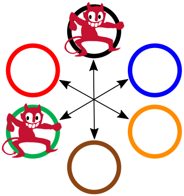
Towards Standardization of Threshold Schemes for Cryptographic Primitives at NIST

Luís Brandão

Joint work with: Apostol Vassilev, Nicky Mouha, Michael Davidson



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National Institute of Standards and Technology (Gaithersburg MD, USA)

Presentation at ICMC19

International Cryptographic Module Conference
May 16, 2019 @ Vancouver, Canada

Outline

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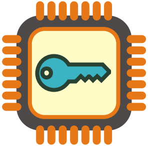

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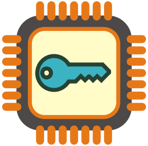

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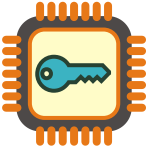

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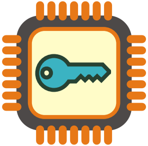

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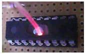

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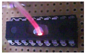
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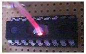

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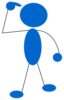
How can we address single-points of failure?

*question-2.html



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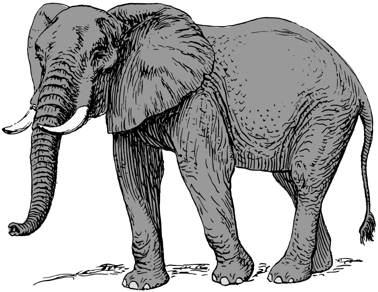

*colored-elephant.html

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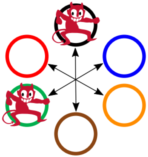
The threshold approach



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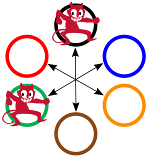
The threshold approach

At high-level:

use redundancy & diversity to mitigate the *compromise* of up to a threshold number (f-out-of-n) of components



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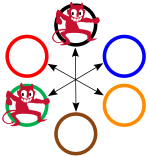

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improve security

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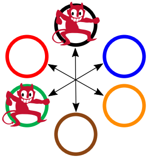

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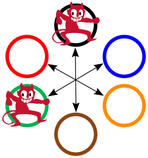

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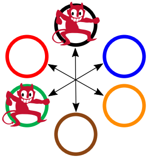

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Some properties:

- **withstands** several *compromised* components;
- **needs** several *uncompromised* components;
- **prevents** secret keys from being in one place;
- **enhances** resistance against side-channel attacks; ...

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Split a secret key into n secret “shares” for storage at rest.

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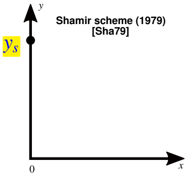

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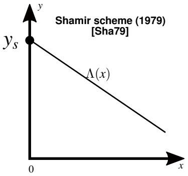

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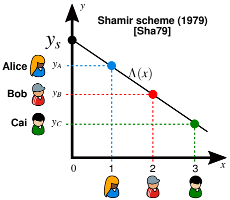

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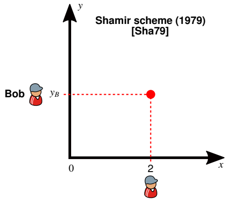

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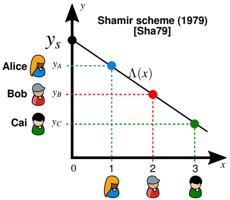

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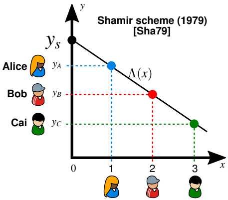

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Alice: *=2478
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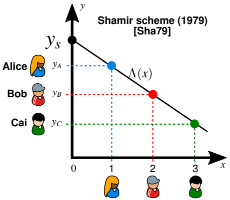

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Use threshold schemes for cryptographic primitives (next)

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1. Convey high-dimensionality of the threshold space

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Overview the NIST effort towards standardization of threshold schemes

1. Convey high-dimensionality of the threshold space
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3. Motivate feedback and engagement from stakeholders

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Conventional scheme $(k = n = 1)$

A 3-out-of-3 threshold scheme $(k = n = 3)$

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  - Public VerKey: $e \;\; (= d^{-1} \pmod{\phi})$
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- KeyGen (by dealer):
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About this threshold scheme:

SignKey d not recombined;

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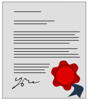
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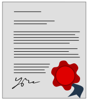

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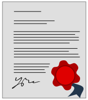

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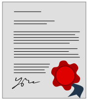

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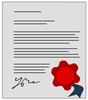

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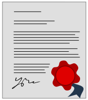

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Depends on attack model (e.g., attack surface, ...), system model (e.g., rejuvenations, ...), ...

Outline

NIST Internal Report (NISTIR) 8214

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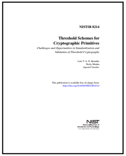
Threshold Schemes for Cryptographic Primitives — Challenges and Opportunities in Standardization and Validation of Threshold Cryptography. [BMV18] doi:10.6028/NIST.IR.8214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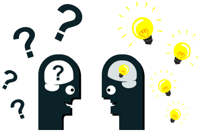
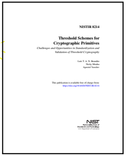


Image adapted from: openclipart.org/detail/283392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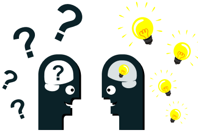

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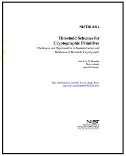

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Past timeline:

- 2018-July: Draft online 3 months for public comments
- 2018-October: Received comments from 13 external sources
- 2019-March: Final version online, along with “diff” and received comments

https://csrc.nist.gov/publications/detail/nistir/8214/final

Characterizing threshold schemes

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To reflect on a threshold scheme, start by characterizing **4 main features**:

- Kinds of threshold 
- Executing platform 
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But there are other factors ...

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- **Conceivable attack types.**
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A threshold scheme **improving** security against an attack in an application **may be powerless or degrade** security for another attack in another application

The validation challenge

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Devise standards of testable and validatable threshold schemes

vs.

devise testing and validation for standardized threshold schemes

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Devise standards of testable and validatable threshold schemes
vs.
devise testing and validation for standardized threshold schemes

Validation is needed in the federal context:

- need to use **validated** implementations [tC96] of **standardized** algorithms
- FIPS 140-2/3 defines, for cryptographic modules, 4 security levels: subsets of applicable security assertions [NIS01]

(FIPS = Federal Information Processing Standards)

Outline

#NTCW2019

NIST Threshold Cryptography Workshop 2019

https://csrc.nist.gov/Events/2019/NTCW19

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March 11–12, 2019 @
NIST Gaithersburg MD, USA

www.nist.gov/image/surfgaithersburgjpg

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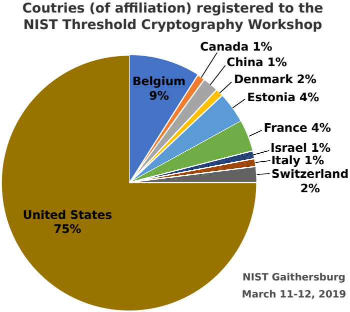

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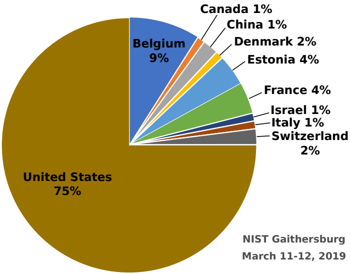
Coutries (of affiliation) registered to the NIST Threshold Cryptography Workshop



About 80 attendees

A platform for open interaction:

- hear about experiences with threshold crypto;
- get to know stakeholders;
- get input to reflect on roadmap and criteria.

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Format and content

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Accepted 15 external submissions:

- 2 panels
- 5 papers
- 8 presentations

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Discussion of diverse topics:

- threshold schemes in general (motivation and implementation feasibility);
- NIST standardization of cryptographic primitives
- a post-quantum threshold public-key encryption scheme;
- threshold signatures (adaptive security; elliptic curve digital signature algorithm);
- validation of cryptographic implementations;
- threshold circuit design (tradeoffs, pitfalls, combined attacks, verification tools);
- secret-sharing with leakage resilience;
- distributed symmetric-key encryption;
- applications and experience with threshold cryptography.

Results

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A step in *driving an open and transparent process towards standardization of threshold schemes for cryptographic primitives*. (See NISTIR 7977)

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Some notes:

- differences in granularity (building blocks vs. full functionalities);
- separation of single-device vs. multi-party;
- importance of envisioning applications;
- stakeholders' willingness to contribute;
- usefulness of explaining rationale (e.g., as complimented for the NISTIR);
- encouragement to move forward.

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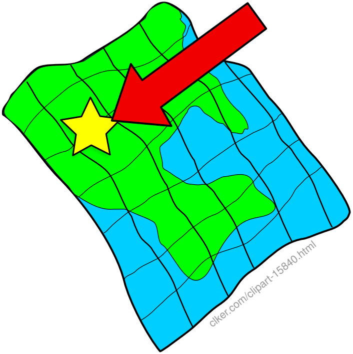
These elements are helpful for the next step ... designing a roadmap

Outline

Preliminary roadmap (ongoing)

We are writing a draft “preliminary roadmap”



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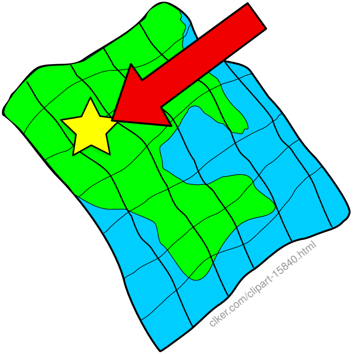

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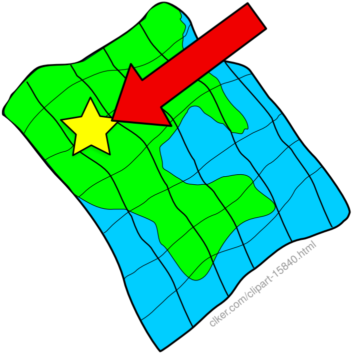

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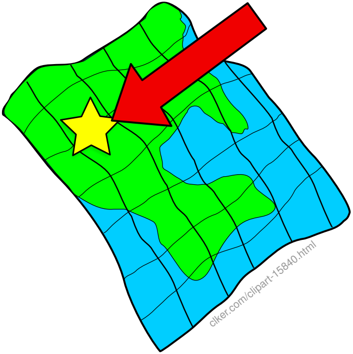

Need: ***mapping layers*** (coordinates) and ***weighing factors***

Disclaimer: the structure suggested in the next slides is still subject to change.

Mapping layers

An abstract layered decomposition of the threshold standardization space

Four layers

$\boxed{\text{Standardization space for threshold schemes for cryptographic primitives}}$

Mapping layers

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Four layers: domains

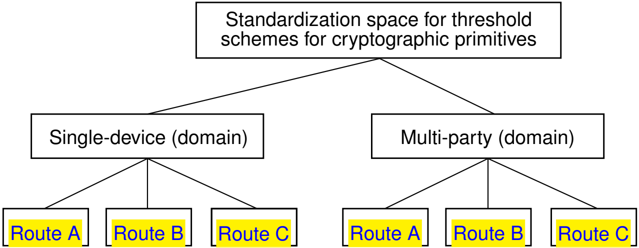
Standardization space for threshold schemes for cryptographic primitives

- Single-device (domain)
- Multi-party (domain)

Mapping layers

An abstract layered decomposition of the threshold standardization space

Four layers: domains, routes



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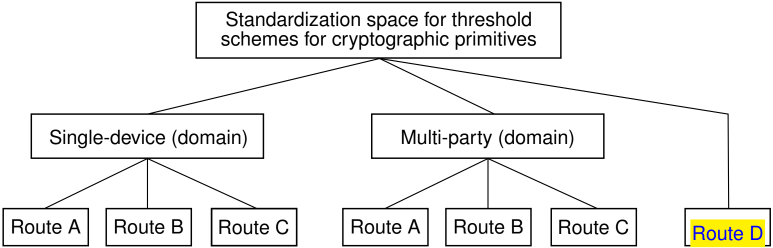

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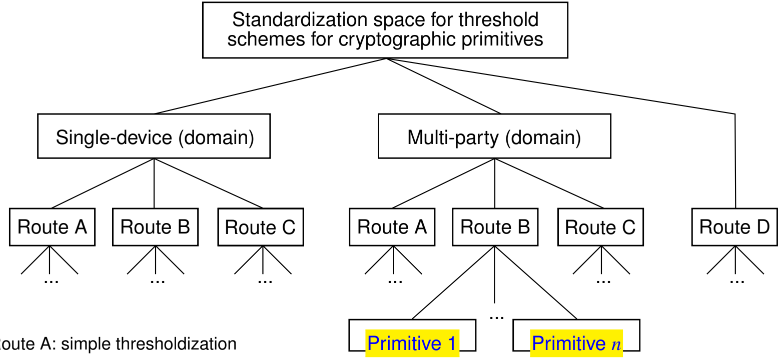

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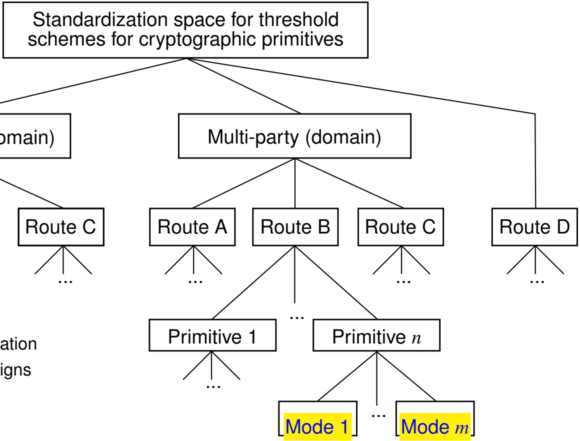

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Not every possible combination needs to be a standardization goal

Weighing factors

The four layers provide a map. But where to look in the map?

openclipart.org/detail/281637

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  - threshold circuit design in single-device (address side-channel leakage)
  - distribute trust across several operators of crypto primitives*
  - multi-signatures in crypto currencies
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*(emphasis on approved conventional primitives)

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- **Useful features:**
  - efficiency and practicality
  - suitability for automated testing
  - ability to rejuvenate components
  - ...

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- elements within *layers*, *application motivations* and *other factors*
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Later: separate criteria for separate focuses; calls for contributions

Example *routes* for calls for contributions:

- algorithms for standardization
- reference implementations and comparisons
- research contributions
- ...

Possibly fit some of these in a 2^{nd} workshop (?)

Outline

Final remarks

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  - in technology ... when crypto implementations have vulnerabilities
  - at the human level ... when crypto operators go rogue

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  - at the human level ... when crypto operators go rogue
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We would like to have a process in collaboration with stakeholders!

- Project webpage: https://csrc.nist.gov/Projects/Threshold-Cryptography
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Word cloud based on the NISTIR 8214

Thank you for your attention

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Presentation at the International Cryptographic Module Conference
May 16, 2019 @ Vancouver, Canada

luis.brandao@nist.gov

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References

[BB12] L. T. A. N. Brandão and A. N. Bessani. *On the reliability and availability of replicated and rejuvenating systems under stealth attacks and intrusions*. Journal of the Brazilian Computer Society, 18(1):61–80, 2012. DOI:10.1007/s13173-012-0062-x.

[BDL97] D. Boneh, R. A. DeMillo, and R. J. Lipton. *On the Importance of Checking Cryptographic Protocols for Faults*. In W. Fumy (ed.), Advances in Cryptology — EUROCRYPT '97, pages 37–51, Berlin, Heidelberg, 1997. Springer Berlin Heidelberg. DOI:10.1007/3-540-69053-0˙4.

[BMV18] L. T. A. N. Brandão, N. Mouha, and A. Vassilev. *Threshold Schemes for Cryptographic Primitives — Challenges and Opportunities in Standardization and Validation of Threshold Cryptography*. Draft NISTIR 8214, July 2018.

[BMW+18] J. v. Bulck, M. Minkin, O. Weisse, D. Genkin, B. Kasikci, F. Piessens, M. Silberstein, T. F. Wenisch, Y. Yarom, and R. Strackx. *Foreshadow: Extracting the Keys to the Intel SGX Kingdom with Transient Out-of-Order Execution*. In 27th USENIX Security Symposium (USENIX Security 18), page 991–1008, Baltimore, MD, 2018. USENIX Association.

[BN06] M. Bellare and G. Neven. *Multi-signatures in the Plain public-Key Model and a General Forking Lemma*. In Proceedings of the 13th ACM Conference on Computer and Communications Security, CCS '06, pages 390–399, New York, NY, USA, 2006. ACM. DOI:10.1145/1180405.1180453.

[Cha00] G. Chaucer. *The Ten Commandments of Love*, 1340–1400. See "For three may kepe counseil if twain be away!" in the "Secretnesse" stanza of the poem. https://sites.fas.harvard.edu/ chaucer/special/lifemann/love/ten-comm.html. Accessed: July 2018.

[DLK+14] Z. Durumeric, F. Li, J. Kasten, J. Amann, J. Beekman, M. Payer, N. Weaver, D. Adrian, V. Paxson, M. Bailey, and J. A. Halderman. *The Matter of Heartbleed*. In Proceedings of the 2014 Conference on Internet Measurement Conference, IMC '14, pages 475–488, New York, NY, USA, 2014. ACM. DOI:10.1145/2663716.2663755.

[Don13] D. Donzai. *Using Cold Boot Attacks and Other Forensic Techniques in Penetration Tests*, 2013. https://www.ethicalhacker.net/features/root/using-cold-boot-attacks-forensic-techniques-penetration-tests/. Accessed: July 2018.

[Gro16] C. T. Group. *NIST Cryptographic Standards and Guidelines Development Process*. NISTIR 7977, March 2016. DOI:10.6028/NIST.IR.7977.

[HSH+09] J. A. Halderman, S. D. Schoen, N. Heninger, W. Clarkson, W. Paul, J. A. Calandrino, A. J. Feldman, J. Appelbaum, and E. W. Felten. *Lest We Remember: Cold-boot Attacks on Encryption Keys*. Commun. ACM, 52(5):91–98, May 2009. DOI:10.1145/1506409.1506429.

[KGG+18] P. Kocher, D. Genkin, D. Gruss, W. Haas, M. Hamburg, M. Lipp, S. Mangard, T. Prescher, M. Schwarz, and Y. Yarom. *Spectre Attacks: Exploiting Speculative Execution*. ArXiv e-prints, January 2018. arXiv:1801.01203.

[LSG+18] M. Lipp, M. Schwarz, D. Gruss, T. Prescher, W. Haas, S. Mangard, P. Kocher, D. Genkin, Y. Yarom, and M. Hamburg. *Meltdown*. ArXiv e-prints, jan 2018. arXiv:1801.01207.

[MDS19] MDS. *RIDL and Fallout: MDS attacks*, 2019. https://mdsattacks.com/.

[NIS01] NIST. *Security Requirements for Cryptographic Modules, Federal Information Processing Standard (FIPS) 140-2*, 2001. DOI:10.6028/NIST.FIPS.140-2.

[RSA78] R. L. Rivest, A. Shamir, and L. Adleman. *A method for obtaining digital signatures and public-key cryptosystems*. Communications of the ACM, 21(2):120–126, 1978. DOI:10.1145/359340.359342.

[RSWO17] E. Ronen., A. Shamir, A.-O. Weingarten, and C. O'Flynn. *IoT Goes Nuclear: Creating a ZigBee Chain Reaction*. IEEE Symposium on Security and Privacy, pages 195–212, 2017. DOI:10.1109/SP.2017.14.

[Sau34] R. Saunders. *Poor Richard's Almanack — 1735*. Benjamin Franklin, 1734.

[Sch90] C. P. Schnorr. *Efficient Identification and Signatures for Smart Cards*. In G. Brassard (ed.), Advances in Cryptology — CRYPTO' 89 Proceedings, pages 239–252, New York, NY, 1990. Springer New York. DOI:10.1007/0-387-34805-0˙22.

[SH07] J.-M. Schmidt and M. Hutter. *Optical and EM Fault-Attacks on CRT-based RSA: Concrete Results*, pages 61–67. Verlag der Technischen Universität Graz, 2007.

[Sha97] W. Shakespeare. *An excellent conceited Tragedie of Romeo and Juliet*. Printed by John Danter, London, 1597.

[Sha79] A. Shamir. *How to Share a Secret*. Communications of the ACM, 22(11):612–613, Nov 1979. DOI:10.1145/359168.359176.

[Sho00] V. Shoup. *Practical Threshold Signatures*. In B. Preneel (ed.), Advances in Cryptology — EUROCRYPT 2000, pages 207–220, Berlin, Heidelberg, 2000. Springer Berlin Heidelberg. DOI:10.1007/3-540-45539-6˙15.

[tC96] U. S. 104th Congress. *Information Technology Management Reform Act. Public Law 104–106, Section 5131*, 1996. https://www.dol.gov/ocfo/media/regs/ITMRA.pdf.

[WBM+18] O. Weisse, J. v. Bulck, M. Minkin, D. Genkin, B. Kasikci, F. Piessens, M. Silberstein, R. Strackx, T. F. Wenisch, and Y. Yarom. *Foreshadow-NG: Breaking the Virtual Memory Abstraction with Transient Out-of-Order Execution*. Technical Report, 2018.

Extra slides

Next follow some extra slides

Reliability $(\mathcal{R})$ — one metric of security

Probability that a security property (e.g., secrecy) never fails during a mission time

Time normalized: $\tau = 1$ is the expected time to failure (ETTF) of a node

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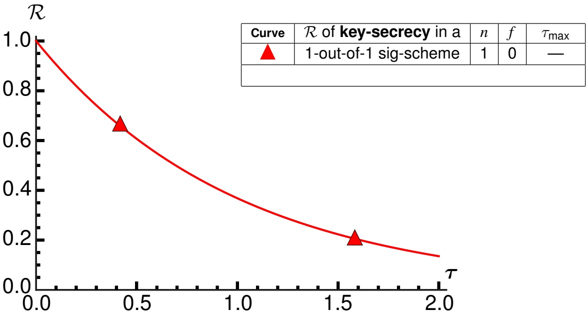
A possible model: each node fails (independently) with constant rate probability



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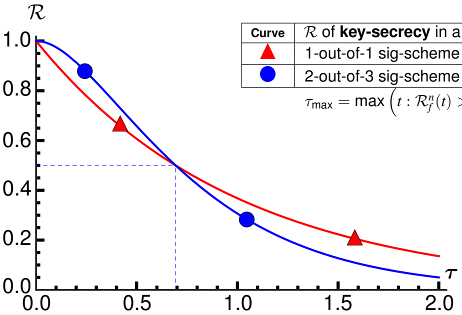

Curve	$\mathcal{R}$ of **key-secrecy** in a	n	f	$\tau_{\max}$
▲	1-out-of-1 sig-scheme	1	0	—
●	2-out-of-3 sig-scheme	3	1	0.693

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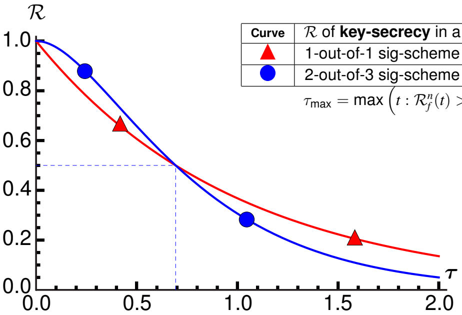

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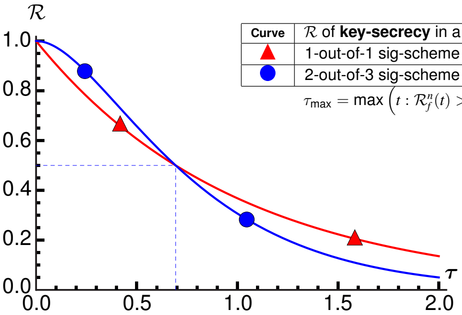
[BB12] Time normalized: $\tau = 1$ is the expected time to failure (ETTF) of a node

Reliability can be degraded when increasing the fault-tolerance threshold f

Reliability $(\mathcal{R})$ — one metric of security

Probability that a security property (e.g., secrecy) never fails during a mission time

A possible model: each node fails (independently) with constant rate probability



Curve	$\mathcal{R}$ of **key-secrecy** in a	n	f	τ_{max}
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Note: rejuvenation of nodes can attenuate the reliability-degradation

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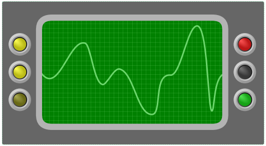

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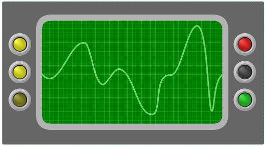

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Challenge questions:

- which models are realistic / match state-of-the-art attacks?
- what concrete parameters (e.g., n) thwart real attacks?

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Threshold Schnorr (multi-)signature [BN06]

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- DL-based homomorphism $\rightarrow$ size equal to 1 signature

(DL = Discrete-Logarithm)

A DL-based example: threshold Schnorr signature

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