



Building Threshold Cryptosystems over a SMR/Blockchain channel

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Threshold Cryptosystem with Honest Majorities

Properties: Robustness, Guaranteed Output
Delivery, Fairness, ...

Typical Communication Model



**Synchronous
Point-to-point
Secure Links**



**Broadcast
Channel**

**Can the Internet be considered synchronous enough to
build Broadcast channels?**

Based on Networking Research

Unless latency per round in minutes is acceptable, the Internet may **not** be considered to be synchronous

Communication Model for the Internet

System Setting

- n parties and an f -limited adversary
- point-to-point links

Asynchrony

- For any message sent, the adversary can delay its delivery by any finite amount of time.
- there is no bound on the time to deliver a message but,
- each message must eventually be delivered.

Partial Synchrony

- **Assumption:** There exists known finite time bound Δ and a special event GST (Global Stabilization Time).
- The adversary must cause the GST event to eventually happen after some unknown finite time.
- Any message sent at time x must be delivered by time $\Delta + \max(x, \text{GST})$.

Byzantine Broadcast

Problem Setting

- n parties and an f -limited adversary
- A distinguished broadcaster p

With bounded synchrony

- **Agreement.**
If two honest parties commit values v and v' respectively, then $v = v'$.
- **Validity.**
If the broadcaster is honest, then all honest parties commit the broadcaster's value.
- **Termination.**
All honest parties commit and terminate.

With partial synchrony

- **Agreement.**
same as above.
- **Validity.**
If the broadcaster is honest and $GST = 0$, then all honest replicas commit the broadcaster's value.
- **Termination.**
All honest replicas commit and terminate after GST .

Threshold Cryptosystem beyond Synchrony

Lower Bound

2/3

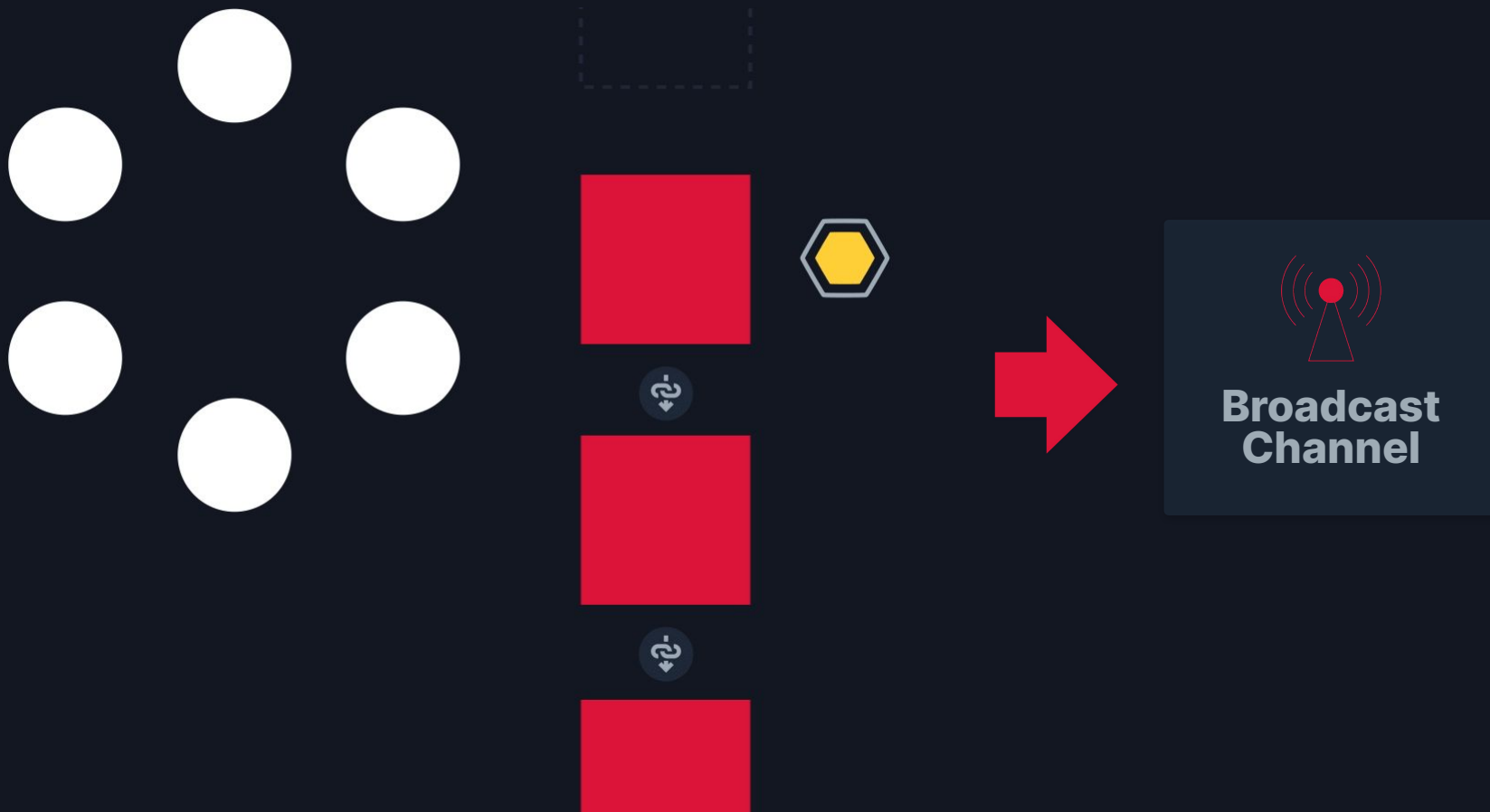
Super Honest Majority required



**Increased complexity
of development**

- dealing with the asynchronous network with common coins
- dealing with timeouts, view-change, responsiveness in partial synchrony

Existing Threshold Crypto based Blockchain



State Machine Replication (SMR)

output: a transactions log = $[tx_0, tx_1, \dots, tx_i]$

Safety:

If $[tx_0, tx_1, \dots, tx_j]$ and $[tx'_0, tx'_1, \dots, tx'_i]$ are output by two honest nodes, then $tx_i = tx'_i$ for all $i \leq \min(j, i')$.

Liveness:

If a transaction tx is input to at least an honest node, then every honest replica eventually outputs a log containing tx .

Informally,

- (i) Senders' messages appear on the blockchain eventually.
- (ii) Different receivers observe messages at different points in time.
- (iii) However, all the nodes eventually observe messages in the exact same total order.



State Machine Replication / Blockchain



Byzantine Broadcast

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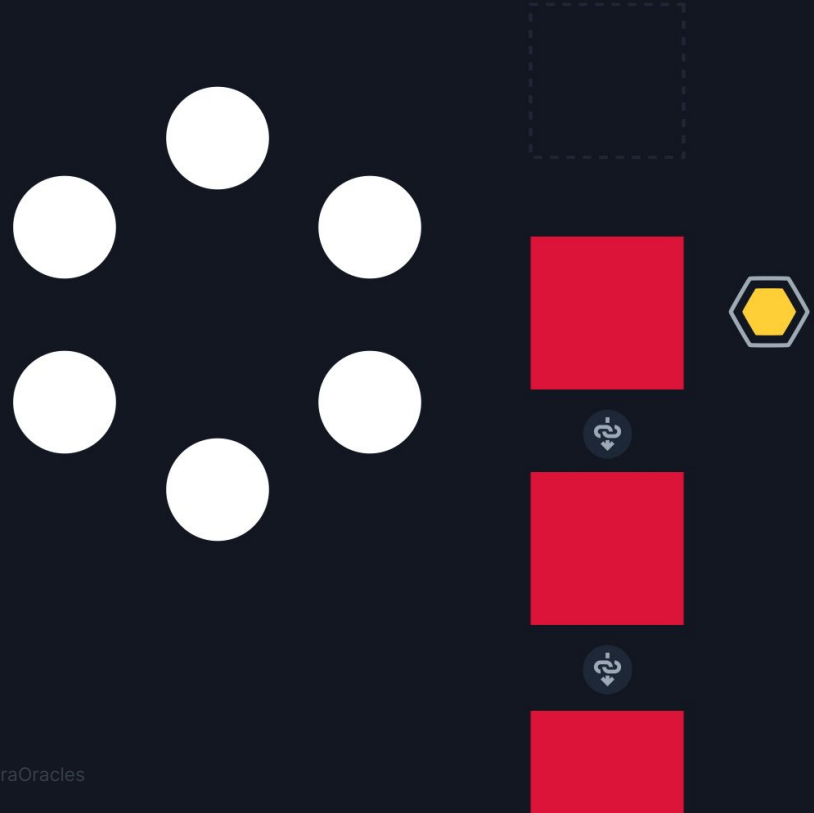
SMRs are not Broadcast Channels

	Employed Broadcast Channel	Efficient SMR / Blockchain
Message Delay	A fixed Δ	Only eventual guarantee
Receivers View	All messages by the end of the round	Only a prefix-order guarantee

A Way Out

SMR-assisted Protocol Design

Making broadcast-based primitives to work
in the environment with SMR



Non-interactive VSS for $n > 2f$



Node 1
(Byzantine)

Encryption Vector, NIZK

Node 2

Node 3
(Slow)



- VSS has termination only for honest dealers
 - we can replace SMR by Bracha's reliable broadcast (RBC)
- Reasonable computational efficiency
- NI-VSS can only offer computational hiding property

Verifiable Secret Sharing (VSS)

VSS with Broadcast Channel

51%

Honest Nodes required

Even for unconditional hiding property

Asynchronous VSS

67%

Honest Nodes required

The bound holds for different AVSS versions

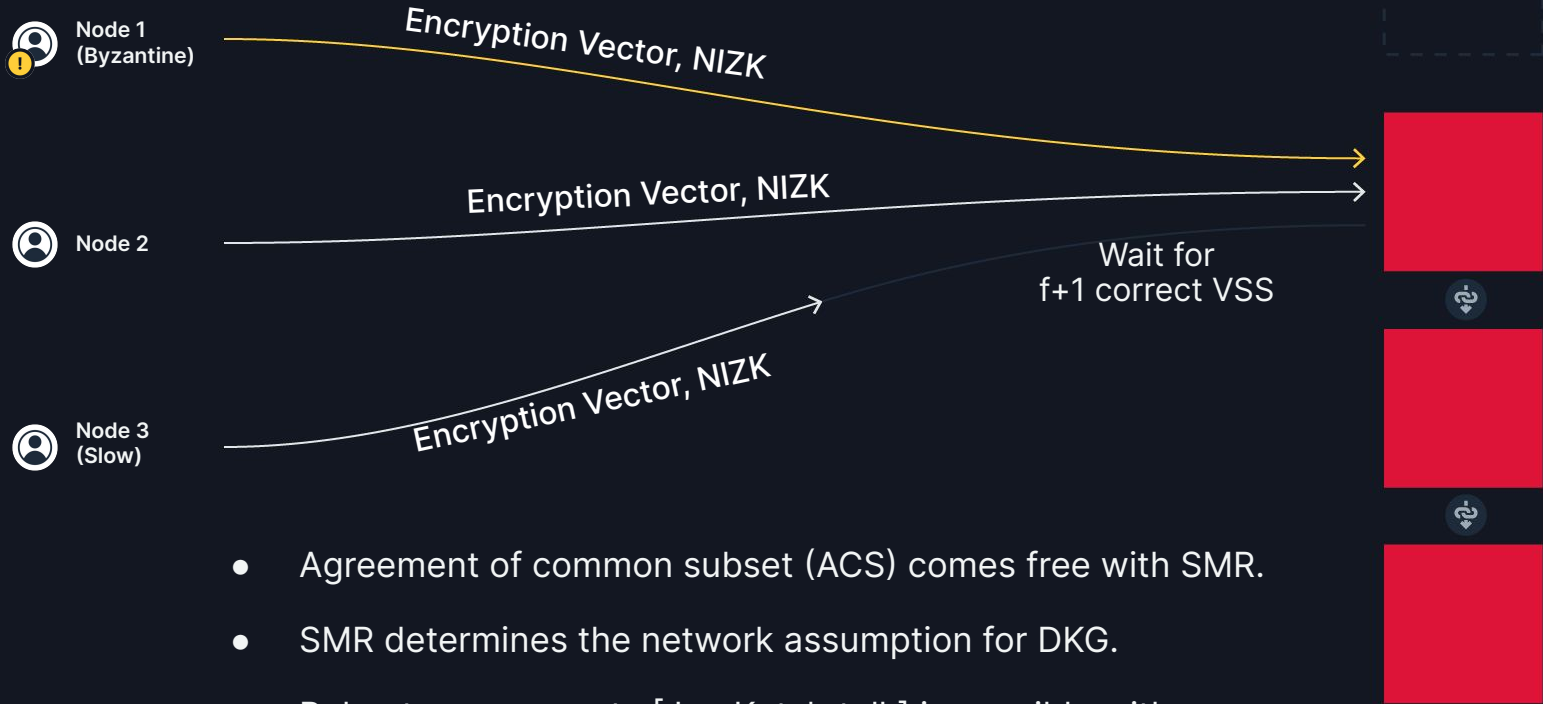
SMR/RBC-Assisted VSS

51%

Honest Nodes required

Unconditional hiding is *not* achievable

Building DKG for $n > 2f$



- Agreement of common subset (ACS) comes free with SMR.
- SMR determines the network assumption for DKG.
- Robustness property [Jon Katz's talk] is possible with an additional asynchronous round.

Distributed Key Generation (DKG)

Partially Synchronous / Async
DKG

67%

Honest Nodes required

Calculation of a shared public key for a random, unknown private key

SMR-assisted DKG

51%

Honest Nodes required

The protocol with a broadcast channel and a SMR channel are almost equivalent

Threshold Cryptography with SMR Channels

Threshold BLS
Signatures / VRF



Purely asynchronous
communication process over
point-to-point links

Threshold
ECDSA / EdDSA



We will need secure
multi-party computation
(MPC) capability

MPC over SMR
Channels for $n > 2f$



CDN MPC Framework
[EUROCRYPT '01] based
On Threshold Linearly
Homomorphic Encryption

MPC over SMR Channels for $n > 2f$

Threshold Linear Homomorphic Encryption Setup

- public key pk
- secret key is shared among the parties

Secure Scalar Operations: (local or on SMR)

- Given $Enc(a)$ and $Enc(b)$
- Compute $Enc(a) \cdot Enc(b)^x = Enc(a+bx)$

Example Setups

Paillier Encryption, Class-group Encryption, Exponentiated ElGamal Encryption

Input (m) Processing:

- Compute ciphertext $c = Enc(pk, m)$

Secure Multiplication:

- Given $Enc(a)$ and $Enc(b)$
- Publish $Enc(d_i)$ and $Enc(b \cdot d_i)$
(wait for $f+1$ tuples)
 - Compute and threshold decrypt $Enc(a + \sum d_i)$
 - Publish decrypted share & compute $(a + \sum d_i)$
 - Compute secure product as
 $(a + \sum d_i) \cdot E(b) - \sum Enc(b \cdot d_i) = E(a \cdot b)$

Conclusion and Unresolved Issues

Proposed VSS/DKG/MPC with
SMR Channel

51%

Honest Nodes required

Computational Hiding
Key Overhead: Encryption + NIZK

Achieving Unconditional
Hiding

67%

Honest Nodes required

Converting Feldman/Pedersen
VSS to work on SMR

Several Open Problems

1. Proving lower-bound $n > 3f$ for unconditional hiding (i.e. using secure and authenticated channels)
 2. MPC for $n > 3f$ with unconditional hiding towards avoiding encryptions and NIZK
- ...

Thanks!

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