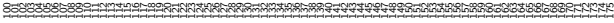


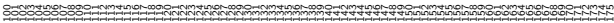

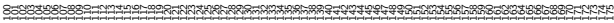

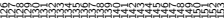

User picks  $\lambda \geq 100$ , requires Core-SVP  $\geq \lambda$ .

$X$	size(sntrup) < size( $X$ ) for $\lambda$ in
frodo	{100, ..., 175}
kyber	{112, ..., 153}
lac	{148, ..., 175}
newhope	{100, ..., 175}
ntru	{107, ..., 129} $\cup$ {146, ..., 175}
round5n1	{100, ..., 175}
round5nd	{}
saber	{126, ..., 153}
threebears	{100, ..., 129} $\cup$ {155, ..., 175}

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# Core-SVP comparison examples

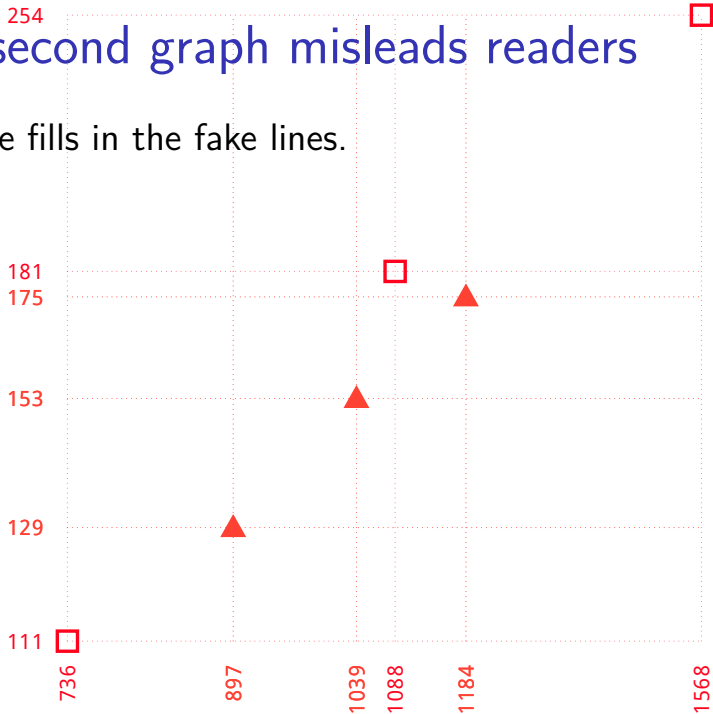
Ciphertext sizes for sntrup: 897, 1039, 1184.

User picks  $S \leq 1280$ , requires ciphertext size  $\leq S$ .

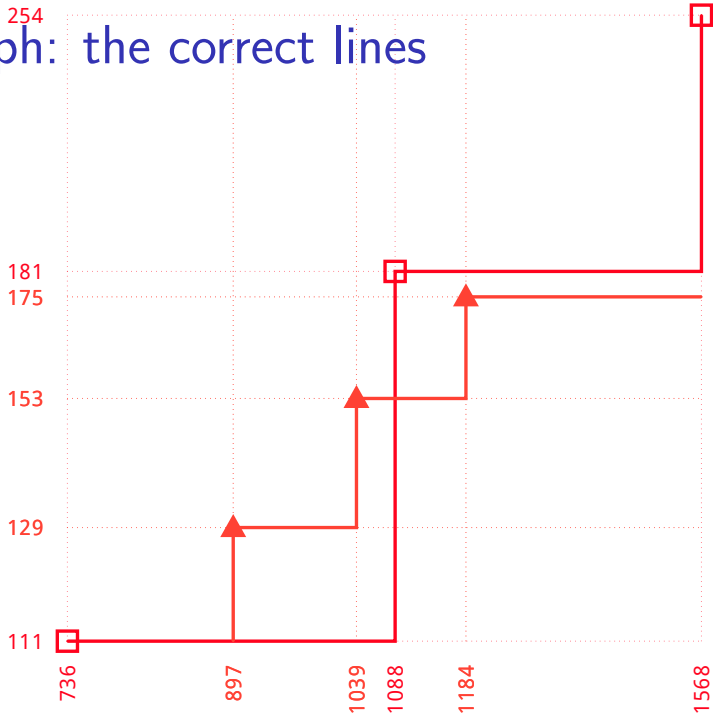
$X$	$\text{sec}(\text{sntrup}) > \text{sec}(X)$ for $S$ in
frodo	$\{897, \dots, 1280\}$
kyber	$\{897, \dots, 1087\}$
lac	$\{1039, \dots, 1187\}$
newhope	$\{897, \dots, 1280\}$
ntnu	$\{897, \dots, 930\} \cup \{1039, \dots, 1229\}$
round5n1	$\{897, \dots, 1280\}$
round5nd	$\{\}$
saber	$\{897, \dots, 1087\}$
threebears	$\{897, \dots, 916\} \cup \{1184, \dots, 1280\}$

# How the second graph misleads readers

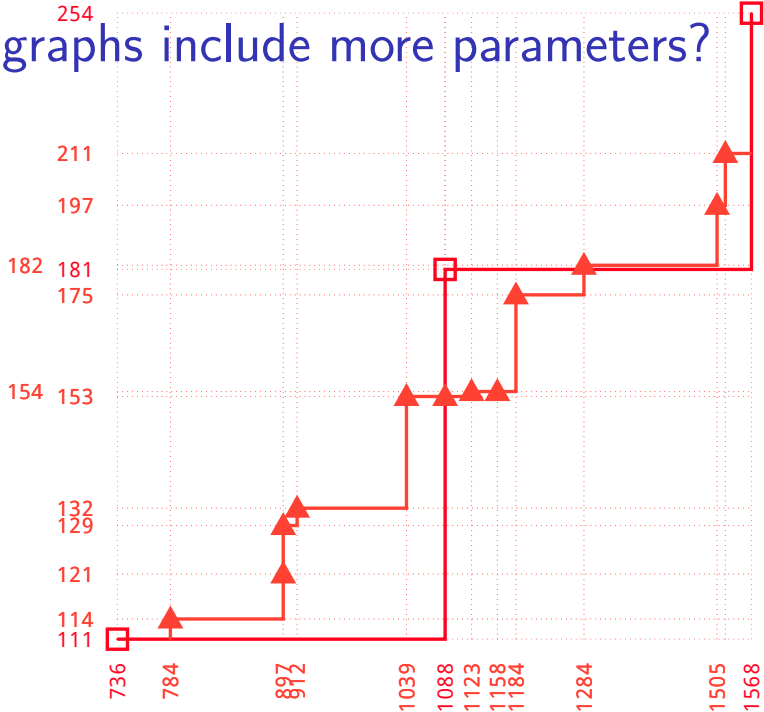
Human eye fills in the fake lines.



# Third graph: the correct lines



# Should graphs include more parameters?



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