A Brief Overview of
Private Set Intersection

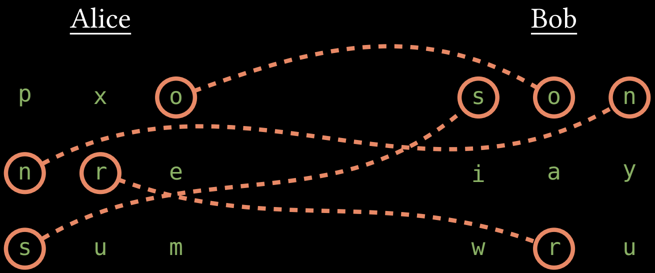
Mike Rosulek, Oregon State University

NIST STPPA, April 19, 2021

what is private set intersection (PSI)?

Alice				Bob		
p	x	o		s	o	n
n	r	e		i	a	y
s	u	m		w	r	u

what is private set intersection (PSI)?



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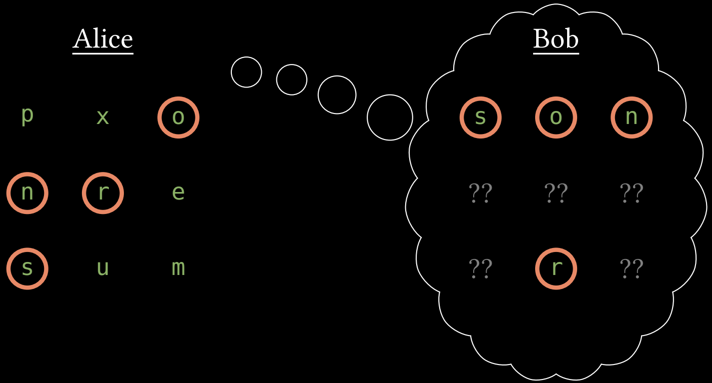
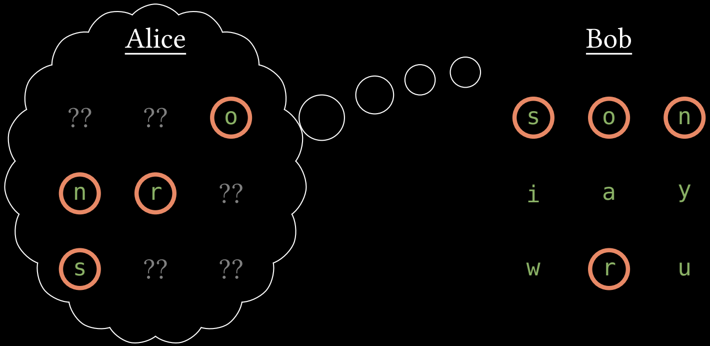
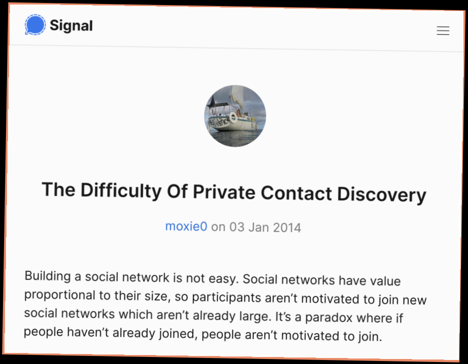


what is private set intersection (PSI)?



why use PSI?



Signal

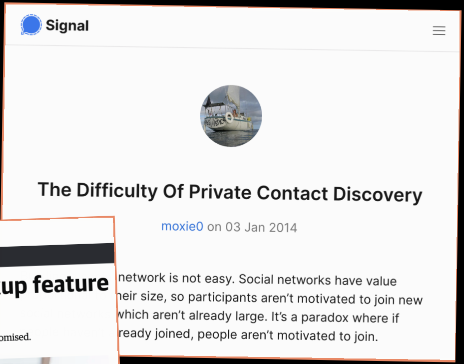
The Difficulty Of Private Contact Discovery

moxie0 on 03 Jan 2014

Building a social network is not easy. Social networks have value proportional to their size, so participants aren't motivated to join new social networks which aren't already large. It's a paradox where if people haven't already joined, people aren't motivated to join.

{my phone contacts} ∩ {users of your service}

why use PSI?



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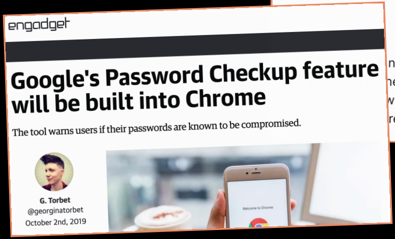


engadget

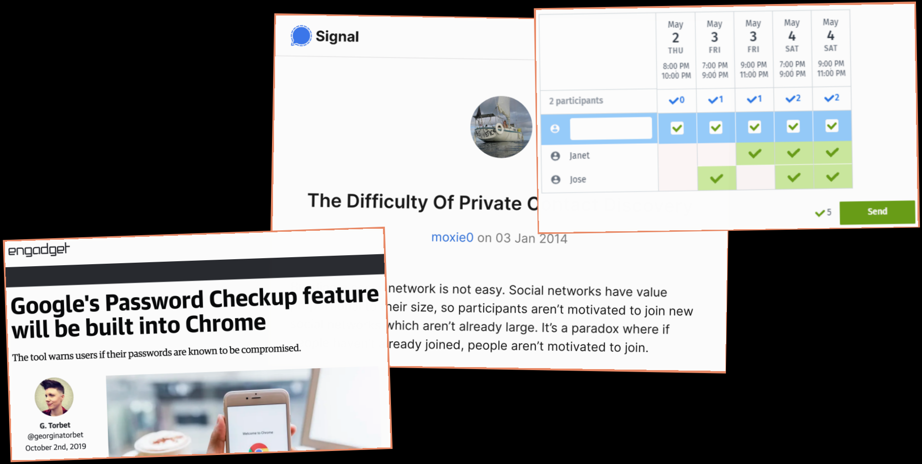
Google's Password Checkup feature will be built into Chrome

The tool warns users if their passwords are known to be compromised.

G. Torbet
@georginatorbet
October 2nd, 2019

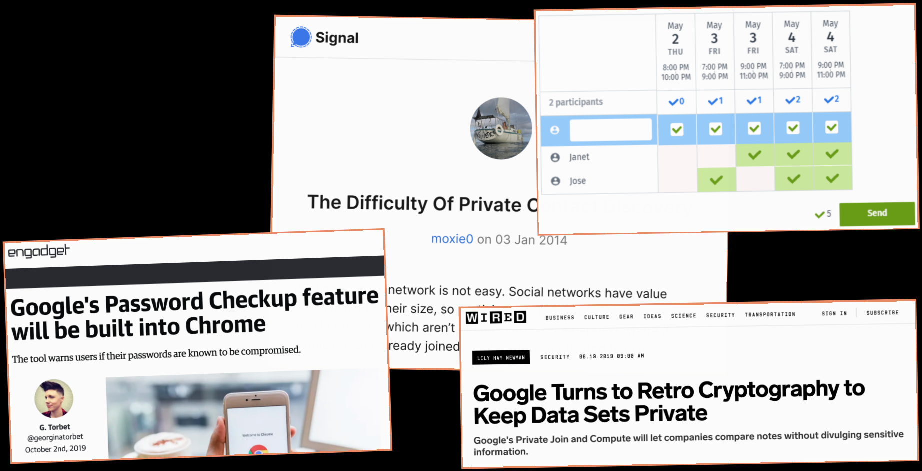
{my passwords} ∩ {passwords found in breaches}

why use PSI?



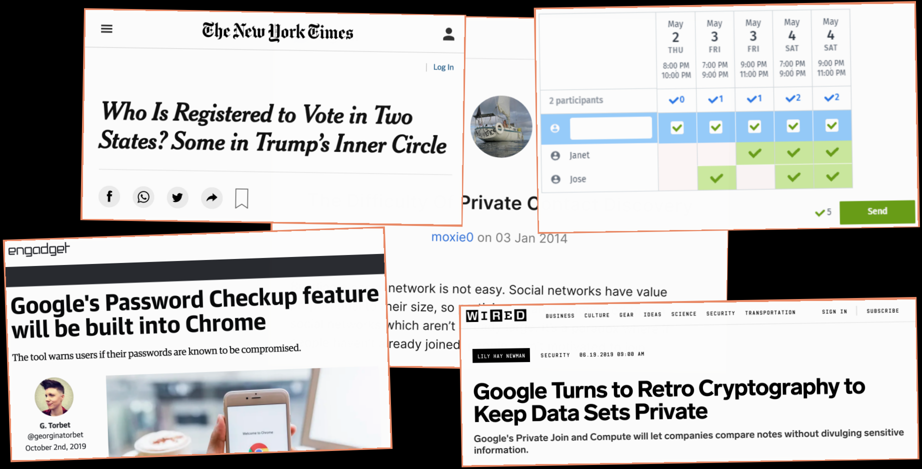
{my availability} ∩ {your availability}

why use PSI?



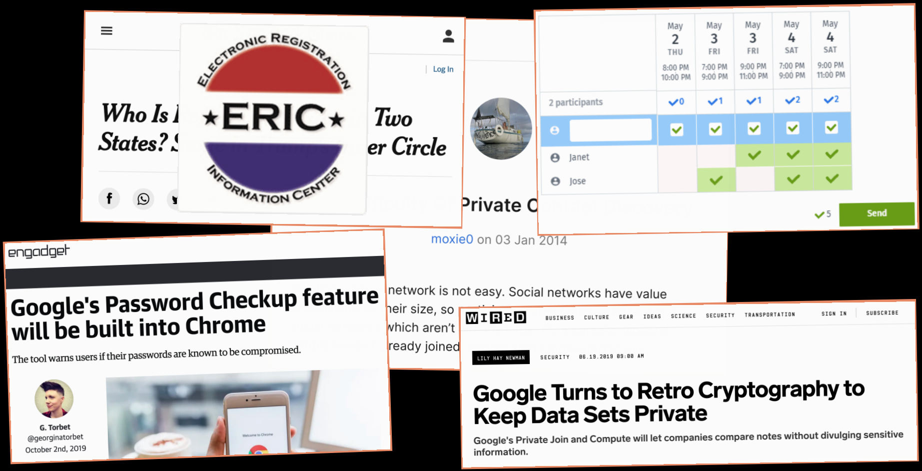
{people who saw ad} ∩ {customers who made purchases}

why use PSI?



$\{\text{voters registered in OR}\} \cap \{\text{voters registered in NY}\}$

why use PSI?



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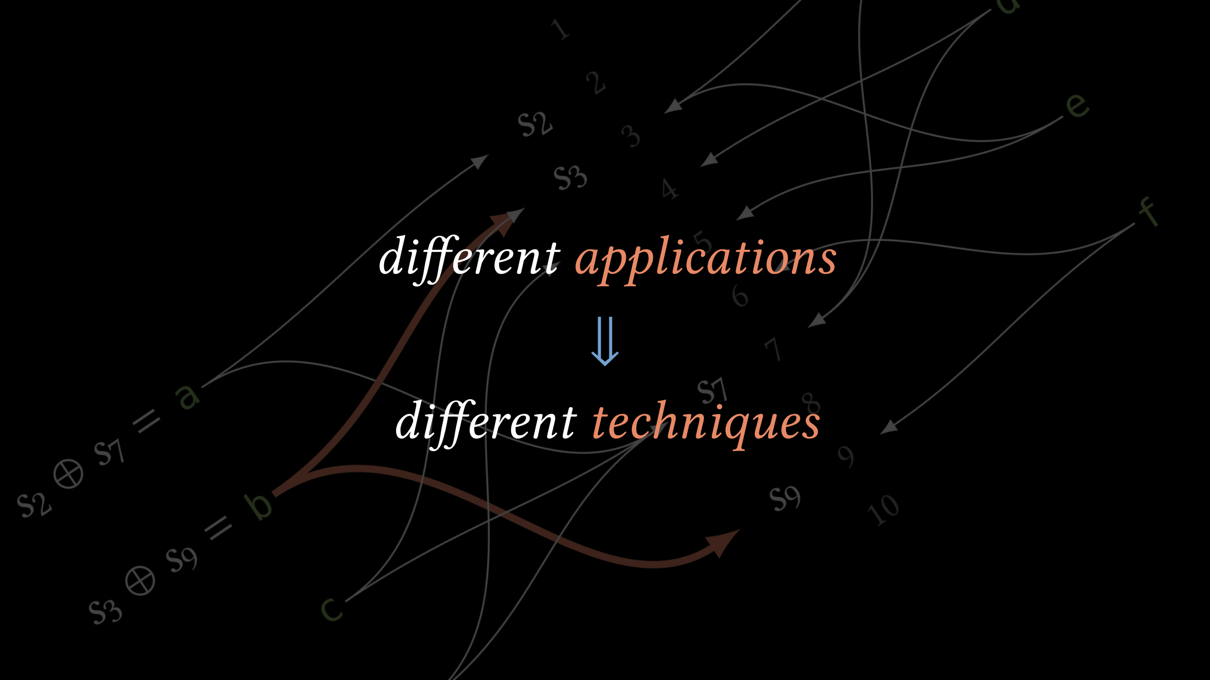

different applications

$\Downarrow$

different techniques

PSI on **small sets** (hundreds)

- private availability poll
- key agreement techniques

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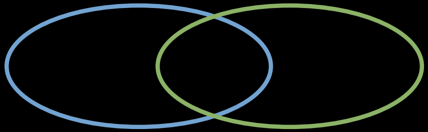

PSI on **large sets** (millions)

- double-registered voters
- OT extension; combinatorial tricks

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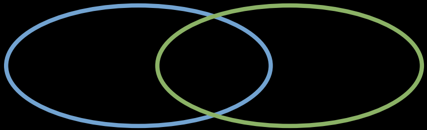
- key agreement techniques



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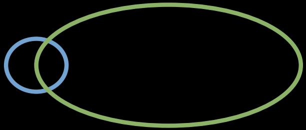
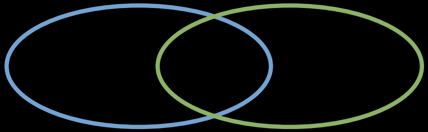


PSI on **asymmetric sets** (100 : billion)

- contact discovery; password checkup
- offline phase; leakage

PSI on **small sets** (hundreds)

- private availability poll
- key agreement techniques



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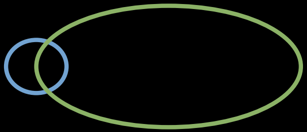

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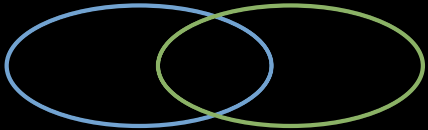
computing on the intersection

- sales statistics about intersection
- generic MPC



PSI on **small sets** (hundreds)

- private availability poll
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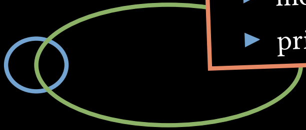
PSI on **large sets** (millions)

- double-registered voters
- n; combinatorial tricks

Not to mention:

- approximate/fuzzy matching
- more than 2 parties/sets
- private set *union*



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computing on the intersection

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PSI on ***small sets***

key agreement techniques

*a **bad** mental model for PSI*

Alice

$x_1, x_2, \ldots$

H = good cryptographic hash function

Bob

$y_1, y_2, \ldots$

$$H(y_1), H(y_2), \ldots \longleftarrow$$

*a **bad** mental model for PSI*

Alice

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$H(y_1), H(y_2), \ldots$

compare

$H(x_1), H(x_2), \ldots$

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$H(x_1), H(x_2), \ldots$

Dictionary attack:

- compute $H(u)$ for every u

*a **bad** mental model for PSI*

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$x_1, x_2, \ldots$

$y_1, y_2, \ldots$


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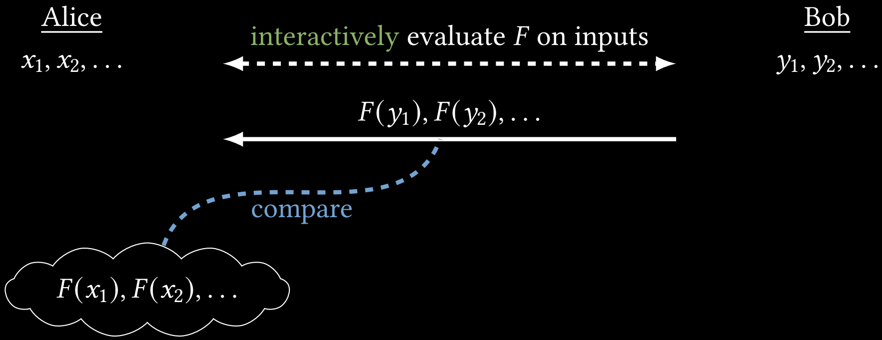
Does Hashing Make Data "Anonymous"?

By: Ed Felten, Chief Technologist | Apr 22, 2012 7:05AM

$H(x_1), H$

every u

*a **better** mental model for PSI*



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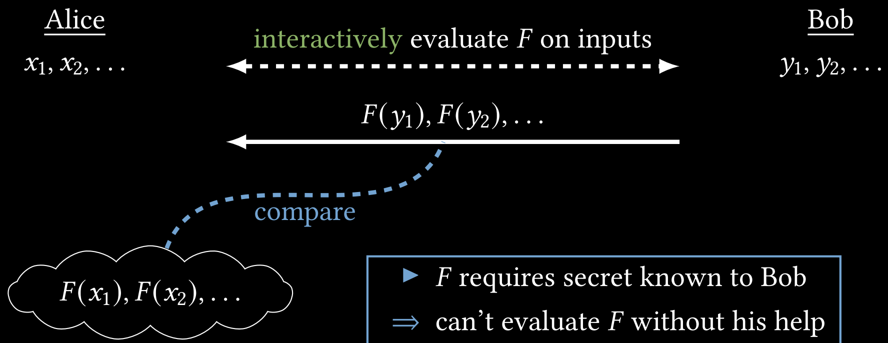


- F requires secret known to Bob

$\Rightarrow$ can’t evaluate F without his help

- Alice “committed” to few x_i’s

Alice

x

Bob

y

Does $x = y$?

[Shamir80, Meadows86, Jablon96]

H = random oracle

Alice		Bob
x		y
	$\xrightarrow{\quad H(x)^a \quad}$	
	$\xleftarrow{\quad H(y)^b \quad}$	

[Shamir80, Meadows86, Jablon96]

Alice x

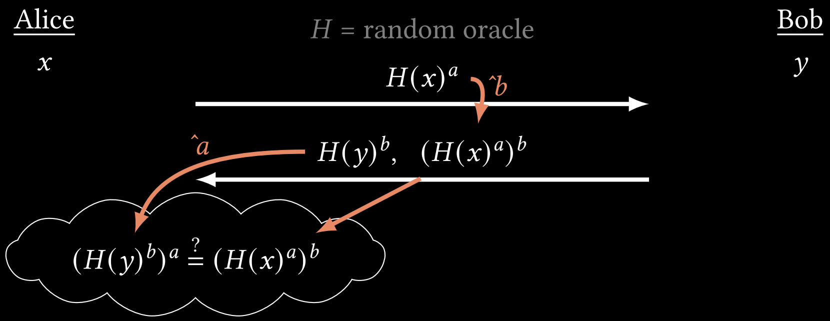
H = random oracle

Bob y

$$\xrightarrow{\quad H(x)^a \quad \hat{}b \quad}$$

$$\xleftarrow{\quad H(y)^b, \quad (H(x)^a)^b \quad}$$

[Shamir80, Meadows86, Jablon96]



[Shamir80, Meadows86, Jablon96]

Alice $\qquad\qquad$ H = random oracle $\qquad\qquad$ **Bob**

x $\qquad\qquad\qquad\qquad$ y

$$H(x)^a \xrightarrow{\hat{}b}$$

$$\hat{}a \qquad H(y)^b, \quad (H(x)^a)^b$$

$$(H(y)^b)^a \stackrel{?}{=} (H(x)^a)^b$$

interactively evaluate

$$F(x) = H(x)^{ab}$$

[Shamir80, Meadows86, Jablon96]

Alice x

H = random oracle

Bob y

$$H(x)^a \xrightarrow{\hat{}\,b}$$

$$\xleftarrow{\hat{}\,a} \quad H(y)^b, \quad (H(x)^a)^b$$

$$(H(y)^b)^a \stackrel{?}{=} (H(x)^a)^b$$

$$\boxed{\text{interactively evaluate} \quad F(x) = H(x)^{ab}}$$

$$x \neq y \overset{\text{RO}}{\Longrightarrow} H(y) \text{ independent of everything else} \overset{\text{DDH}}{\Longrightarrow} H(y)^b \approx \$$$

[Shamir80, Meadows86, Jablon96]

Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

What is $X \cap Y$*?*

[HubermanFranklinHogg99]

$$\begin{array}{ccc} \underline{\text{Alice}} & & \underline{\text{Bob}} \\ x_1, x_2, \ldots & \xrightarrow{H(x_1)^a,\, H(x_2)^a,\, \ldots} & y_1, y_2, \ldots \\ & \xleftarrow{H(y_1)^b,\, H(y_2)^b,\, \ldots,\, (H(x_1)^a)^b,\, (H(x_2)^a)^b,\, \ldots} & \end{array}$$

[HubermanFranklinHogg99]

Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

$$\xrightarrow{H(x_1)^a,\, H(x_2)^a,\, \ldots}$$

$$\xleftarrow{H(y_1)^b,\, H(y_2)^b,\, \ldots,\, (H(x_1)^a)^b,\, (H(x_2)^a)^b,\, \ldots}$$

[HubermanFranklinHogg99]

- Malicious security via ZK [DeCristofaroKimTsudik10,JareckiLiu09]
- Authenticated items [DeCristofaroKimTsudik10]
- From generic key agreement [RosulekTrieu21]

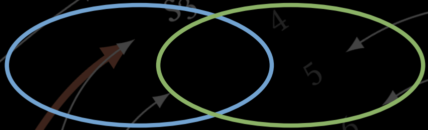
overview: PSI on small sets

for 256 items:

0.1 seconds; 10 KB

with malicious security!

[RosulekTrieu21]



PSI on ***large sets***

OT & hashing techniques; scaling to 1M items

scaling to 1 million items?

$$\xrightarrow{H(x_1)^a, H(x_2)^a, \ldots, H(x_{1000000})^a}$$

scaling to 1 million items?

$$\underbrace{H(x_1)^a, H(x_2)^a, \ldots, H(x_{1000000})^a}_{> \text{4 minutes!}}$$

batch oblivious PRF (OPRF)

Alice | Bob

1
2
3
4
5
6
7
8
9
⋮

batch oblivious PRF (OPRF)

Alice | Bob

x_1 1
x_2 2
x_3 3
x_4 4
x_5 5
x_6 6
x_7 7
x_8 8
x_9 9
$\vdots$

batch oblivious PRF (OPRF)

Alice		Bob
$F_1(x_1)$	1	$F_1(\cdot)$
$F_2(x_2)$	2	$F_2(\cdot)$
$F_3(x_3)$	3	$F_3(\cdot)$
$F_4(x_4)$	4	$F_4(\cdot)$
$F_5(x_5)$	5	$F_5(\cdot)$
$F_6(x_6)$	6	$F_6(\cdot)$
$F_7(x_7)$	7	$F_7(\cdot)$
$F_8(x_8)$	8	$F_8(\cdot)$
$F_9(x_9)$	9	$F_9(\cdot)$
	$\vdots$	

batch oblivious PRF (OPRF)

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	$F_1(x_1)$	1	$F_1(\cdot)$	
	$F_2(x_2)$	2	$F_2(\cdot)$	
	$F_3(x_3)$	3	$F_3(\cdot)$	
	$F_4(x_4)$	4	$F_4(\cdot)$	
	$F_5(x_5)$	5	$F_5(\cdot)$	learns nothing about x_i's
	$F_6(x_6)$	6	$F_6(\cdot)$	
	$F_7(x_7)$	7	$F_7(\cdot)$	
	$F_8(x_8)$	8	$F_8(\cdot)$	
	$F_9(x_9)$	9	$F_9(\cdot)$	
		$\vdots$		

batch oblivious PRF (OPRF)

Alice				Bob
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	$F_4(x_4)$	4	$F_4(\cdot)$	
all other $F_i(x^*)$ look random	$F_5(x_5)$	5	$F_5(\cdot)$	learns nothing about x_i's
	$F_6(x_6)$	6	$F_6(\cdot)$	
	$F_7(x_7)$	7	$F_7(\cdot)$	
	$F_8(x_8)$	8	$F_8(\cdot)$	
	$F_9(x_9)$	9	$F_9(\cdot)$	
		$\vdots$		

batch oblivious PRF (OPRF)

Alice			Bob

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	$F_9(x_9)$	9	$F_9(\cdot)$	
		$\vdots$		

achieved very efficiently from OT extension

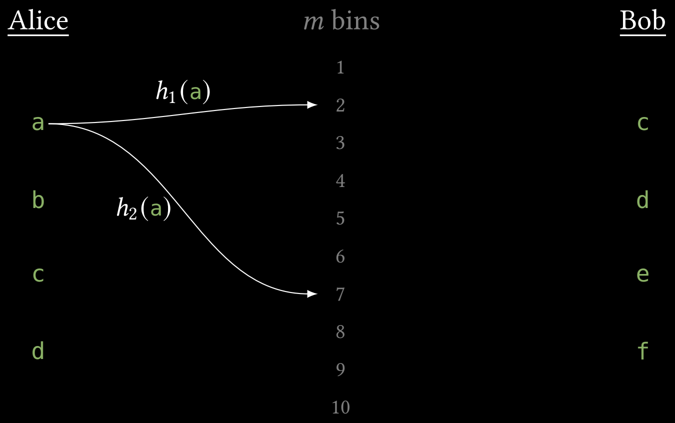
Alice	Bob
a	c
b	d
c	e
d	f

[PinkasSchneiderZohner14, KolesnikovKumaresanRosulekTrieu16]

Alice	m bins	Bob
	1	
	2	
a	3	c
b	4	d
	5	
c	6	e
	7	
d	8	f
	9	
	10	

1. Agree on random $h_1, h_2 : \{0,1\}^* \to [m]$

[PinkasSchneiderZohner14, KolesnikovKumaresanRosulekTrieu16]



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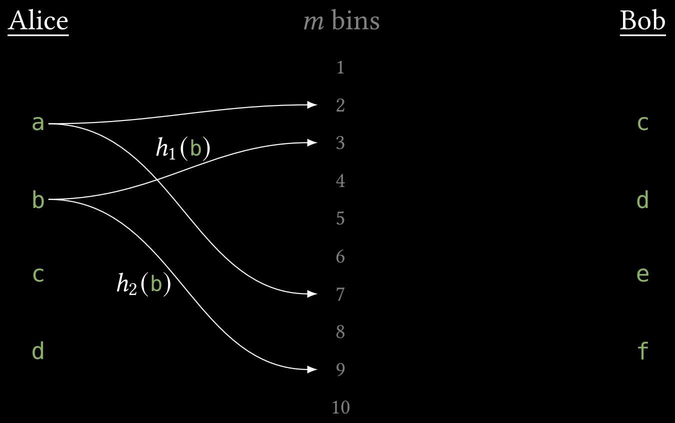

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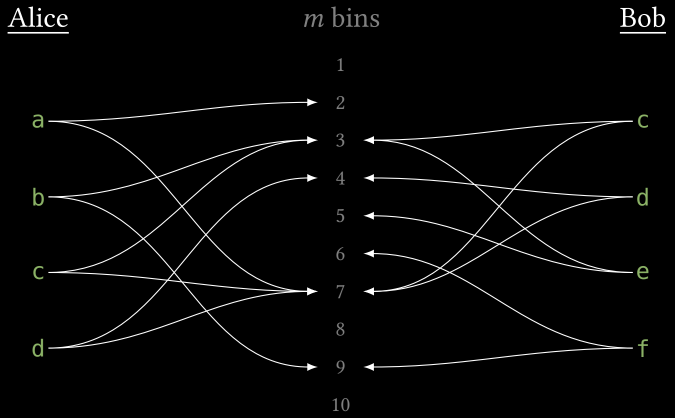

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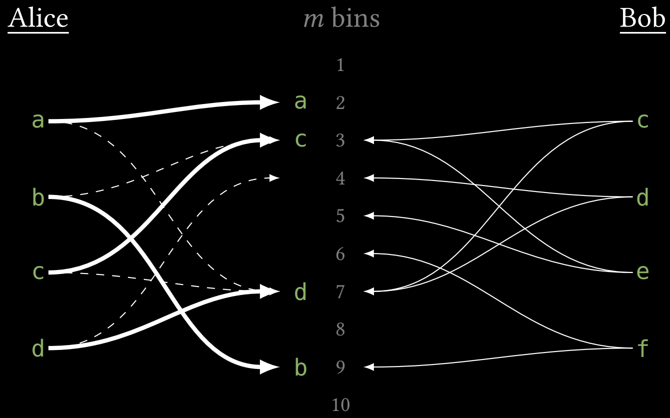

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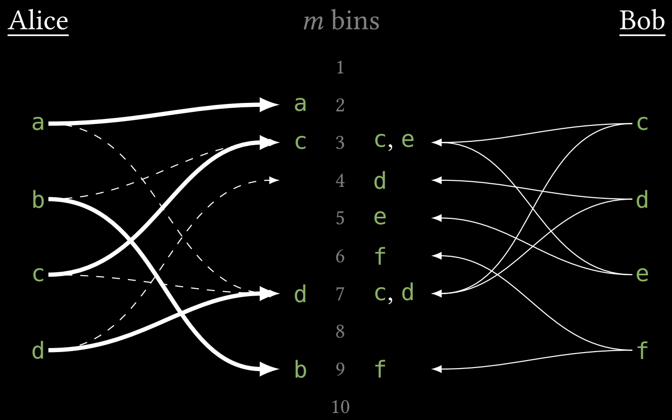

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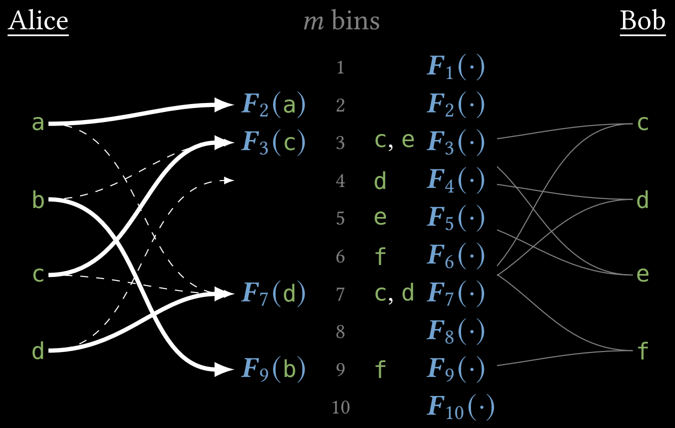

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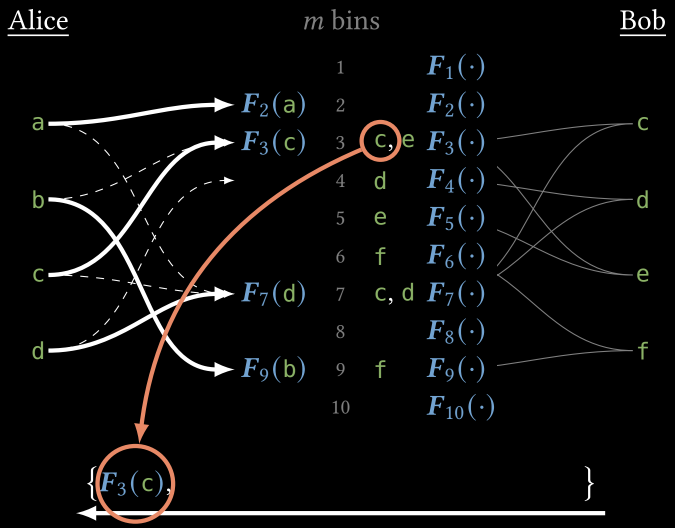
[PinkasSchneiderZohner14, KolesnikovKumaresanRosulekTrieu16]



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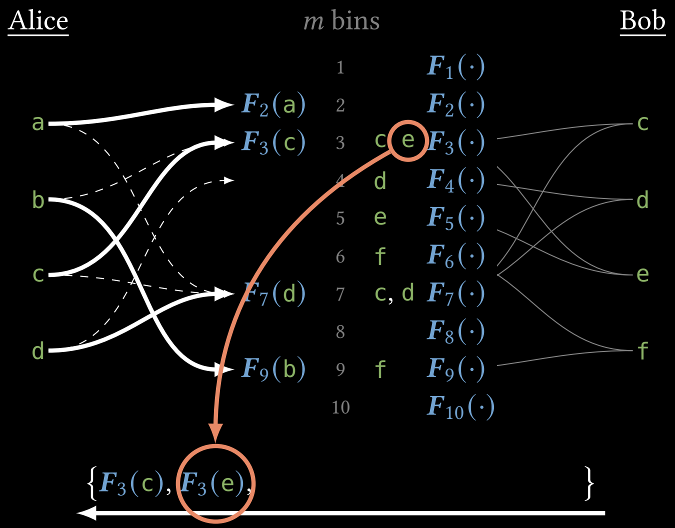

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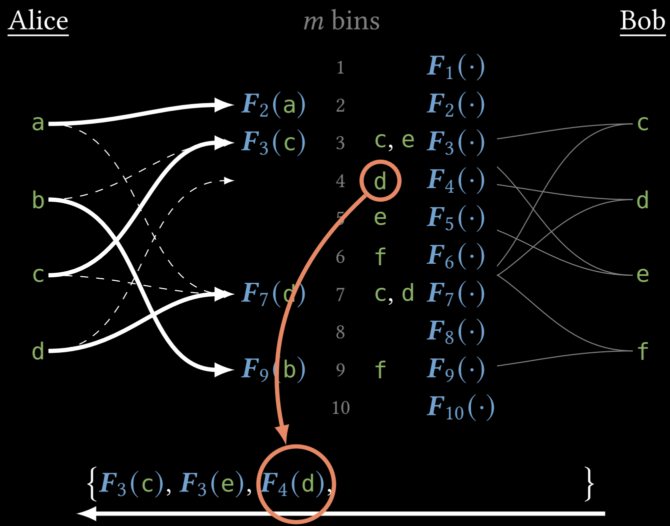

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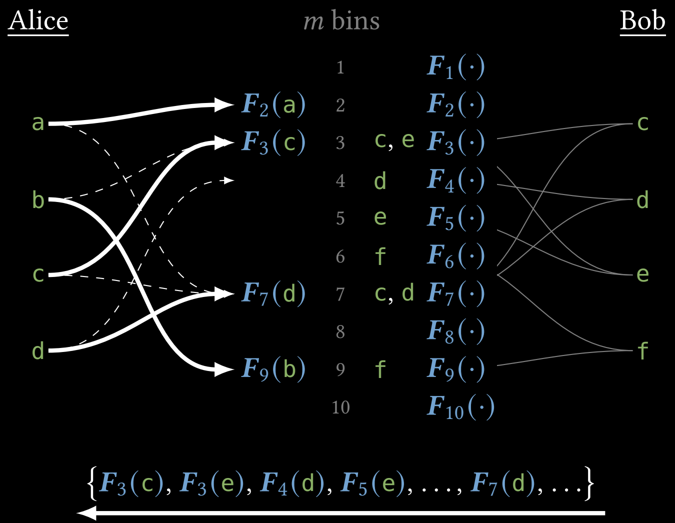

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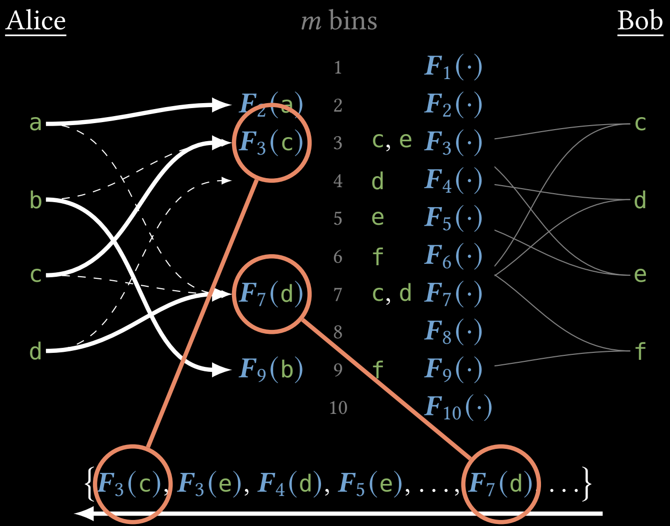
[PinkasSchneiderZohner14, KolesnikovKumaresanRosulekTrieu16]



1. Agree on random $h_1, h_2 : \{0,1\}^* \to [m]$
2. Alice places each x into bin $h_1(x)$ or $h_2(x)$
3. Bob places each x into bins $h_1(x)$ and $h_2(x)$
4. OPRF in each bin: Alice learns one $F_i(x)$; Bob learns entire $F_i(\cdot)$
5. Bob sends all $F_i(x)$ values

[PinkasSchneiderZohner14, KolesnikovKumaresanRosulekTrieu16]

why isn’t it secure against malicious parties?

why isn't it secure against malicious parties?

Alice

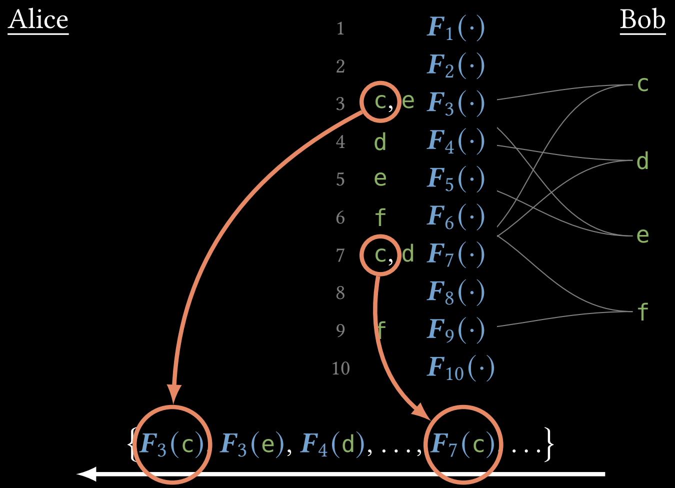
Bob

1		$F_1(\cdot)$	
2		$F_2(\cdot)$	c
3	c, e	$F_3(\cdot)$	
4	d	$F_4(\cdot)$	d
5	e	$F_5(\cdot)$	
6	f	$F_6(\cdot)$	e
7	c, d	$F_7(\cdot)$	
8		$F_8(\cdot)$	f
9	f	$F_9(\cdot)$	
10		$F_{10}(\cdot)$	

$\{F_3(\mathsf{c}), F_3(\mathsf{e}), F_4(\mathsf{d}), \ldots, F_7(\mathsf{c}), \ldots\}$

$\longleftarrow$

why isn't it secure against malicious parties?

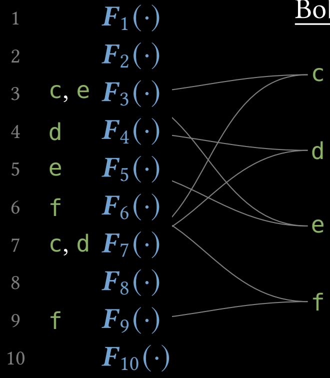


Bob should send two F-values per item

why isn't it secure against malicious parties?

Alice

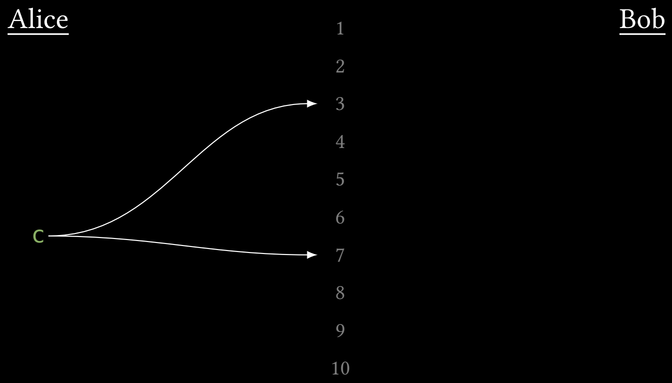
Bob



Bob should send two F-values per item , what if he sends only one?

$\{F_3(\mathsf{c}), F_3(\mathsf{e}), F_4(\mathsf{d}), \ldots, \cancel{F_7(\mathsf{c})}, \ldots\}$

why isn't it secure against malicious parties?



$$\{F_3(\mathtt{c}), F_3(\mathtt{e}), F_4(\mathtt{d}), \ldots, \cancel{F_7(\mathtt{c})}, \ldots\}$$

Bob should send two F-values per item , what if he sends only one?

Alice has c; does she include it in output?

why isn't it secure against malicious parties?

<u>Alice</u>

<u>Bob</u>

1

2

$F_3(c)$ 3

4

5

6

c

7

8

9

10

$\{F_3(c), F_3(e), F_4(d), \ldots, \cancel{F_7(c)}, \ldots\}$

Bob should send two F-values per item , what if he sends only one?

Alice has c; does she include it in output?

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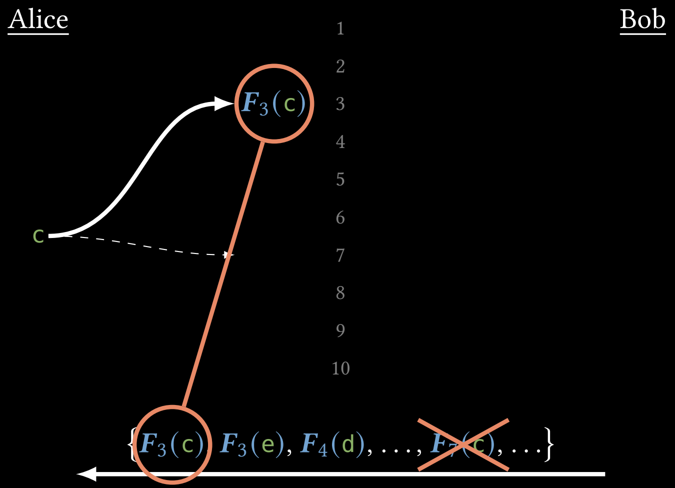


Bob should send two F-values per item , what if he sends only one?

Alice has c; does she include it in output?

why isn't it secure against malicious parties?

Alice

Bob

1
2
3
4
5
6
7
8
9
10

$\mathtt{c}$

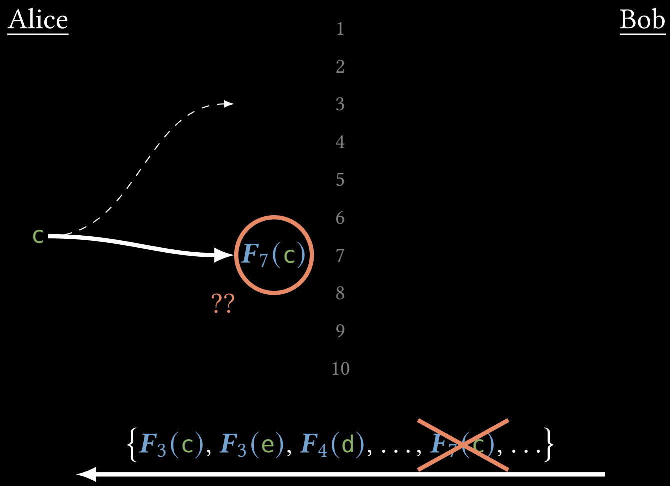
$\boldsymbol{F}_7(\mathtt{c})$

$\{\boldsymbol{F}_3(\mathtt{c}), \boldsymbol{F}_3(\mathtt{e}), \boldsymbol{F}_4(\mathtt{d}), \ldots, \cancel{\boldsymbol{F}_7(\mathtt{c})}, \ldots\}$

Bob should send two $\boldsymbol{F}$-values per item , what if he sends only one?

Alice has $\mathtt{c}$; does she include it in output?

why isn't it secure against malicious parties?

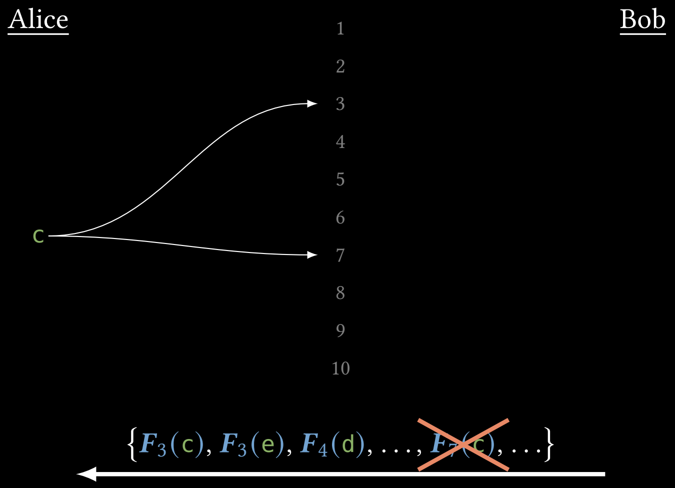


Bob should send two F-values per item , what if he sends only one?

Alice has c; does she include it in output?

Only if c placed in bin 3!

why isn't it secure against malicious parties?



Bob should send two F-values per item , what if he sends only one?

Alice has c; does she include it in output?

Only if c placed in bin 3!

- Depends on Alice's **entire input**!

$\Rightarrow$ can't simulate!

how do we overcome this problem?

[PinkasRosulekTrieuYanai20]

batch OPRF for malicious PSI

Alice		Bob
$F_1(x_1)$	1	$F_1(\cdot)$
$F_2(x_2)$	2	$F_2(\cdot)$
$F_3(x_3)$	3	$F_3(\cdot)$
$F_4(x_4)$	4	$F_4(\cdot)$
$F_5(x_5)$	5	$F_5(\cdot)$
$F_6(x_6)$	6	$F_6(\cdot)$
$F_7(x_7)$	7	$F_7(\cdot)$
$F_8(x_8)$	8	$F_8(\cdot)$
$F_9(x_9)$	9	$F_9(\cdot)$
	$\vdots$	

batch OPRF for malicious PSI

Alice		Bob
$F_1(x_1)$	1	$F_1(\cdot)$
$F_2(x_2)$	2	$F_2(\cdot)$
$F_3(x_3)$	3	$F_3(\cdot)$
$F_4(x_4)$	4	$F_4(\cdot)$
$F_5(x_5)$	5	$F_5(\cdot)$
$F_6(x_6)$	6	$F_6(\cdot)$
$F_7(x_7)$	7	$F_7(\cdot)$
$F_8(x_8)$	8	$F_8(\cdot)$
$F_9(x_9)$	9	$F_9(\cdot)$
	$\vdots$	

State of the art malicious batch OPRF [OrrùOrsiniScholl17]

- essentially same cost as semi-honest

batch OPRF for malicious PSI

Alice		Bob
$F_1(x_1)$	1	$F_1(\cdot)$
$F_2(x_2)$	2	$F_2(\cdot)$
$F_3(x_3)$	3	$F_3(\cdot)$
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$F_7(x_7)$	7	$F_7(\cdot)$
$F_8(x_8)$	8	$F_8(\cdot)$
$F_9(x_9)$	9	$F_9(\cdot)$
	$\vdots$	

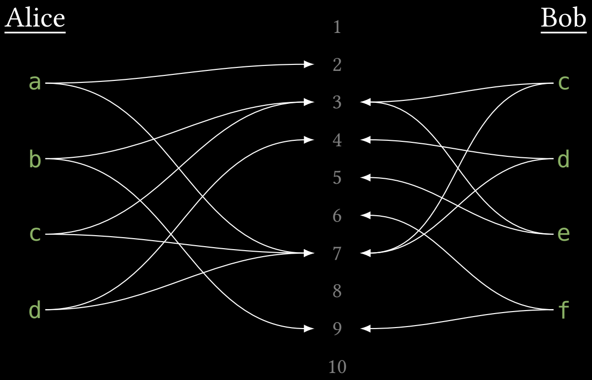
State of the art malicious batch OPRF [OrrùOrsiniScholl17]

- essentially same cost as semi-honest
- consistency check relies on an additive homomorphism:

$$F_i(x) \oplus F_j(y) = F_{ij}(x \oplus y)$$

*: *a gross oversimplification*

[PinkasRosulekTrieuYanai20] *protocol main idea:*



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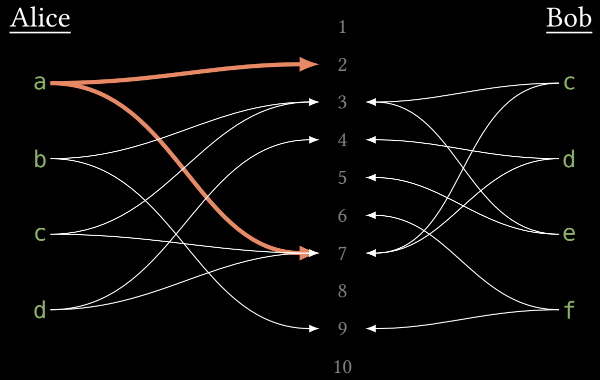

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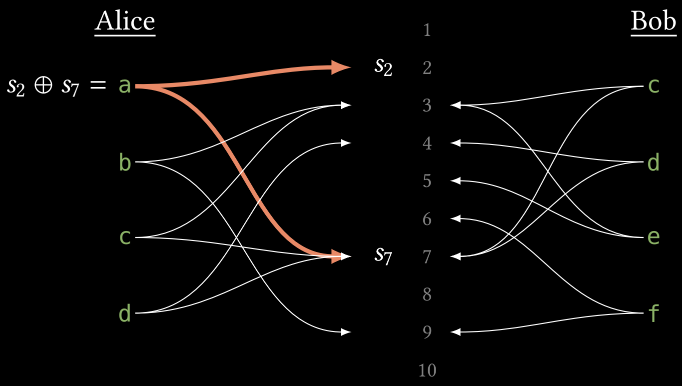
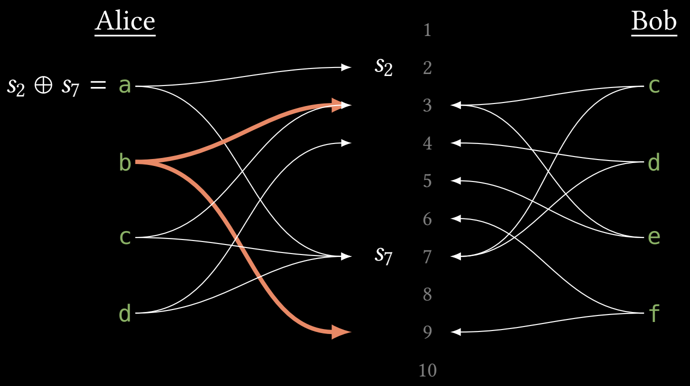


Alice secret-shares x into bins $h_1(x)$ and $h_2(x)$

[PinkasRosulekTrieuYanai20] *protocol main idea:*



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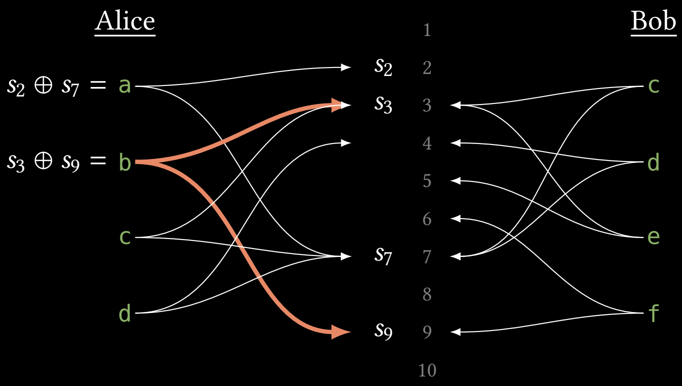

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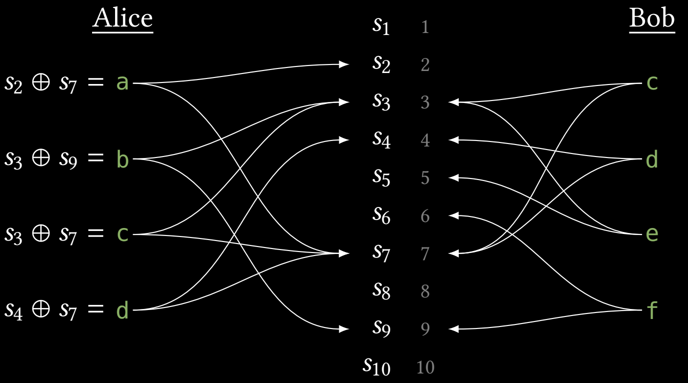

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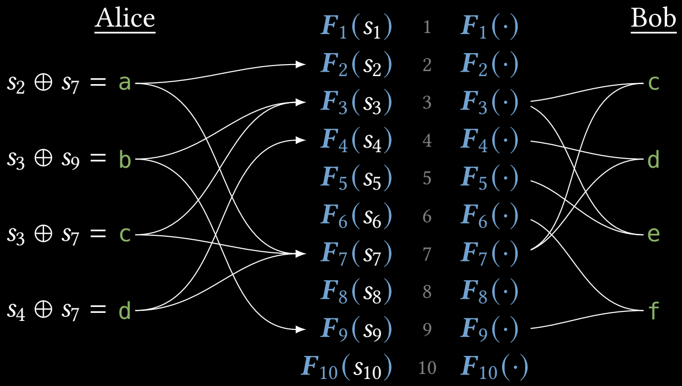

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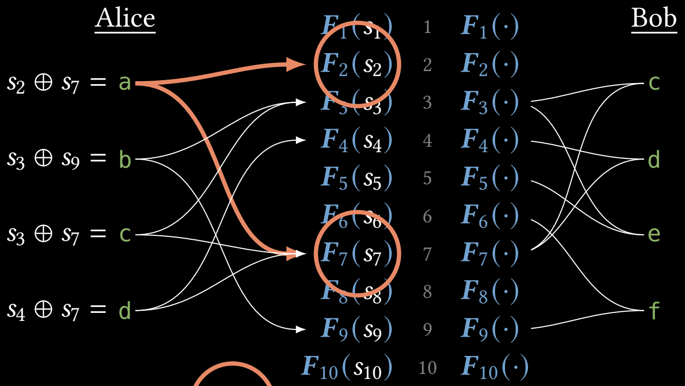
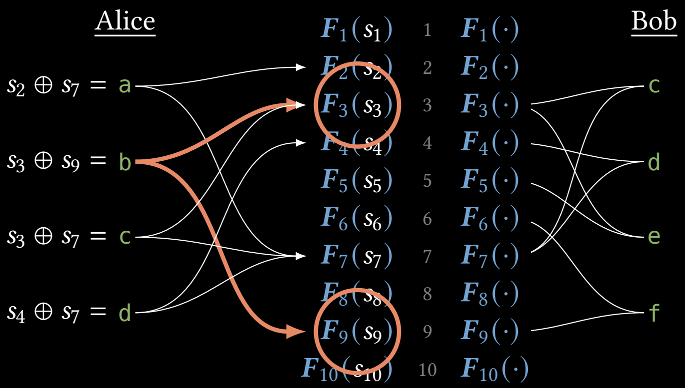


Alice secret-shares x into bins $h_1(x)$ and $h_2(x)$

$$F_2(s_2) \oplus F_7(s_7) = F_{27}(\mathsf{a})$$

[PinkasRosulekTrieuYanai20] *protocol main idea:*



Alice secret-shares x into bins $h_1(x)$ and $h_2(x)$

$$F_2(s_2) \oplus F_7(s_7) = F_{27}(\mathsf{a})$$
$$F_3(s_3) \oplus F_9(s_9) = F_{39}(\mathsf{b})$$

[PinkasRosulekTrieuYanai20] *protocol main idea:*

Alice

$s_2 \oplus s_7 = \mathsf{a}$

$s_3 \oplus s_9 = \mathsf{b}$

$s_3 \oplus s_7 = \mathsf{c}$

$s_4 \oplus s_7 = \mathsf{d}$

$F_1(s_1)$	1	$F_1(\cdot)$
$F_2(s_2)$	2	$F_2(\cdot)$
$F_3(s_3)$	3	$F_3(\cdot)$
$F_4(s_4)$	4	$F_4(\cdot)$
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$F_6(s_6)$	6	$F_6(\cdot)$
$F_7(s_7)$	7	$F_7(\cdot)$
$F_8(s_8)$	8	$F_8(\cdot)$
$F_9(s_9)$	9	$F_9(\cdot)$
$F_{10}(s_{10})$	10	$F_{10}(\cdot)$

Bob

c

d

e

f

Alice secret-shares x into bins $h_1(x)$ and $h_2(x)$

$F_2(s_2) \oplus F_7(s_7) = F_{27}(\mathsf{a})$

$F_3(s_3) \oplus F_9(s_9) = F_{39}(\mathsf{b})$

$F_3(s_3) \oplus F_7(s_7) = F_{37}(\mathsf{c})$

$F_4(s_4) \oplus F_7(s_7) = F_{47}(\mathsf{d})$

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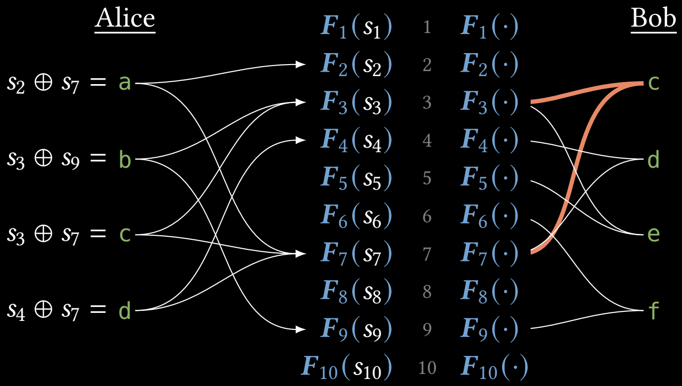

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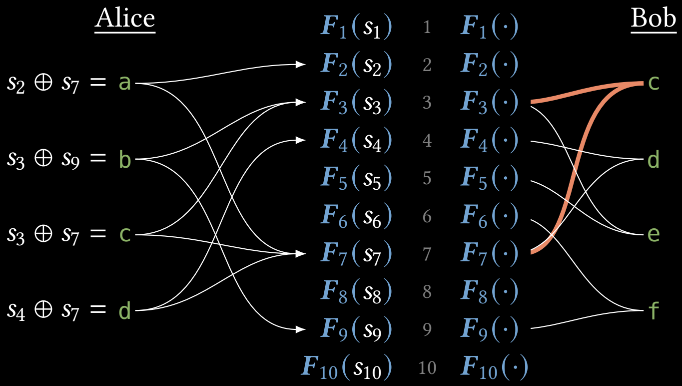

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$F_4(s_4) \oplus F_7(s_7) = F_{47}(\mathsf{d})$

$\{F_{37}(\mathsf{c}), \qquad\qquad \}$ $\longleftarrow$

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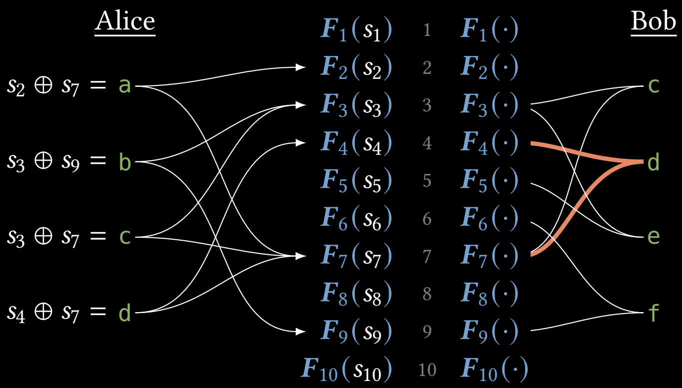

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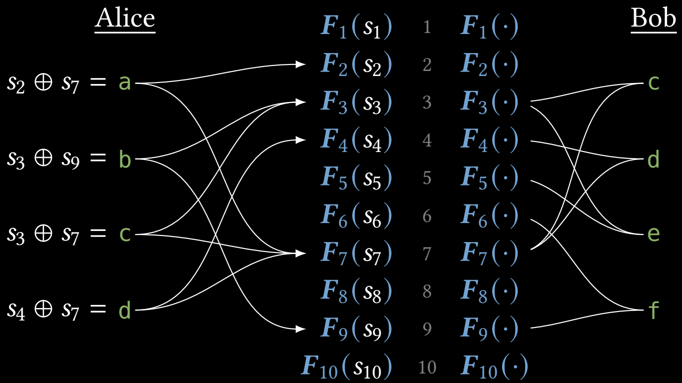
$$\{F_{37}(\mathsf{c}), F_{47}(\mathsf{d}), \quad \}$$
$$\longleftarrow$$

[PinkasRosulekTrieuYanai20] *protocol main idea:*



Alice secret-shares x into bins $h_1(x)$ and $h_2(x)$

Bob sends only one F-value per item

$$F_2(s_2) \oplus F_7(s_7) = F_{27}(\mathsf{a})$$
$$F_3(s_3) \oplus F_9(s_9) = F_{39}(\mathsf{b})$$
$$F_3(s_3) \oplus F_7(s_7) = F_{37}(\mathsf{c})$$
$$F_4(s_4) \oplus F_7(s_7) = F_{47}(\mathsf{d})$$

$$\{F_{37}(\mathsf{c}), F_{47}(\mathsf{d}), F_{35}(\mathsf{e}), F_{69}(\mathsf{f})\} \longleftarrow$$

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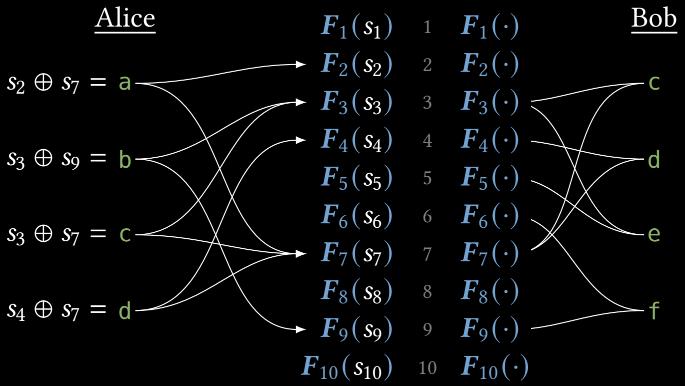

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$$\{F_{37}(\mathsf{c}), F_{47}(\mathsf{d}), F_{35}(\mathsf{e}), F_{69}(\mathsf{f})\} \longleftarrow$$

overview: PSI on large sets

for 1 million items:

4.5 – 5 seconds; 128 – 145 MB

[KolesnikovKumaresanRosulekTrieu16]

[GarimellaPinkasRosulekTrieuYanai21]

overview: PSI on large sets

for 1 million items:

4.5 – 5 seconds; 128 – 145 MB

semi-honest security

[KolesnikovKumaresanRosulekTrieu16]

[GarimellaPinkasRosulekTrieuYanai21]

overview: PSI on large sets

for 1 million items:

4.5 – 5 seconds; 128 – 145 MB

malicious security

[KolesnikovKumaresanRosulekTrieu16]

[GarimellaPinkasRosulekTrieuYanai21]

*PSI on **asymmetric sets***

offline preprocessing techniques and leakage; scaling to billions of items

how to scale to billions of items?

THE VERGE TECH REVIEWS SCIENCE CREATORS ENTERTAINMENT

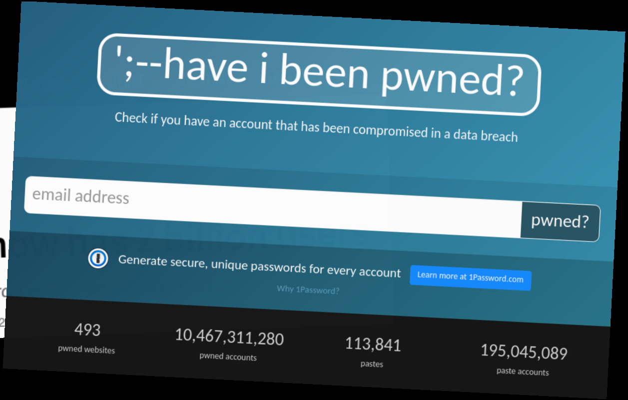
APPS TECH FACEBOOK

WhatsApp now has 2 billion users

And it has no plans to drop end-to-end encryption

By Jon Porter | @JonPorty | Feb 12, 2020, 10:50am EST

how to scale to billions of items?



idea #1: offline preprocessing

Alice		Bob
$x_1, x_2, \ldots$	$H(x_1)^a, H(x_2)^a, \ldots$ $\longrightarrow$	$y_1, y_2, \ldots$
(hundreds)		*(billions!)*
	$H(y_1)^b, H(y_2)^b, \ldots,\ (H(x_1)^a)^b, (H(x_2)^a)^b, \ldots$ $\longleftarrow$	

see [KalesRechbergerSchneiderSenkerWeinert19]

idea #1: offline preprocessing

Alice

$x_1, x_2, \ldots$

(hundreds)

Bob

$y_1, y_2, \ldots$

(billions!)

$$\xrightarrow{H(x_1)^a, H(x_2)^a, \ldots}$$

$$\xleftarrow{H(y_1)^b, H(y_2)^b, \ldots, \; (H(x_1)^a)^b, (H(x_2)^a)^b, \ldots}$$

see [KalesRechbergerSchneiderSenkerWeinert19]

idea #1: offline preprocessing

Alice

$x_1, x_2, \ldots$

(hundreds)

Bob

$y_1, y_2, \ldots$

(billions!)

$$H(x_1)^a, H(x_2)^a, \ldots \longrightarrow$$

$$H(y_1)^b, H(y_2)^b, \ldots, \quad (H(x_1)^a)^b, (H(x_2)^a)^b, \ldots \longleftarrow$$

only dependence on Bob's set

see [KalesRechbergerSchneiderSenkerWeinert19]

idea #1: offline preprocessing

$H(y_1)^b, H(y_2)^b, \ldots$ ←

offline phase

Alice

$x_1, x_2, \ldots$

(hundreds)

Bob

$y_1, y_2, \ldots$

(billions!)

$H(x_1)^a, H(x_2)^a, \ldots$ →

$(H(x_1)^a)^b, (H(x_2)^a)^b, \ldots$ ←

see [KalesRechbergerSchneiderSenkerWeinert19]

idea #1: offline preprocessing

$H(y_1)^b, H(y_2)^b, \ldots$ ←

offline phase, shared among all clients

Alice

$x_1, x_2, \ldots$

(hundreds)

Bob

$y_1, y_2, \ldots$

(billions!)

$H(x_1)^a, H(x_2)^a, \ldots$ →

$(H(x_1)^a)^b, (H(x_2)^a)^b, \ldots$ ←

- Safe to reuse b for many PSIs $\Longrightarrow$ reuse offline phase for all clients!

see [KalesRechbergerSchneiderSenkerWeinert19]

idea #1: offline preprocessing

$H(y_1)^b, H(y_2)^b, \ldots$

offline phase, shared among all clients

Alice

$x_1, x_2, \ldots$

(hundreds)

Bob

$y_1, y_2, \ldots$

(billions!)

$H(x_1)^a, H(x_2)^a, \ldots$

$(H(x_1)^a)^b, (H(x_2)^a)^b, \ldots$

- Safe to reuse b for many PSIs $\Longrightarrow$ reuse offline phase for all clients!
- Clever encodings for offline message: 4GB / 1B items

see [KalesRechbergerSchneiderSenkerWeinert19]

idea #2: allow some leakage

100 items

Alice:

Bob:

1 billion items

see [LiPalAliSullivanChatterjeeRistenpart19]

idea #2: allow some leakage

Alice:

100 items

1 million bins

Bob:

1 billion items

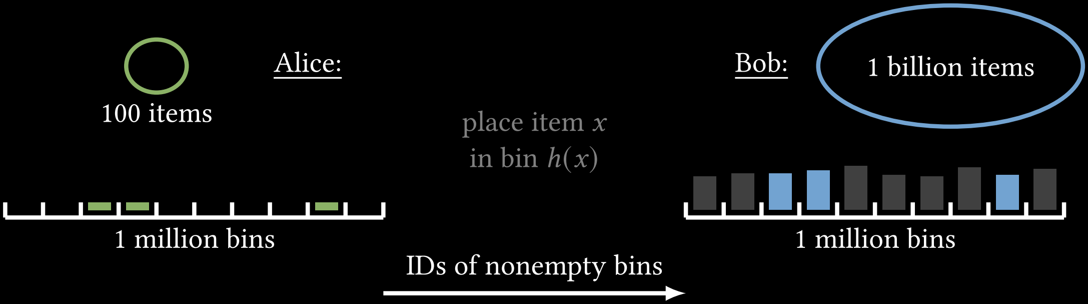
1 million bins

see [LiPalAliSullivanChatterjeeRistenpart19]

idea #2: allow some leakage

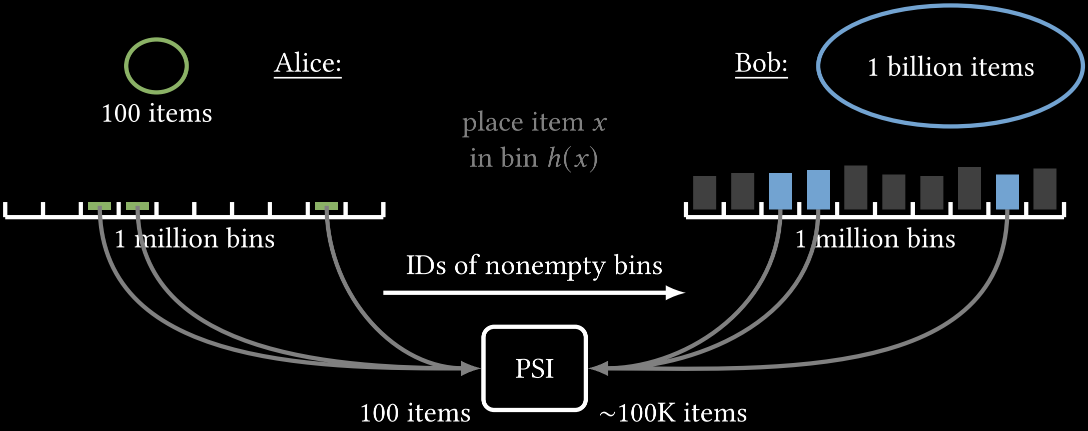
see [LiPalAliSullivanChatterjeeRistenpart19]

idea #2: allow some leakage



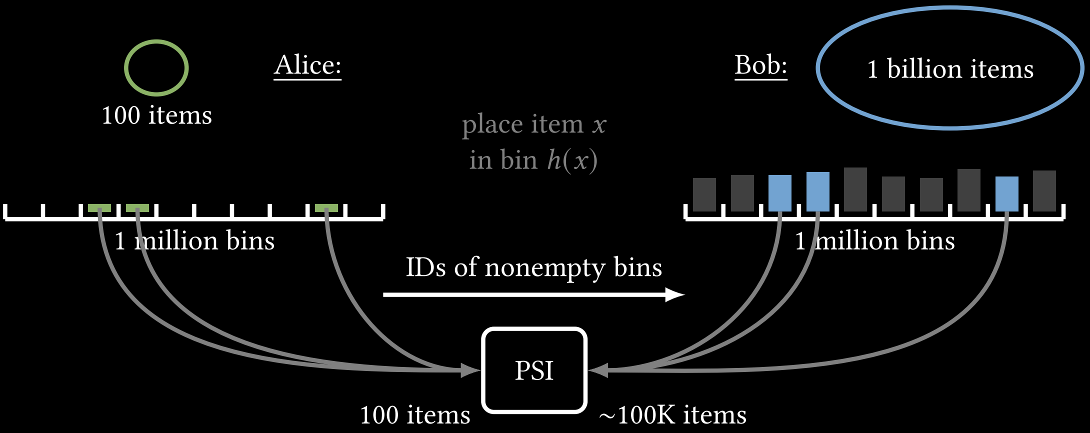
see [LiPalAliSullivanChatterjeeRistenpart19]

idea #2: allow some leakage



see [LiPalAliSullivanChatterjeeRistenpart19]

idea #2: allow some leakage



choice of h? see [LiPalAliSullivanChatterjeeRistenpart19]

overview: PSI on asymmetric sets

for 256 million vs 1000 items (no leakage):

offline setup: 33 seconds; 1 GB

discovery: 3 seconds; 6 MB

for 1 billion vs 100 items (under previous **leakage** scenario):

0.2 seconds; 1 MB

computing on the intersection

next frontier

Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

$$H(x_1)^a, H(x_2)^a, \ldots \longrightarrow$$

$$H(y_1)^b, H(y_2)^b, \ldots, (H(x_1)^a)^b, (H(x_2)^a)^b, \ldots \longleftarrow$$

what is $X \cap Y$?

[HubermanFranklinHogg99]

Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

$$H(x_1)^a, H(x_2)^a, \ldots$$

$$H(y_1)^b, H(y_2)^b, \ldots, \boxed{(H(x_1)^a)^b, (H(x_2)^a)^b, \ldots}$$

shuffle

what is $|X \cap Y|$*?*

[HubermanFranklinHogg99]

Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

$$\text{Alice} \xrightarrow{H(x_1)^a,\, H(x_2)^a,\, \ldots} \text{Bob}$$

$$\text{Alice} \xleftarrow{H(y_1)^b,\, H(y_2)^b,\, \ldots,\, \boxed{(H(x_1)^a)^b,\, (H(x_2)^a)^b,\, \ldots}} \text{Bob}$$

shuffle

what is $|X \cap Y|$?

what about computing other functions of the intersection? what about large sets?

[HubermanFranklinHogg99]

state of the art

Alice	Bob
$x_1, x_2, \ldots$	$y_1, y_2, \ldots$

- Using $O(n)$ communication, reduce PSI to $O(n)$ comparisons (vs n^2)

[PinkasSchneiderTkachenkoYanai19]

state of the art

Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

interactive preprocessing

$r_1, r_2, \ldots$

$s_1, s_2, \ldots$

$$x_i \in Y \iff r_i = s_i$$

- Using $O(n)$ communication, reduce PSI to $O(n)$ comparisons (vs n^2)

[PinkasSchneiderTkachenkoYanai19]

state of the art

Alice	Bob
$x_1, x_2, \ldots$	$y_1, y_2, \ldots$

$\boxed{\text{interactive preprocessing}}$

Alice	Bob
$r_1, r_2, \ldots$	$s_1, s_2, \ldots$

$\boxed{\text{compare } r_i \stackrel{?}{=} s_i, (\forall i)}$

generic MPC

- Using $O(n)$ communication, reduce PSI to $O(n)$ comparisons (vs n^2)
- Perform the comparisons inside generic MPC $\rightsquigarrow$ compute on the result

[PinkasSchneiderTkachenkoYanai19]

overview: computing on the intersection

for 1 million items:

2 minutes ; 2.5 GB

[PinkasSchneiderTkachenkoYanai19]

overview: computing on the intersection

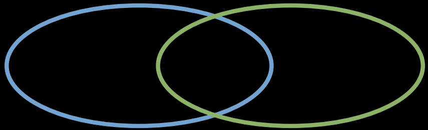
for 1 million items:

$$\underbrace{\text{2 minutes}}_{30\times\text{ plain PSI}} \; ; \; \underbrace{\text{2.5 GB}}_{20\times\text{ plain PSI}}$$

[PinkasSchneiderTkachenkoYanai19]

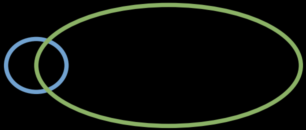
PSI on **small sets** (hundreds)

- efficient! 0.1sec / 256 items
- based on Diffie-Hellman KA



PSI on **large sets** (millions)

- fast! 4sec / 1M items
- OT extension & hashing techniques



PSI on **asymmetric sets**

- huge challenges for practice
- allow leakage, preprocessing?

computing on the intersection

- many open problems
- 20-30× performance gap

PSI on **small sets** (hundreds)

- efficient! 0.1sec / 256 items
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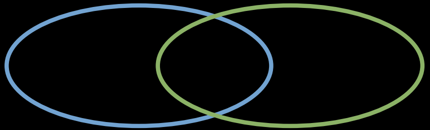
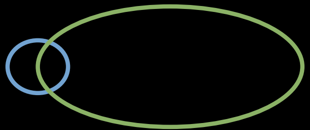


PSI on **large sets** (millions)

- fast! 4sec / 1M items
- OT extension & hashing techniques

thank you!



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computing on the intersection

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