Breaking Category Five SPHINCS$^+$ with SHA-256

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Summary of Result

- SPHINCS$^+$ is a stateless hash-based signature selected for standardization by NIST
- We present a forgery attack that reduces classical security by 40 bits
  - For submitted parameter sets:
    - That target Category 5
    - While using SHA-256
- Our attack builds on a previous attack by Antonov on the DM-SPR property of SHA-256 (a security assumption for SPHINCS$^+$)
- The SPHINCS$^+$ team has proposed a tweak which defeats our attack by using SHA-512 instead of SHA-256 (where necessary)

Outline

- DM-SPR Property and Antonov's Attack
- Using Antonov's Attack to Forge $WOTS^+$ (This Paper)
- Optimizations (This Paper)
- The $SPHINCS^+$ Tweak
- Conclusion

Prefixes and *Distinct Function Multitarget Preimage Resistance* (DM-SPR)

- Many places in hypertree where a preimage can create a forgery:
  - Hashes in Merkle Trees
  - Hash Chains in $WOTS^+$
  - Hash trees in FORS
  - FORS public key hash
  - $WOTS^+$ public key hash (Our attack here)
- New targets are revealed with every honest signature
- To avoid a 1 out of t multi-target preimage attack:
  - Make sure hash input at each hypertree location has a distinct prefix
  - Formalized as a tweakable hash function with DM-SPR property

Antonov's Attack on SHA-256 DM-SPR [Antonov 2022]

- Collect t target hashes with different prefixes
- Find preimage with the same prefix for 1 of them
  - Use Herding to reach same state from all prefixes at the penultimate block
  - Use Multi-Target preimage search on compression function to find a block to append and reach a target
- Longest hash input in SPHINCS$^+$ is WOTS$^+$ public key hash
- That's still pretty short (34 blocks)
  - To balance cost of herding, multi-target preimage search, use some compression-function 3-collisions
    - Let t be $2^{10}3^{23} \approx 2^{46}$ instead of 2^{33}
    - 3-Collision search cost: $1.5 \cdot 3^{23} \cdot 2^{170.7} \approx 2^{208}$
    - Multi-Target preimage cost: $2^{256}/2^{46} \approx 2^{210}$

What's Left to Do?

- Antonov's attack lets us create a validly-signed WOTS$^+$ public key preimage
- But we need to know the corresponding private key to forge a SPHINCS$^+$ signature
  - This involves knowing preimages of parts of WOTS$^+$ public key
  - For validity, prefix must match hypertree location
  - But hypertree location depends which target we reached
  - No way to force correct prefix for all targets
- Or at least part of it…
  - As long as we can sign more than one possible digest with our WOTS$^+$ key
  - Can graft a forged Merkle-Tree root to the hypertree for less than 2^{256} work!

Our Attack: Outline

- Find a preimage of some $WOTS^+$ public key with enough private key info to sign **some** digests
- Brute-force search for a valid Merkle/FORS tree whose root has signable digest
- Sign the tree root with the attacked $WOTS^+$ key
- To forge a signature, try message randomization strings until the hypertree address is a descendent address of the tree root

WOTS+ Signature

- Write digest as base-w (16) number
- Append a base-w checksum
  - (960 – <sum of digits>)
- Sign each digit d_i of digest plus checksum by:
  - Hash $sk_{i,0}$ (with prefix) d_i times
  - Put the result in the signature
- Note: The signature of 0xF is just pk_i

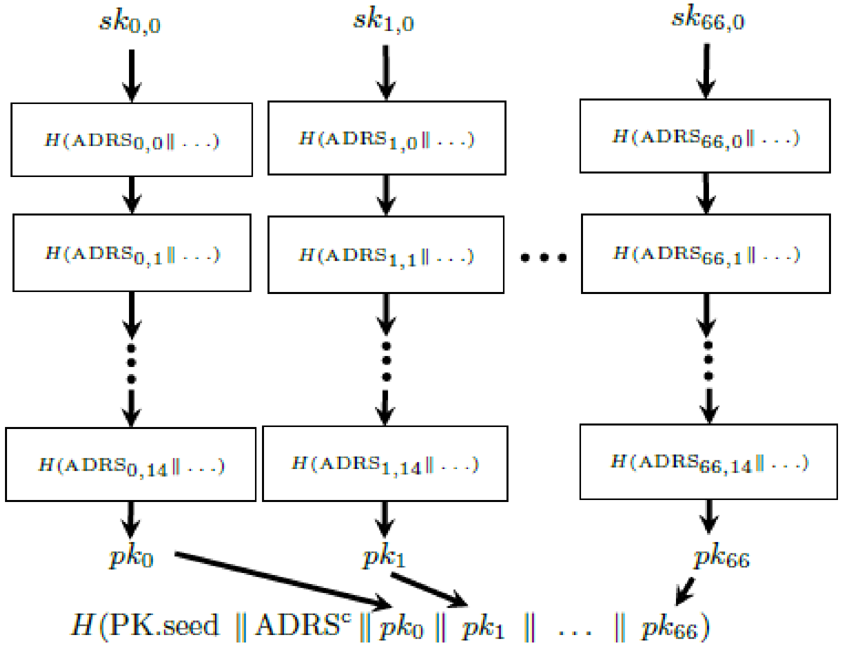


Fig. 2. A WOTS+ public key.

Finding a Merkle/FORS Root We Can Sign

- Aim to sign a digest like:

  xxxxxxxx xxxxxxxx xxxxxxxF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF

- Modify Antonov's multi-target preimage search to find a WOTS⁺ public key that can sign this
  - Treat the part that signs xxxx… as prefix – so we know $sk_{i,0}$ for this part
  - Use the last block of the prefix and the part that signs FFFF… for herding and multitarget preimage search
  - Target the SHA-256 state immediately before the first block that signs checksum
  - The part that signs the checksum will come from the target honest signature
- Can forge a signature on any Merkle/FORS root of the above form as long as checksum works out

Making Sure the Checksum Works Out

- For a digest like:

  xxxxxxxx xxxxxxxx xxxxxxxF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF

  - Checksum is $960 - 41 \cdot 15 - \sum x$
- We can increment, but not decrement, digits of honest checksum
  - Increment a digit by hashing (with prefix) sk_{i,d_i}
- Can choose targets with unusually small checksums
- Need $\sum x$ to be small enough with high enough probability

Optimization: Batched Multicollision Search

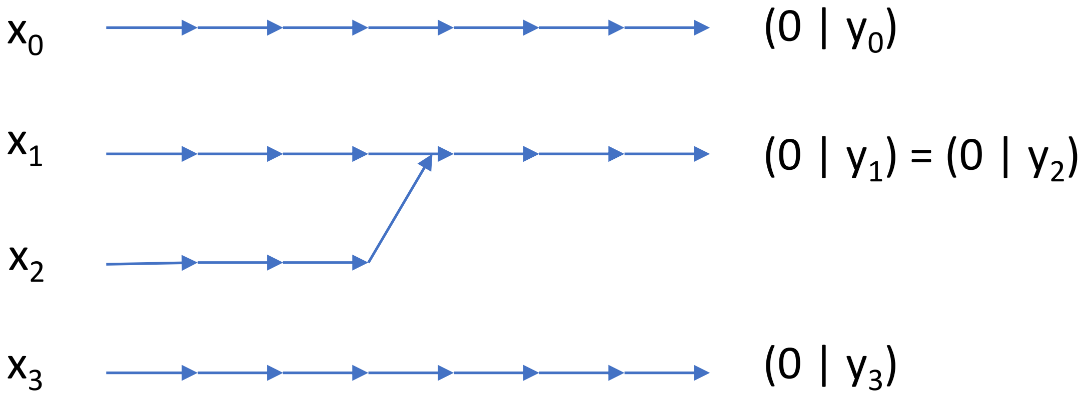
- Best parameterization of our attack involves finding lots of 4-way collisions with distinct prefixes
- It is cheaper to search for lots of collisions at once
  - Finding a single 4-way collision costs $\sim 2^{192}$
  - Finding t 4-way collisions costs $\sim 2^{192} t^{1/4}$
  - (Ignoring prefixes and memory access costs)
- To get good memory access costs, use parallel collision search techniques
- To avoid wasting time colliding already-used prefixes
  - Compute collisions in smaller batches of size αt
- More detail in paper

Memory Considerations for our Attack

- Sometimes memory intensive attacks or attacks that don't parallelize well are more expensive in practice than simple time-complexity suggests
- To show these considerations do not undermine our attack:
  - We analyze highly parallel implementations of all steps of our attack
  - in a computation model, where reading or writing a bit to a memory of size M costs $2^{-5}\sqrt{M}$ bit operations (probably somewhat pessimistic)
- This only changed the cost of our attack by about 2 bits of security

Memory/Parallelization-Friendly Collision Search [V-OW 1994]

- Each of M threads picks, and remembers, a random starting point x_i.
- Each thread iteratively hashes x_i until it reaches a "distinguished point" with $n/2 - \log_2(M)$ leading zeroes.
- The threads then collectively sort their outputs to find colliding distinguished points, y_i, y_j.
- Two threads iteratively recompute hashes of x_i, x_j to find the hash collision.
- Time = $\sim 2^{n/2}$/ M; Space = ~M.



Adapting V-OW to k-collisions [JL 2009]

- Similar to V-OW, but distinguished points need $\frac{k-1}{k}n - \log_2(M)$ leading zeroes
- M needs to be at least as large as the expected number of 2-collisions: $2^{\frac{k-2}{k}n}$

Important Memory Costs

- When using V-OW like techniques to find collisions with different prefixes
  - need to define a function on a 256-bit input that will pick each of t prefixes $\frac{1}{t}$ of the time
  - In order to deal with different prefixes either
    a. Recompute state including hash-chains every time
    b. Store precomputed values (about $512t$ bits)
  - Option b turns out to be cheaper, even with memory costs for reasonable values t
- Sorting distinguished points costs $O(M^{1.5})$
  - Dominant cost when looking for 4-way or bigger collisions

Attack Complexity

Table 1. Summary of Our Results on SPHINCS$^+$ Category Five Parameters

Parameter Set	Cost				Reference
	Herd	Link	Signable	Total	
SPHINCS$^+$-256f	$2^{214.8}$	$2^{216.4}$	$2^{215.7}$	$\approx 2^{217.4}$	Section 4.3
SPHINCS$^+$-256s	$2^{214.8}$	$2^{216.4}$	$2^{215.7}$	$\approx 2^{217.4}$	Section 4.3

SPHINCS$^+$ Tweak [Hülsing 2022]

- In response to Antonov's attack on DM-SPR the SPHINCS $^+$ team issued a tweak to the SPHINCS$^+$ specification
  - Replaced SHA-256 with SHA-512, for hashing multi-block inputs in Category 3 and 5 parameters
  - Still some use of SHA-256, but doesn't seem exploitable

Conclusion

- Our attack shows that some submitted parameter sets of SPHINCS$^+$ are not as strong as claimed
- The problem is not the security proof for the SPHINCS$^+$ construction, but how its tweakable hash functions are instantiated
- Lesson: need to be very careful trying to get more than 128 bits of security from SHA-256

- On the upside:
  - SPHINCS$^+$'s proposed tweak seems to address these issues
  - SHA-256 on fixed-length inputs pretty reliably gets 128 bits of security, so it's unlikely this sort of oversight leads to a practical break

Thank You!

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