

Practical Applications of Combinatorial Testing

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Tutorial Overview

1. Why are we doing this?
2. What is combinatorial testing?
3. What tools are available?
4. How do I use this in the real world?

Differences from yesterday's talk:

Less history

More applications

More code

Plus, ad for undergrad research fellowship program

What is NIST and why are we doing this?

- US Government agency, whose mission is to support US industry through developing better measurement and test methods
- 3,000 scientists, engineers, and support staff including 3 Nobel laureates
- Research in physics, chemistry, materials, manufacturing, computer science
- Trivia: NIST is one of the only federal agencies chartered in the Constitution (also DoD, Treasury, Census)



NIST

National Institute of
Standards and Technology

Interaction Testing and Design of Experiments (DOE) Where did these ideas come from?



Scottish physician James Lind determined cure of scurvy

Ship HM Bark Salisbury in 1747

12 sailors “were as similar as I could have them”

6 treatments 2 sailors for each – cider, sulfuric acid, vinegar, seawater, orange/lemon juice, barley water

Principles used ([blocking](#), [replication](#), [randomization](#))

Did not consider interactions, but otherwise used basic Design of Experiments principles

Father of DOE:

R A Fisher, 1890-1962, British geneticist

Key features of DoE

- Blocking
- Replication
- Randomization
- Orthogonal arrays to test interactions between factors

Test	P1	P2	P3
1	1	1	3
2	1	2	2
3	1	3	1
4	2	1	2
5	2	2	1
6	2	3	3
7	3	1	1
8	3	2	3
9	3	3	2

Each combination occurs same number of times, usually once.

Example: P1, P2 = 1,2

Orthogonal Arrays for Software Interaction Testing

Functional (black-box) testing

Hardware-software systems

Identify single and 2-way combination faults

Early papers

Taguchi followers (mid1980's)

Mandl (1985) Compiler testing

Tatsumi et al (1987) Fujitsu

Sacks et al (1989) Computer experiments

Brownlie et al (1992) AT&T

Generation of test suites using OAs

OATS (Phadke, AT&T-BL)

Interaction Failure Internals

How does an interaction fault manifest itself in code?

Example: `altitude_adj == 0 && volume < 2.2` (2-way interaction)

```
if (altitude_adj == 0 ) {  
    // do something  
    if (volume < 2.2) { faulty code! BOOM! }  
    else { good code, no problem}  
} else {  
    // do something else  
}
```

A test that included `altitude_adj == 0` and `volume = 1` would trigger this failure

What's different about software?

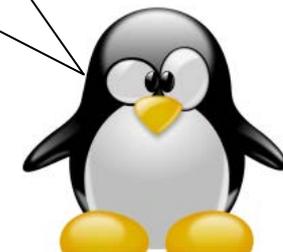
Traditional DoE

- Continuous variable results
- Small number of parameters
- Interactions typically increase or decrease output variable

DoE for Software

- Binary result (pass or fail)
- Large number of parameters
- Interactions affect path through program

Does this difference make any difference?



So how did testing interactions work in practice for software?

- Pairwise testing commonly applied to software
- Intuition: some problems only occur as the result of an interaction between parameters/components
- Tests all pairs (2-way combinations) of variable values
- Pairwise testing finds about 50% to 90% of flaws

90% of flaws!
Sounds pretty good!



Finding 90% of flaws is pretty good, right?



"Relax, our engineers found 90 percent of the flaws."

I don't think I want to get on that plane.



Software Failure Analysis

- NIST studied software failures in a variety of fields including 15 years of FDA medical device recall data
- What **causes** software failures?
 - logic errors?
 - calculation errors?
 - inadequate input checking?
 - interaction faults? Etc.



Interaction faults: e.g., failure occurs if

`pressure < 10 && volume > 300` (interaction between 2 factors)

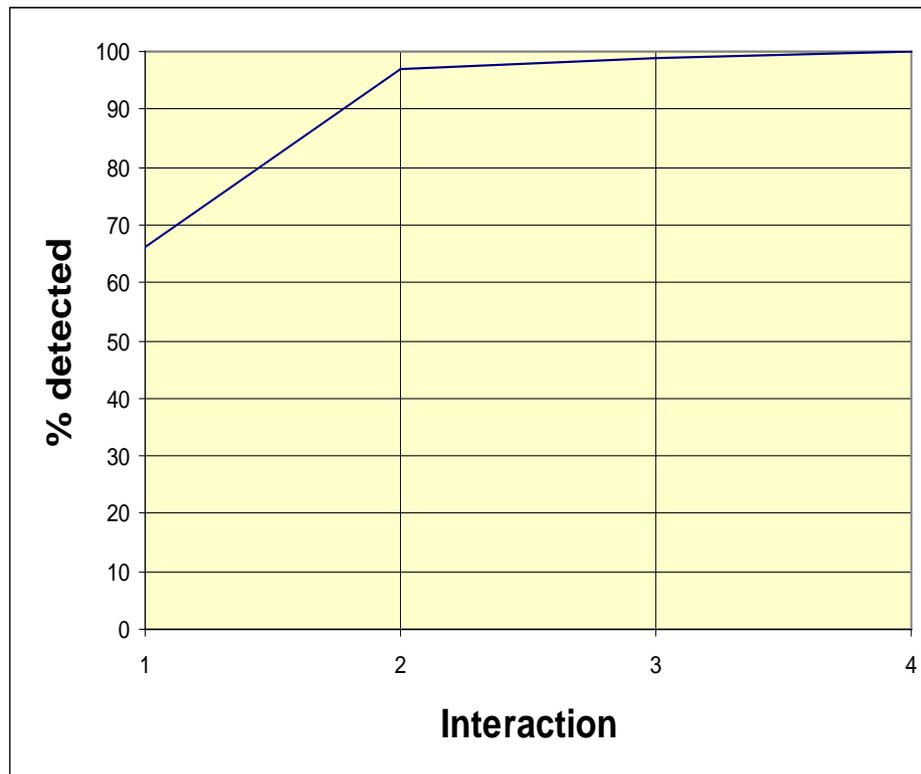
Example from FDA failure analysis:

Failure when “altitude adjustment set on 0 meters
and total flow volume set at delivery rate of less than 2.2 liters per minute.”

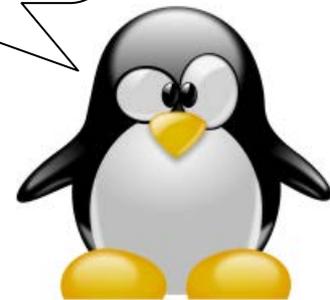
So this is a **2-way interaction** – maybe pairwise testing would be effective?

So interaction testing ought to work, right?

- Interactions e.g., failure occurs if
 - pressure < 10 (1-way interaction)
 - pressure < 10 & volume > 300 (2-way interaction)
 - pressure < 10 & volume > 300 & velocity = 5 (3-way interaction)
- Surprisingly, no one had looked at interactions beyond 2-way before
- **The most complex failure reported required 4-way interaction to trigger. Traditional DoE did not consider this level of interaction.**

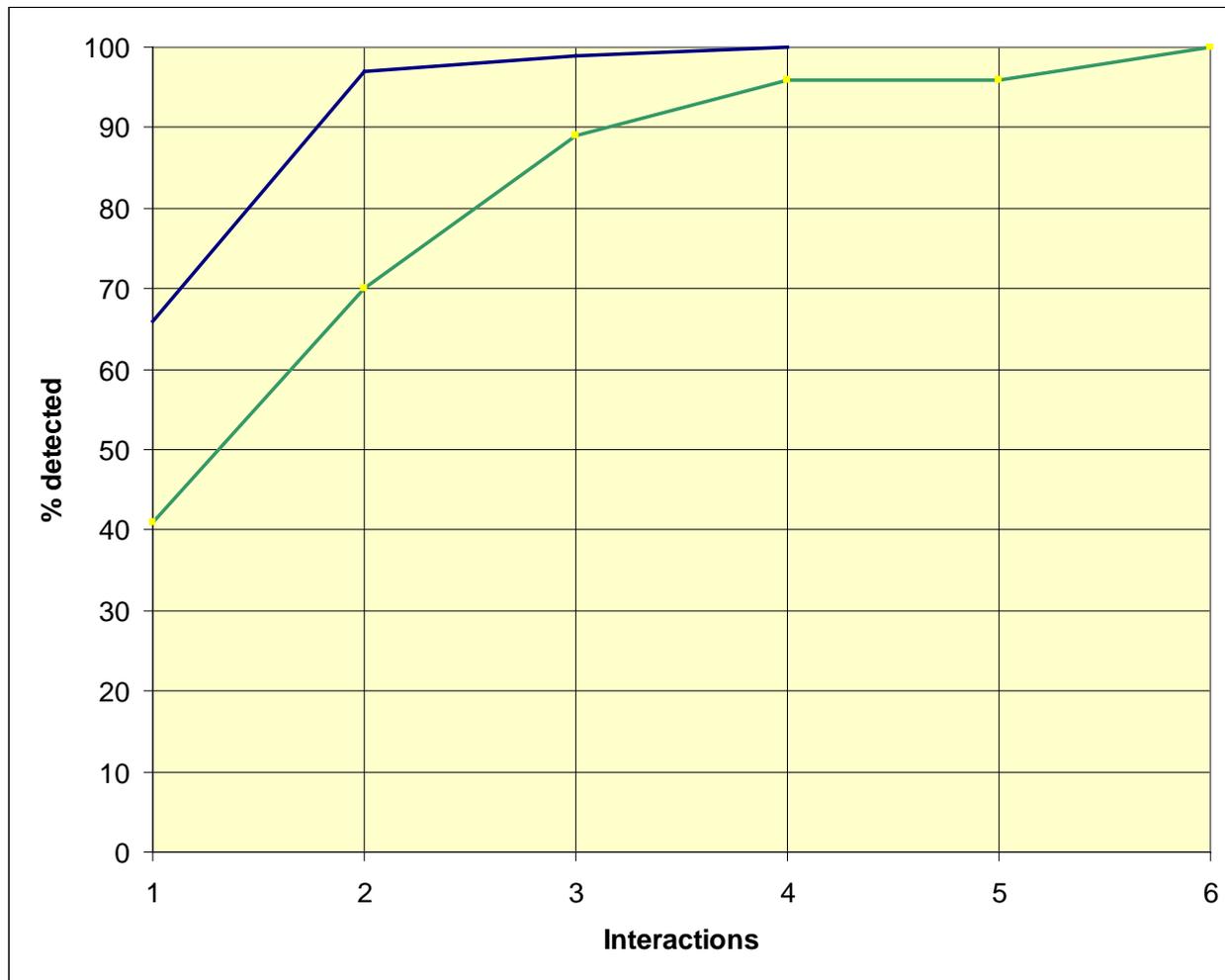


Interesting, but that's just one kind of application!



What about other applications?

Server (green)

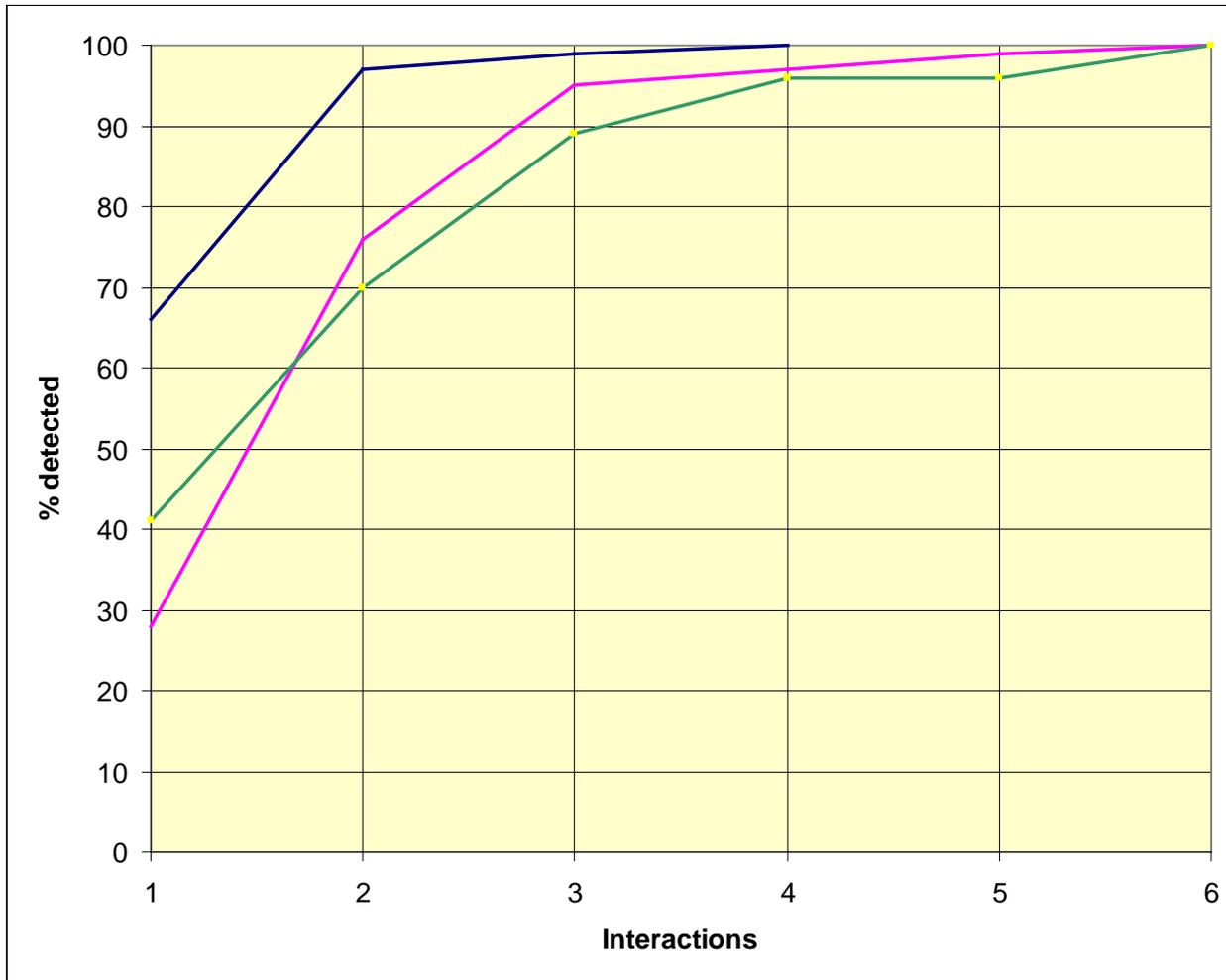


These faults more complex than medical device software!!

Why?

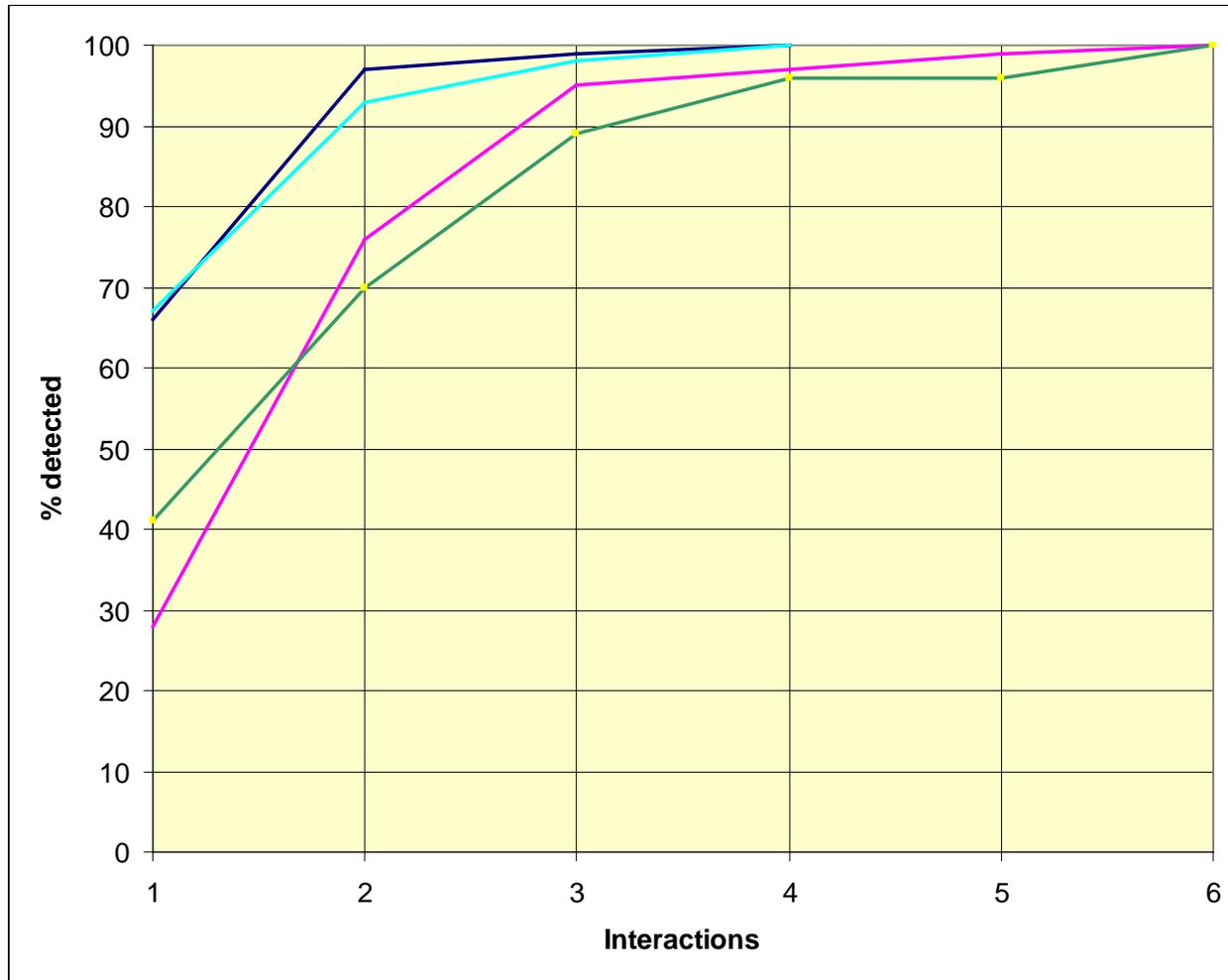
Others?

Browser (magenta)



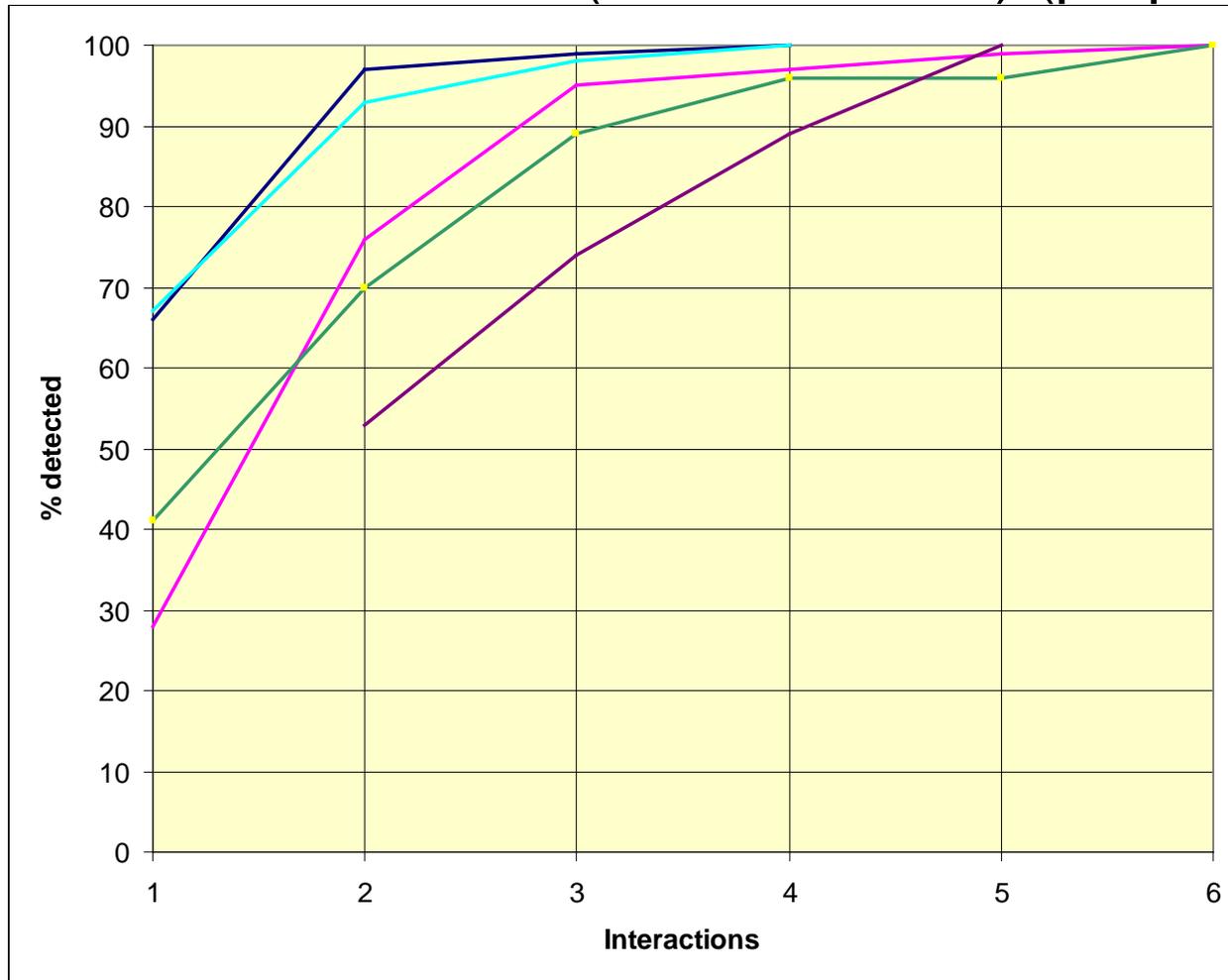
Still more?

NASA Goddard distributed database (light blue)



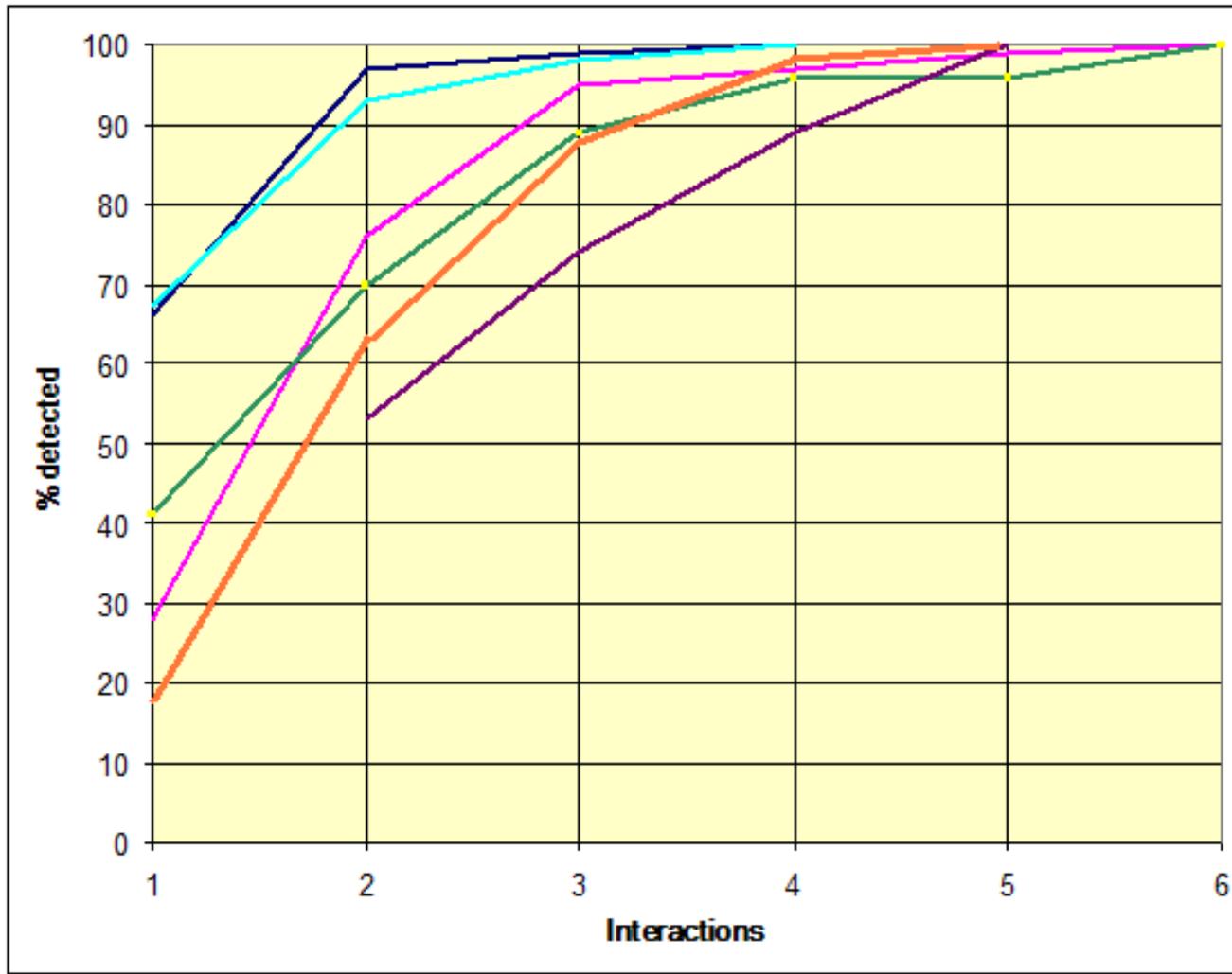
Even more?

FAA Traffic Collision Avoidance System module
(seeded errors) (purple)



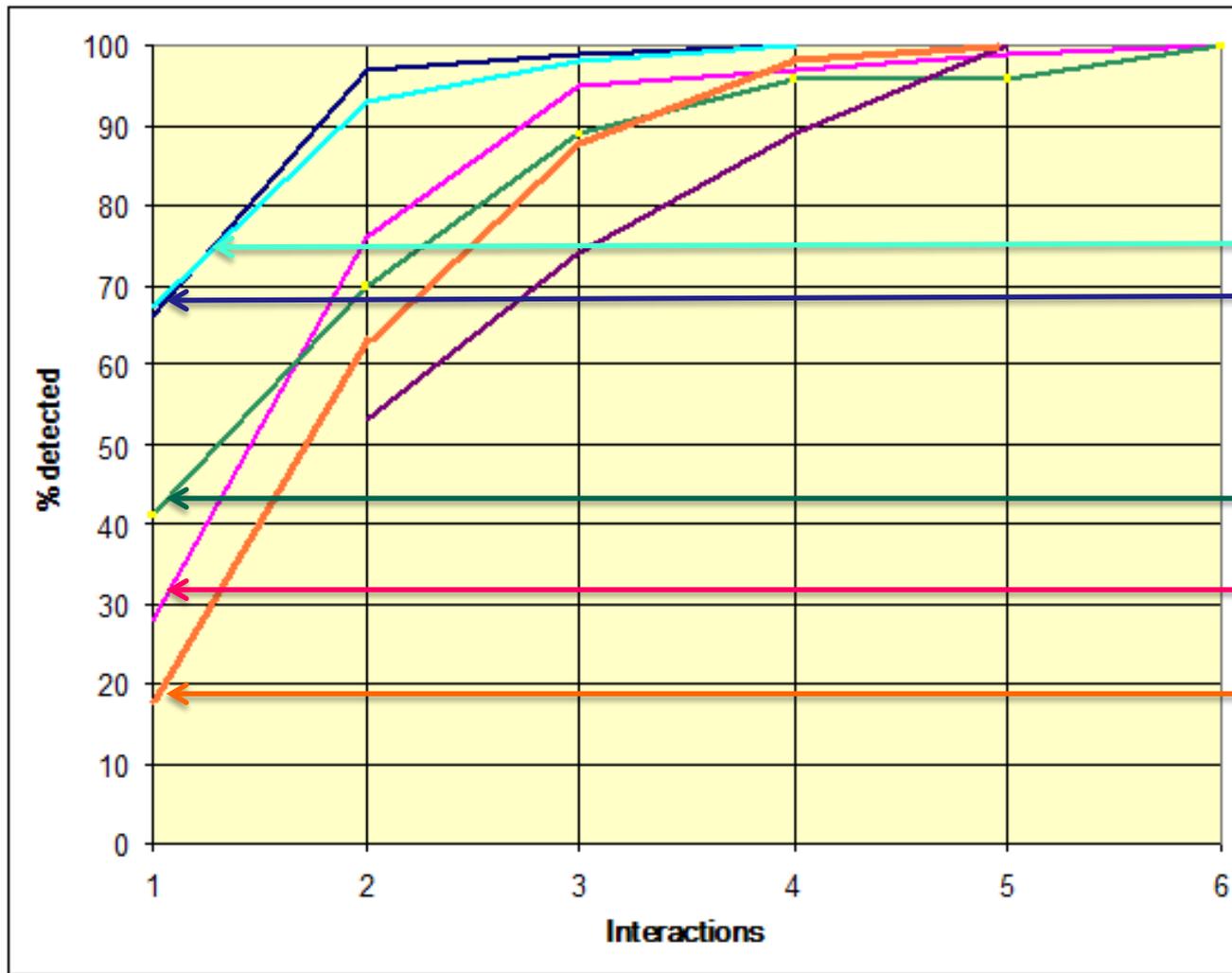
Finally

Network security (Bell, 2006) (orange)



Curves appear to be similar across a variety of application domains.

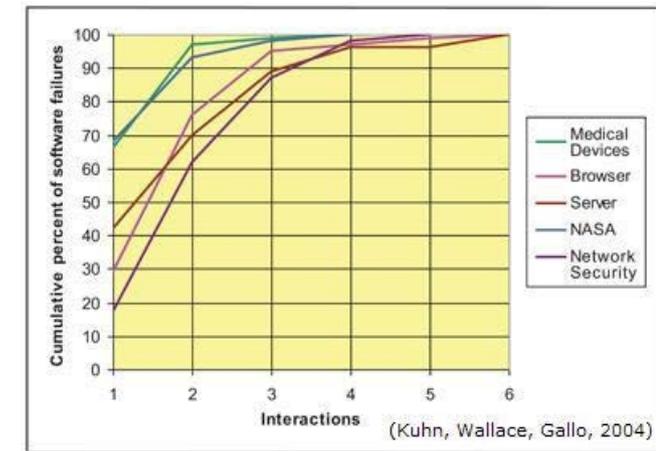
Fault curve pushed down and right as faults detected and removed?



App	users
NASA	10s (testers)
Med.	100s to 1000s
Server	10s of mill.
Browser	10s of mill.
TCP/IP	100s of mill.

Interaction Rule

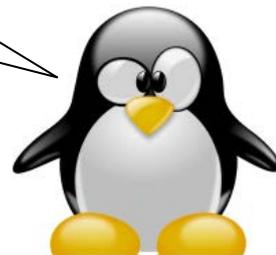
- So, how many parameters are involved in faults?



Interaction rule: most failures are triggered by one or two parameters, and progressively fewer by three, four, or more parameters, and the maximum interaction degree is small.

- **Maximum interactions** for fault triggering was 6
- Popular “pairwise testing” not enough
- More empirical work needed
- Reasonable evidence that maximum interaction strength for fault triggering is **relatively small**

How does it help me to know this?

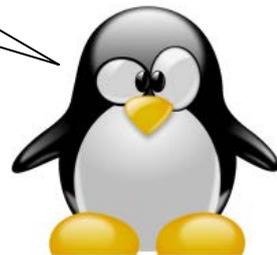


How does this knowledge help?

If all faults are triggered by the interaction of t or fewer variables, then testing all t -way combinations can provide strong assurance.

(taking into account: value propagation issues, equivalence partitioning, timing issues, more complex interactions, . . .)

Still no silver
bullet. Rats!

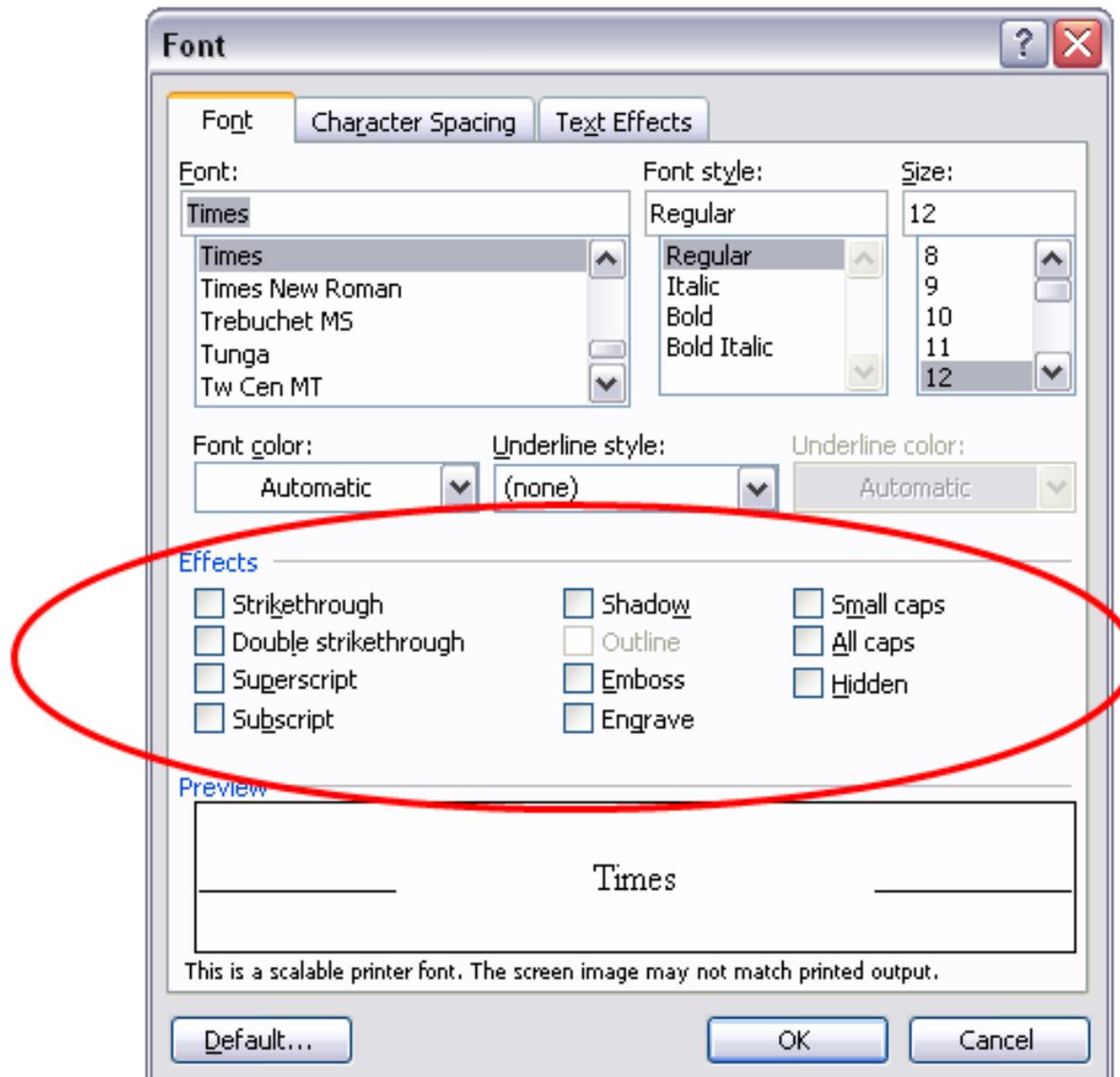


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How do we use this knowledge in testing?

A simple example

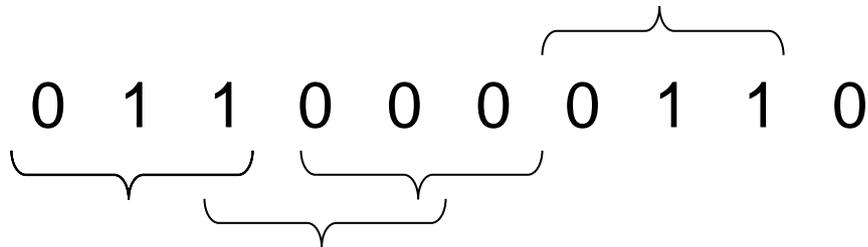


How Many Tests Would It Take?

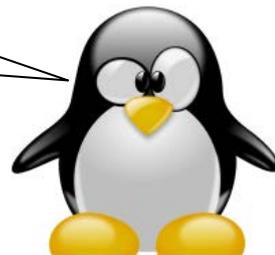
- There are 10 effects, each can be **on** or **off**
- All combinations is $2^{10} = 1,024$ tests
- What if our budget is too limited for these tests?
- Instead, let's look at all **3-way interactions** ...

Now How Many Would It Take?

- There are $\binom{10}{3} = 120$ 3-way interactions.
- Naively $120 \times 2^3 = 960$ tests.
- Since we can pack 3 triples into each test, we need no more than 320 tests.
- Each test exercises many triples:



OK, OK, what's the **smallest** number of tests we need?



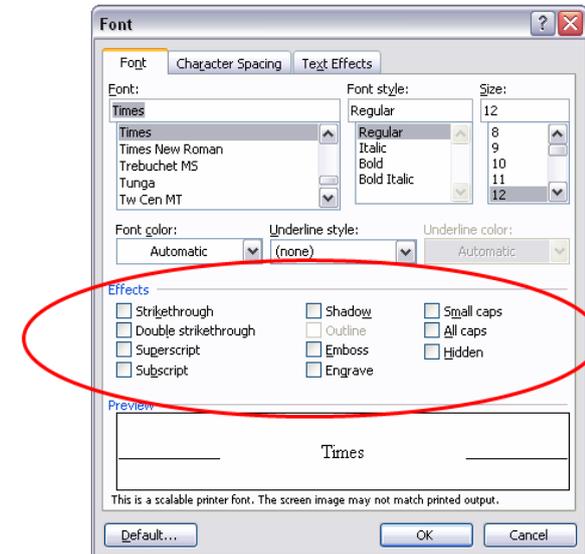
A covering array

All triples in only 13 tests, covering $\binom{10}{3} 2^3 = 960$ combinations

Each row is a test:

0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1
1	1	1	0	1	0	0	0	0	1
1	0	1	1	0	1	0	1	0	0
1	0	0	0	1	1	1	0	0	0
0	1	1	0	0	1	0	0	1	0
0	0	1	0	1	0	1	1	1	0
1	1	0	1	0	0	1	0	1	0
0	0	0	1	1	1	0	0	1	1
0	0	1	1	0	0	1	0	0	1
0	1	0	1	1	0	0	0	1	0
1	0	0	0	0	0	0	1	1	1
0	1	0	0	0	1	1	0	1	1

Each column is a parameter:



- Developed 1990s
- Extends Design of Experiments concept
- NP hard problem but good algorithms now

Summary

Design of Experiments for Software Testing

Not orthogonal arrays, but Covering arrays: Fixed-value $CA(N, v^k, t)$ has four parameters N, k, v, t : It is a matrix covers every t-way combination at least once

Key differences

orthogonal arrays:

- Combinations occur same number of times
- Not always possible to find for a particular configuration

covering arrays:

- Combinations occur at least once
- Always possible to find for a particular configuration
- Always smaller than orthogonal array (or same size)

A larger example

Suppose we have a system with on-off switches. Software must produce the right response for any combination of switch settings:



How do we test this?

34 switches = $2^{34} = 1.7 \times 10^{10}$ possible inputs = 1.7×10^{10} tests



What if we knew no failure involves more than 3 switch settings interacting?

- 34 switches = $2^{34} = 1.7 \times 10^{10}$ possible inputs = **1.7×10^{10}** tests
- If only 3-way interactions, need only **33** tests
- For 4-way interactions, need only **85** tests



Two ways of using combinatorial testing

Use combinations here

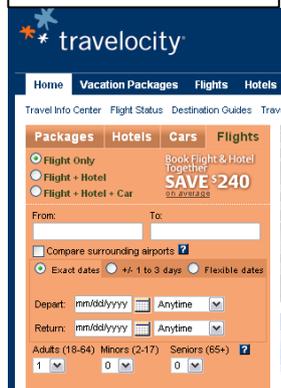
or here

Configuration

Test case	OS	CPU	Protocol
1	Windows	Intel	IPv4
2	Windows	AMD	IPv6
3	Linux	Intel	IPv6
4	Linux	AMD	IPv4

Test data inputs

System under test



Testing Configurations

- Example: app must run on any configuration of OS, browser, protocol, CPU, and DBMS
- Very effective for interoperability testing, being used by NIST for DoD Android phone testing

Test	OS	Browser	Protocol	CPU	DBMS
1	XP	IE	IPv4	Intel	MySQL
2	XP	Firefox	IPv6	AMD	Sybase
3	XP	IE	IPv6	Intel	Oracle
4	OS X	Firefox	IPv4	AMD	MySQL
5	OS X	IE	IPv4	Intel	Sybase
6	OS X	Firefox	IPv4	Intel	Oracle
7	RHL	IE	IPv6	AMD	MySQL
8	RHL	Firefox	IPv4	Intel	Sybase
9	RHL	Firefox	IPv4	AMD	Oracle
10	OS X	Firefox	IPv6	AMD	Oracle

Testing Smartphone Configurations

Some Android configuration options:

```
int HARDKEYBOARDHIDDEN_NO;  
int HARDKEYBOARDHIDDEN_UNDEFINED;  
int HARDKEYBOARDHIDDEN_YES;  
int KEYBOARDHIDDEN_NO;  
int KEYBOARDHIDDEN_UNDEFINED;  
int KEYBOARDHIDDEN_YES;  
int KEYBOARD_12KEY;  
int KEYBOARD_NOKEYS;  
int KEYBOARD_QWERTY;  
int KEYBOARD_UNDEFINED;  
int NAVIGATIONHIDDEN_NO;  
int NAVIGATIONHIDDEN_UNDEFINED;  
int NAVIGATIONHIDDEN_YES;  
int NAVIGATION_DPAD;  
int NAVIGATION_NONAV;  
int NAVIGATION_TRACKBALL;  
int NAVIGATION_UNDEFINED;  
int NAVIGATION_WHEEL;  
  
int ORIENTATION_LANDSCAPE;  
int ORIENTATION_PORTRAIT;  
int ORIENTATION_SQUARE;  
int ORIENTATION_UNDEFINED;  
int SCREENLAYOUT_LONG_MASK;  
int SCREENLAYOUT_LONG_NO;  
int SCREENLAYOUT_LONG_UNDEFINED;  
int SCREENLAYOUT_LONG_YES;  
int SCREENLAYOUT_SIZE_LARGE;  
int SCREENLAYOUT_SIZE_MASK;  
int SCREENLAYOUT_SIZE_NORMAL;  
int SCREENLAYOUT_SIZE_SMALL;  
int SCREENLAYOUT_SIZE_UNDEFINED;  
int TOUCHSCREEN_FINGER;  
int TOUCHSCREEN_NOTOUCH;  
int TOUCHSCREEN_STYLUS;  
int TOUCHSCREEN_UNDEFINED;
```

Configuration option values

Parameter Name	Values	# Values
HARDKEYBOARDHIDDEN	NO, UNDEFINED, YES	3
KEYBOARDHIDDEN	NO, UNDEFINED, YES	3
KEYBOARD	12KEY, NOKEYS, QWERTY, UNDEFINED	4
NAVIGATIONHIDDEN	NO, UNDEFINED, YES	3
NAVIGATION	DPAD, NONAV, TRACKBALL, UNDEFINED, WHEEL	5
ORIENTATION	LANDSCAPE, PORTRAIT, SQUARE, UNDEFINED	4
SCREENLAYOUT_LONG	MASK, NO, UNDEFINED, YES	4
SCREENLAYOUT_SIZE	LARGE, MASK, NORMAL, SMALL, UNDEFINED	5
TOUCHSCREEN	FINGER, NOTOUCH, STYLUS, UNDEFINED	4

Total possible configurations:

$$3 \times 3 \times 4 \times 3 \times 5 \times 4 \times 4 \times 5 \times 4 = 172,800$$

Number of configurations generated for t -way interaction testing, $t = 2..6$

t	# Configs	% of Exhaustive
2	29	0.02
3	137	0.08
4	625	0.4
5	2532	1.5
6	9168	5.3

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5. What's next?

Available Tools

- **Covering array generator** – basic tool for test input or configurations;
- **Sequence covering array generator** – new concept; applies combinatorial methods to event sequence testing
- **Combinatorial coverage measurement** – detailed analysis of combination coverage; automated generation of supplemental tests; helpful for integrating c/t with existing test methods
- **Domain/application specific tools:**
 - Access control policy tester
 - .NET config file generator

New algorithms

- Smaller test sets faster, with a more advanced user interface
- First parallelized covering array algorithm
- **More information per test**

T-Way	IPOG		ITCH (IBM)		Jenny (Open Source)		TConfig (U. of Ottawa)		TVG (Open Source)	
	Size	Time	Size	Time	Size	Time	Size	Time	Size	Time
2	100	0.8	120	0.73	108	0.001	108	>1 hour	101	2.75
3	400	0.36	2388	1020	413	0.71	472	>12 hour	9158	3.07
4	1363	3.05	1484	5400	1536	3.54	1476	>21 hour	64696	127
5	4226	18s	NA	>1 day	4580	43.54	NA	>1 day	313056	1549
6	10941	65.03	NA	>1 day	11625	470	NA	>1 day	1070048	12600

Traffic Collision Avoidance System (TCAS): $2^7 3^2 4^1 10^2$

Times in seconds

ACTS - Defining a new system

New System Form

Parameters Relations Constraints

System Name TCAS

System Parameter

Parameter Name

Parameter Type Boolean

Parameter Values

Selected Parameter **Boolean**

Simple Value

Range Value

Saved Parameters

Parameter Name	Parameter Value
Cur_Vertical_Sep	[299,300,601]
High_Confidence	[true,false]
Two_of_Three_Reports	[true,false]
Own_Tracked_Alt	[1,2]
Other_Track_Alt	[1,2]
Own_Tracked_Alt_Rate	[600,601]
Alt_Layer_Value	[0,1,2,3]
Up_Separation	[0,399,400,499,500,639,640,7...
Down_Separation	[0,399,400,499,500,639,640,7...
Other_RAC	[NO_INTENT,DO_NOT_CLIMB,...
Other_Capability	[TCAS_CA,Other]
Climb_Inhibit	[true,false]

Variable interaction strength

New System Form

Parameters Relations Constraints

Parameters

- Cur_Vertical_Sep
- High_Confidence
- Two_of_Three_Reports
- Own_Tracked_Alt
- Other_Track_Alt
- Own_Tracked_Alt_Rate
- Alt_Layer_Value
- Up_Separation
- Down_Separation
- Other_RAC
- Other_Capability
- Climb_Inhibit

Strength

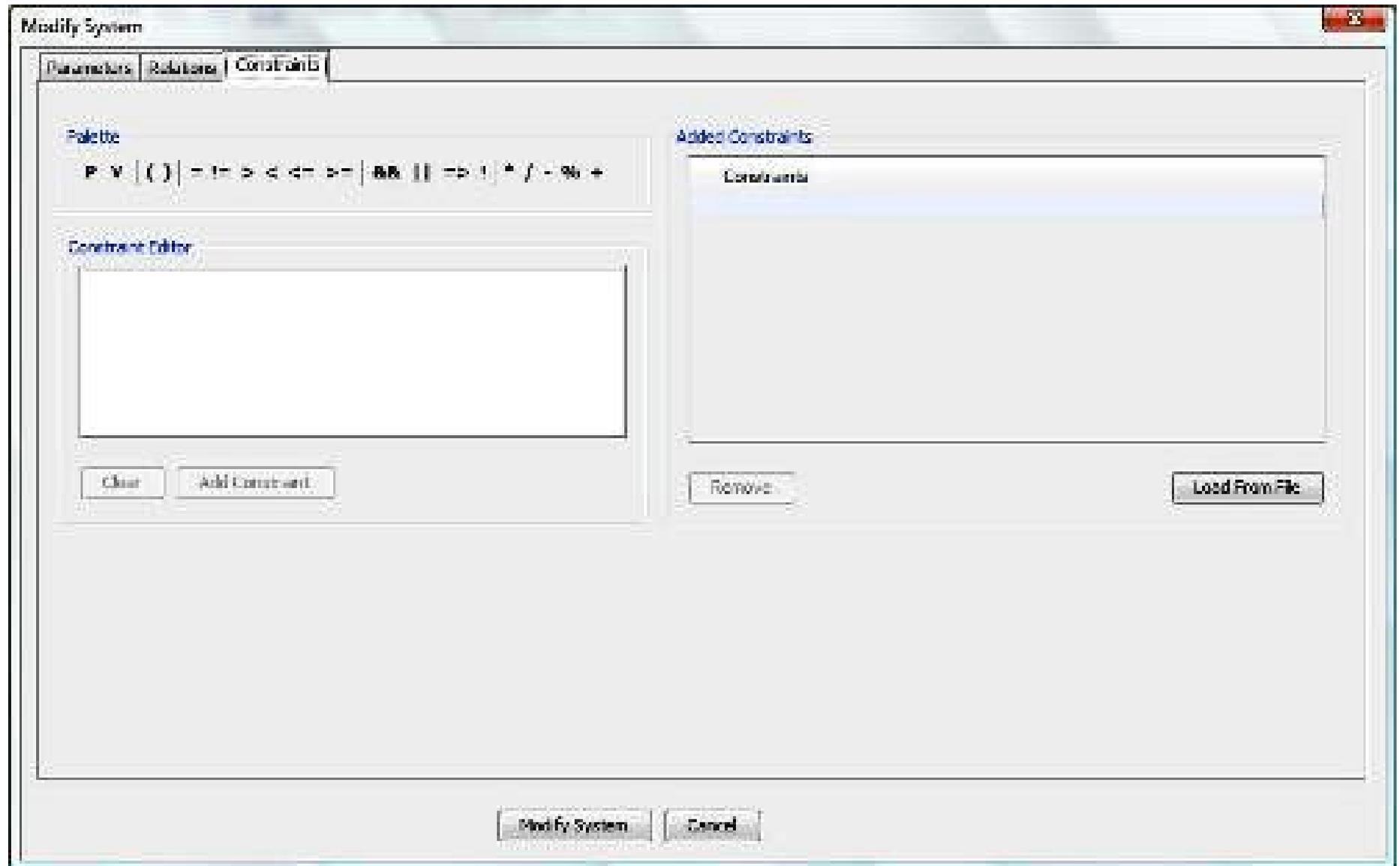
4

Add ->>

Remove

Parameter Names	Strength
Cur_Vertical_Sep,High_Confidence,Two_of_...	2
Alt_Layer_Value,Up_Separation,Down_Sepa...	3

Constraints



Covering array output

FireEye 1.0- FireEye Main Window

System Edit Operations Help

Algorithm IPOG Strength 2

System View

- [Root Node]
 - [SYSTEM-TCAS]
 - Cur_Vertical_Sep
 - 299
 - 300
 - 601
 - High_Confidence
 - true
 - false
 - Two_of_Three_Reports
 - true
 - false
 - Own_Tracked_Alt
 - 1
 - 2
 - Other_Tracked_Alt
 - 1
 - 2
 - Own_Tracked_Alt_Rate
 - 600
 - 601
 - Alt_Layer_Value
 - 0
 - 1
 - 2
 - 3
 - Up_Separation
 - 0
 - 399
 - 400
 - 499
 - 500
 - 639
 - 640

Test Result **Statistics**

	CUR_V...	HIGH...	TWO...	OWN...	OTHER...	OWN...	ALT_L...	UP_SE...	DOWN...	OTHE...	OTHER...	CLIMB...
1	299	true	true	1	1	600	0	0	0	NO_INT...	TCAS_TA	true
2	300	false	false	2	2	601	1	0	399	DO_NO...	OTHER	false
3	601	true	false	1	2	600	2	0	400	DO_NO...	OTHER	true
4	299	false	true	2	1	601	3	0	499	DO_NO...	TCAS_TA	false
5	300	false	true	1	1	601	0	0	500	DO_NO...	OTHER	true
6	601	false	true	2	2	600	1	0	639	NO_INT...	TCAS_TA	false
7	299	false	false	2	1	601	2	0	640	NO_INT...	TCAS_TA	true
8	300	true	false	1	2	600	3	0	739	NO_INT...	OTHER	false
9	601	true	false	2	1	601	0	0	740	DO_NO...	TCAS_TA	true
10	299	true	true	1	2	600	1	0	840	DO_NO...	OTHER	false
11	300	false	true	1	2	600	2	399	0	DO_NO...	TCAS_TA	false
12	601	true	false	2	1	601	3	399	399	DO_NO...	TCAS_TA	true
13	299	false	true	2	1	601	0	399	400	NO_INT...	OTHER	false
14	300	true	false	1	2	600	1	399	499	DO_NO...	OTHER	true
15	601	true	false	2	2	600	2	399	500	DO_NO...	TCAS_TA	false
16	299	true	false	1	1	601	3	399	639	DO_NO...	OTHER	true
17	300	true	true	1	2	600	0	399	640	DO_NO...	OTHER	false
18	601	false	true	2	1	601	1	399	739	DO_NO...	TCAS_TA	true
19	299	false	true	1	2	600	2	399	740	NO_INT...	OTHER	false
20	300	false	false	2	1	601	3	399	840	NO_INT...	TCAS_TA	true
21	601	true	false	2	1	601	1	400	0	DO_NO...	OTHER	true
22	299	false	true	1	2	600	0	400	399	NO_INT...	TCAS_TA	false
23	300	*	*	*	*	*	3	400	400	DO_NO...	TCAS_TA	*
24	601	*	*	*	*	*	2	400	499	NO_INT...	*	*
25	299	*	*	*	*	*	1	400	500	NO_INT...	*	*
26	300	*	*	*	*	*	0	400	639	DO_NO...	*	*
27	601	*	*	*	*	*	3	400	640	DO_NO...	*	*
28	299	*	*	*	*	*	2	400	739	DO_NO...	*	*
29	300	*	*	*	*	*	1	400	740	DO_NO...	*	*
30	601	*	*	*	*	*	0	400	840	DO_NO...	*	*
31	299	true	true	1	1	600	3	499	0	NO_INT...	OTHER	true
32	300	false	false	2	2	601	2	499	399	DO_NO...	TCAS_TA	false

Output options

Mappable values

Degree of interaction coverage: 2
Number of parameters: 12
Number of tests: 100

```
0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 1 1 1 1 0 1 1 1 1
2 0 1 0 1 0 2 0 2 2 1 0
0 1 0 1 0 1 3 0 3 1 0 1
1 1 0 0 0 1 0 0 4 2 1 0
2 1 0 1 1 0 1 0 5 0 0 1
0 1 1 1 0 1 2 0 6 0 0 0
1 0 1 0 1 0 3 0 7 0 1 1
2 0 1 1 0 1 0 0 8 1 0 0
0 0 0 0 1 0 1 0 9 2 1 1
1 1 0 0 1 0 2 1 0 1 0 1
Etc.
```

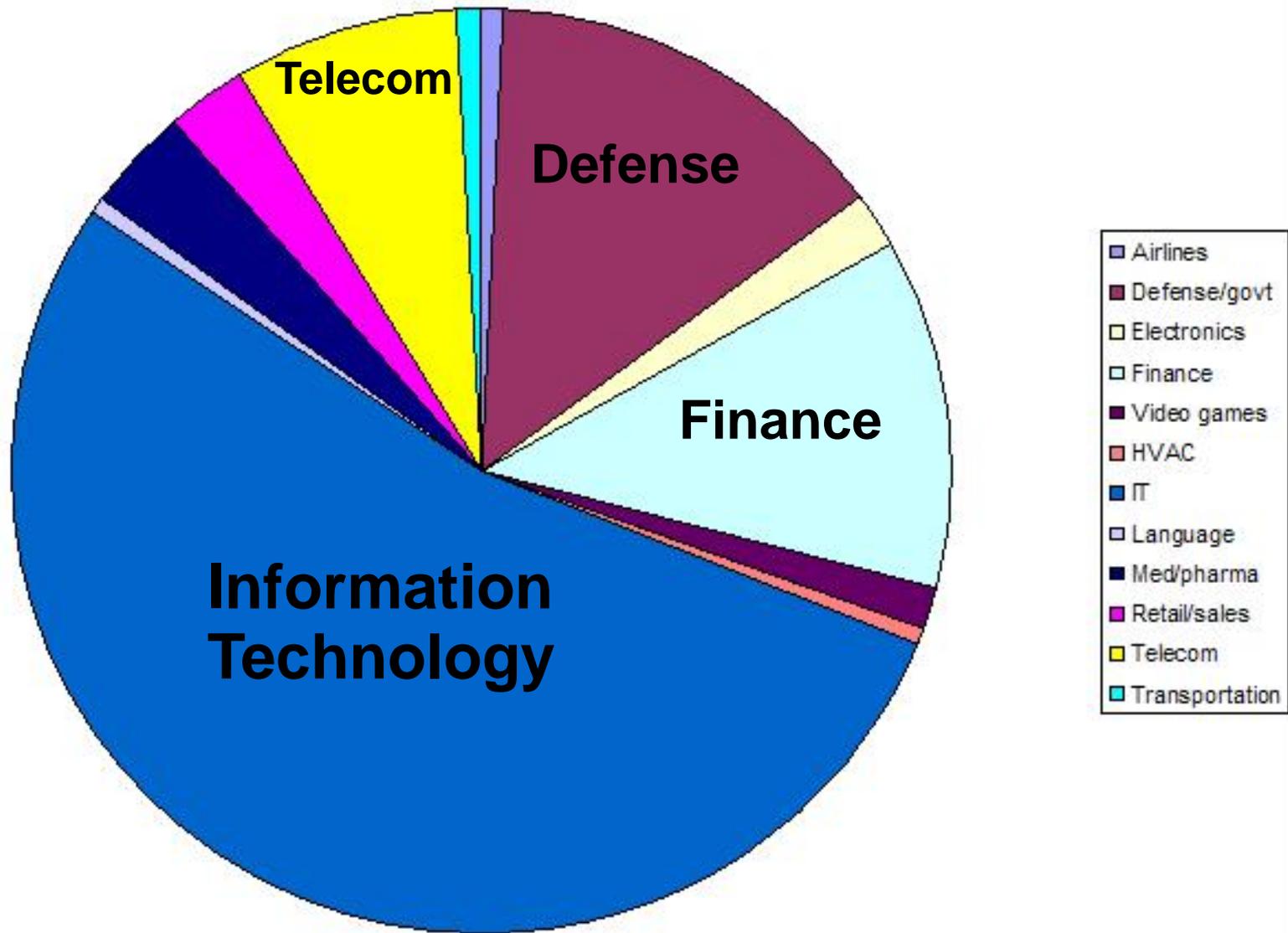
Human readable

Degree of interaction coverage: 2
Number of parameters: 12
Maximum number of values per parameter: 10
Number of configurations: 100

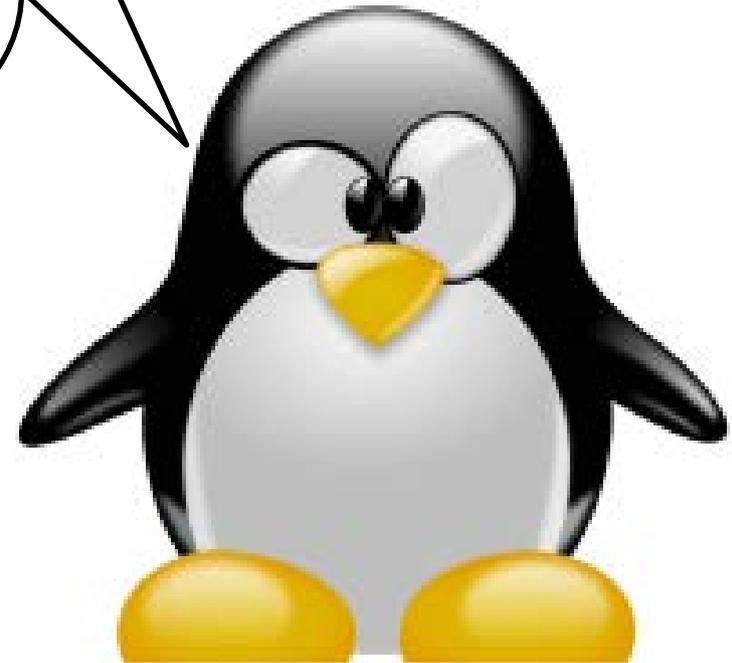
Configuration #1:

```
1 = Cur_Vertical_Sep=299
2 = High_Confidence=true
3 = Two_of_Three_Reports=true
4 = Own_Tracked_Alt=1
5 = Other_Tracked_Alt=1
6 = Own_Tracked_Alt_Rate=600
7 = Alt_Layer_Value=0
8 = Up_Separation=0
9 = Down_Separation=0
10 = Other_RAC=NO_INTENT
11 = Other_Capability=TCAS_CA
12 = Climb_Inhibit=true
```

ACTS Users

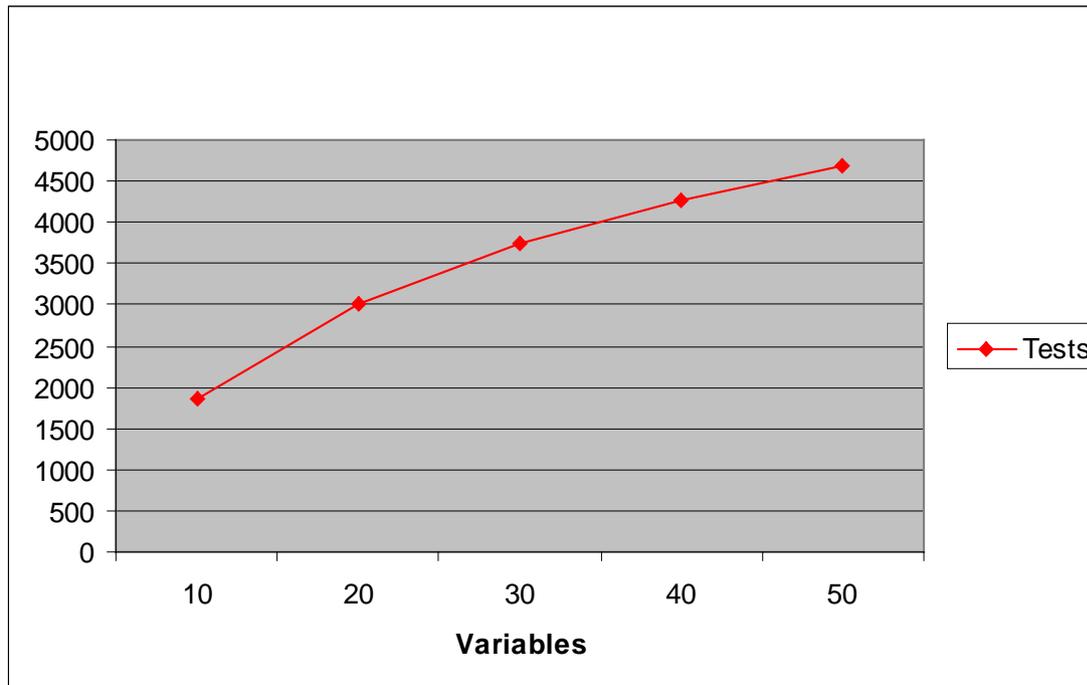


How to I use
this in the real
world ??

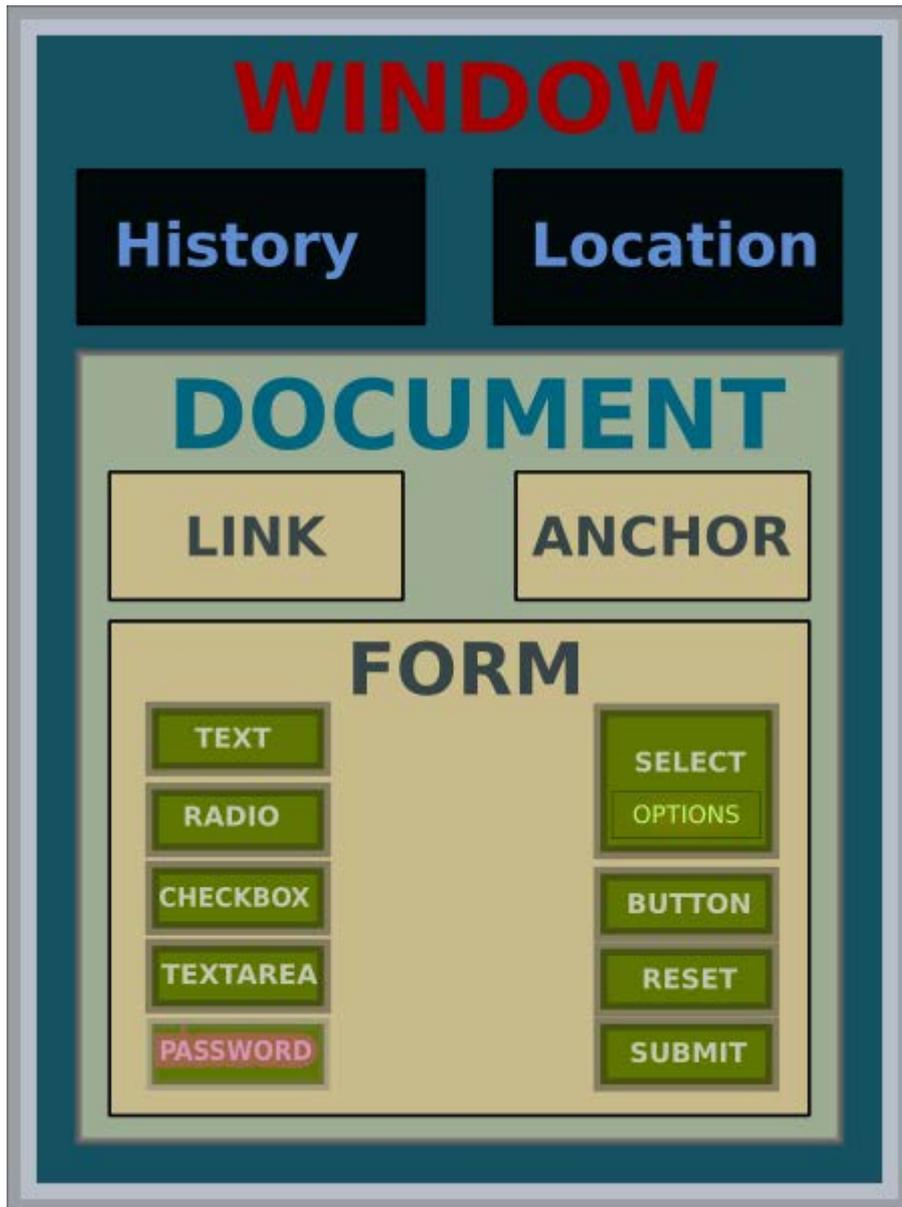


Cost and Volume of Tests

- Number of tests: proportional to $v^t \log n$
for v values, n variables, t -way interactions
- *Thus:*
 - Tests increase *exponentially* with interaction strength t
 - But *logarithmically* with the number of parameters
- Example: suppose we want all 4-way combinations of n parameters, 5 values each:



Real world use - Document Object Model Events



- DOM is a World Wide Web Consortium standard for representing and interacting with browser objects
- NIST developed conformance tests for DOM
- Tests covered all possible combinations of discretized values, >36,000 tests
- Question: can we use the Interaction Rule to increase test effectiveness the way we claim?

Document Object Model Events

Original test set:

Event Name	Param.	Tests
Abort	3	12
Blur	5	24
Click	15	4352
Change	3	12
dblClick	15	4352
DOMActivate	5	24
DOMAttrModified	8	16
DOMCharacterDataModified	8	64
DOMElementNameChanged	6	8
DOMFocusIn	5	24
DOMFocusOut	5	24
DOMNodeInserted	8	128
DOMNodeInsertedIntoDocument	8	128
DOMNodeRemoved	8	128
DOMNodeRemovedFromDocument	8	128
DOMSubTreeModified	8	64
Error	3	12
Focus	5	24
KeyDown	1	17
KeyUp	1	17

Load	3	24
MouseDown	15	4352
MouseMove	15	4352
MouseOut	15	4352
MouseOver	15	4352
MouseUp	15	4352
MouseWheel	14	1024
Reset	3	12
Resize	5	48
Scroll	5	48
Select	3	12
Submit	3	12
TextInput	5	8
Unload	3	24
Wheel	15	4096
Total Tests		36626

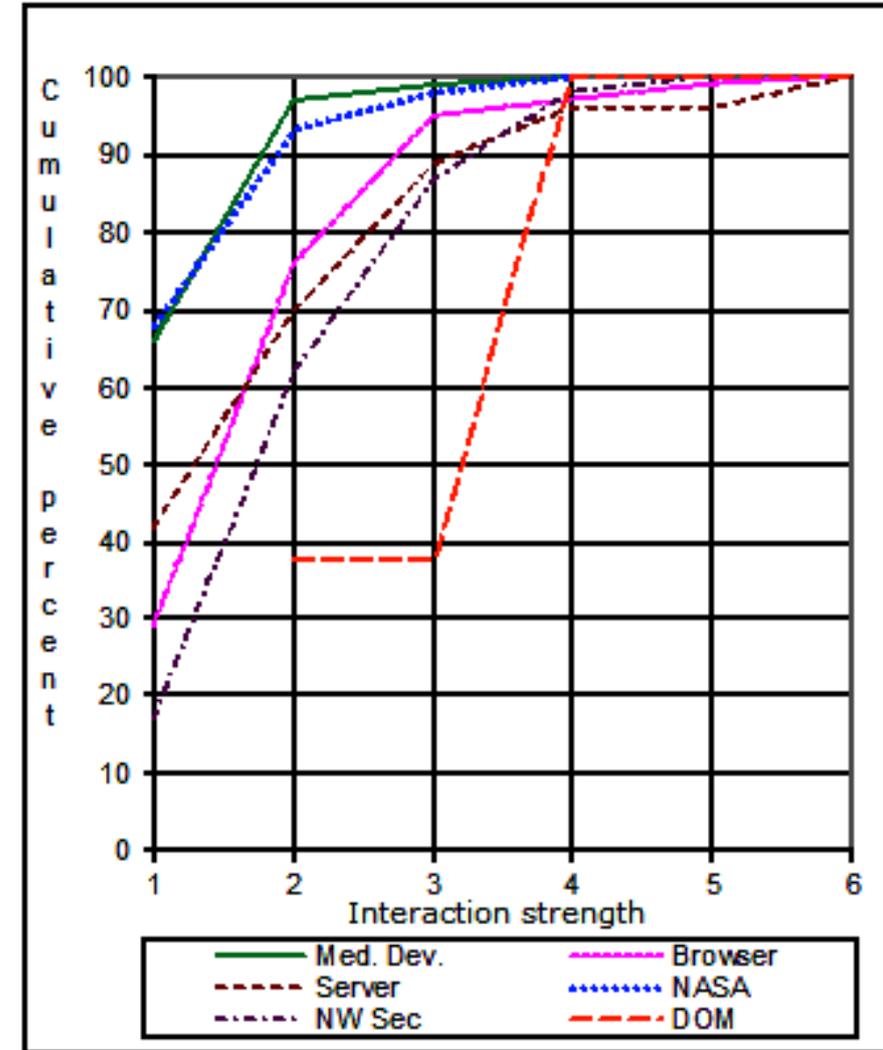
Exhaustive testing of
equivalence class values

Document Object Model Events

Combinatorial test set:

t	Tests	% of Orig.	Test Results		
			Pass	Fail	Not Run
2	702	1.92%	202	27	473
3	1342	3.67%	786	27	529
4	1818	4.96%	437	72	1309
5	2742	7.49%	908	72	1762
6	4227	11.54%	1803	72	2352

All failures found using < 5% of original exhaustive test set



Modeling & Simulation

- 1. Aerospace - Lockheed Martin – analyze structural failures for aircraft design**
- 2. Network defense/offense operations - NIST – analyze network configuration for vulnerability to deadlock**

Problem: unknown factors causing failures of F-16 ventral fin



LANTIRN =
Low Altitude
Navigation &
Targeting
Infrared for
Night

LANTIRN Pod
Location

Ventral Fin A04-14639006

Figure 1. LANTIRN pod carriage on the F-16.

It's not supposed to look like this:



Figure 2. F-16 ventral fin damage on flight with LANTIRN

Can the problem factors be found efficiently?

Original solution: Lockheed Martin engineers spent many months with wind tunnel tests and expert analysis to consider interactions that could cause the problem

Combinatorial testing solution: modeling and simulation using ACTS

Parameter	Values
Aircraft	15, 40
Altitude	5k, 10k, 15k, 20k, 30k, 40k, 50k
Maneuver	hi-speed throttle, slow accel/dwell, L/R 5 deg side slip, L/R 360 roll, R/L 5 deg side slip, Med accel/dwell, R-L-R-L banking, Hi-speed to Low, 360 nose roll
Mach (100 th)	40, 50, 60, 70, 80, 90, 100, 110, 120

Results

- Interactions causing problem included Mach points .95 and .97; multiple side-slip and rolling maneuvers
- Solution analysis tested interactions of Mach points, maneuvers, and multiple fin designs
- Problem could have been found much more efficiently and quickly
- Less expert time required
- Spreading use of combinatorial testing in the corporation:
 - Community of practice of 200 engineers
 - Tutorials and guidebooks
 - Internal web site and information forum

Modeling & Simulation - Networks

- “Simured” network simulator
 - Kernel of ~ 5,000 lines of C++ (not including GUI)
- Objective: detect configurations that can produce deadlock:
 - Prevent connectivity loss when changing network
 - Attacks that could lock up network
- Compare effectiveness of random vs. combinatorial inputs
- Deadlock combinations discovered
- Crashes in >6% of tests w/ valid values (Win32 version only)

Simulation Input Parameters

Parameter		Values
1	DIMENSIONS	1,2,4,6,8
2	NODOSDIM	2,4,6
3	NUMVIRT	1,2,3,8
4	NUMVIRTINJ	1,2,3,8
5	NUMVIRTEJE	1,2,3,8
6	LONBUFFER	1,2,4,6
7	NUMDIR	1,2
8	FORWARDING	0,1
9	PHYSICAL	true, false
10	ROUTING	0,1,2,3
11	DELFIFO	1,2,4,6
12	DELCROSS	1,2,4,6
13	DELCHANNEL	1,2,4,6
14	DELSWITCH	1,2,4,6

$5 \times 3 \times 4 \times 4 \times 4 \times 4 \times 2 \times 2$
 $\times 2 \times 4 \times 4 \times 4 \times 4 \times 4$
 $= 31,457,280$
configurations

Are any of them dangerous?

If so, how many?

Which ones?

Network Deadlock Detection

Deadlocks Detected: combinatorial

t	Tests	500 pkts	1000 pkts	2000 pkts	4000 pkts	8000 pkts
2	28	0	0	0	0	0
3	161	2	3	2	3	3
4	752	14	14	14	14	14

Average Deadlocks Detected: random

t	Tests	500 pkts	1000 pkts	2000 pkts	4000 pkts	8000 pkts
2	28	0.63	0.25	0.75	0.50	0.75
3	161	3	3	3	3	3
4	752	10.13	11.75	10.38	13	13.25

Network Deadlock Detection

Detected 14 configurations that can cause deadlock:

$$14 / 31,457,280 = 4.4 \times 10^{-7}$$

Combinatorial testing found more deadlocks than random, including some that might never have been found with random testing

Why do this testing? Risks:

- accidental deadlock configuration: low
- deadlock config discovered by attacker: **much higher**
(because they are looking for it)

Buffer Overflows

- Empirical data from the National Vulnerability Database
 - Investigated > 3,000 denial-of-service vulnerabilities reported in the NIST NVD for period of 10/06 – 3/07
 - Vulnerabilities triggered by:
 - Single variable – 94.7%
example: *Heap-based buffer overflow in the SFTP protocol handler for Panic Transmit ... allows remote attackers to execute arbitrary code via a long ftps:// URL.*
 - 2-way interaction – 4.9%
example: *single character search string in conjunction with a single character replacement string, which causes an "off by one overflow"*
 - 3-way interaction – 0.4%
example: *Directory traversal vulnerability when register_globals is enabled and magic_quotes is disabled and .. (dot dot) in the page parameter*

Finding Buffer Overflows

```
1.  if (strcmp(conn[sid].dat->in_RequestMethod, "POST")==0) {
2.      if (conn[sid].dat->in_ContentLength<MAX_POSTSIZE) {
3.          .....
4.          conn[sid].PostData=calloc(conn[sid].dat->in_ContentLength+1024,
5.          sizeof(char));
6.          .....
7.          pPostData=conn[sid].PostData;
8.          do {
9.              rc=recv(conn[sid].socket, pPostData, 1024, 0);
10.             .....
11.             pPostData+=rc;
12.             x+=rc;
13.         } while ((rc==1024) || (x<conn[sid].dat->in_ContentLength));
14.     conn[sid].PostData[conn[sid].dat->in_ContentLength]='\0';
15. }
```

Interaction: request-method="POST", content-length = -1000, data= a string > 24 bytes

```
1.  if (strcmp(conn[sid].dat->in_RequestMethod, "POST")==0) {
2.      if (conn[sid].dat->in_ContentLength<MAX_POSTSIZE) {
3.          .....
4.          conn[sid].PostData=calloc(conn[sid].dat->in_ContentLength+1024,
5.          sizeof(char));
6.          .....
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8.          do {
9.              rc=recv(conn[sid].socket, pPostData, 1024, 0);
10.             .....
11.             pPostData+=rc;
12.             x+=rc;
13.         } while ((rc==1024) || (x<conn[sid].dat->in_ContentLength));
14.     conn[sid].PostData[conn[sid].dat->in_ContentLength]='\0';
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sizeof(char));
        .....
4.      pPostData=conn[sid].PostData;
5.      do {
6.          rc=recv(conn[sid].socket, pPostData, 1024, 0);
        .....
7.          pPostData+=rc;
8.          x+=rc;
9.      } while ((rc==1024) || (x<conn[sid].dat->in_ContentLength));
10. conn[sid].PostData[conn[sid].dat->in_ContentLength]='\0';
11. }

```

true branch

Interaction: request-method="POST", content-length = -1000, data= a string > 24 bytes

```

1.   if (strcmp(conn[sid].dat->in_RequestMethod, "POST")==0) {
2.       if (conn[sid].dat->in_ContentLength<MAX_POSTSIZE) {
3.           conn[sid].PostData=calloc(conn[sid].dat->in_ContentLength+1024,
sizeof(char));
4.           pPostData=conn[sid].PostData;
5.           do {
6.               rc=recv(conn[sid].socket, pPostData, 1024, 0);
7.               pPostData+=rc;
8.               x+=rc;
9.           } while ((rc==1024) || (x<conn[sid].dat->in_ContentLength));
10.  conn[sid].PostData[conn[sid].dat->in_ContentLength]='\0';
11.  }

```

true branch

Interaction: request-method="POST", content-length = -1000, data= a string > 24 bytes

```
1.  if (strcmp(conn[sid].dat->in_RequestMethod, "POST")==0) {
2.      if (conn[sid].dat->in_ContentLength<MAX_POSTSIZE) { true branch
    .....
3.      conn[sid].PostData=calloc(conn[sid].dat->in_ContentLength+1024,
sizeof(char));
    Allocate -1000 + 1024 bytes = 24 bytes
    .....
4.      pPostData=conn[sid].PostData;
5.      do {
6.          rc=recv(conn[sid].socket, pPostData, 1024, 0);
    .....
7.          pPostData+=rc;
8.          x+=rc;
9.      } while ((rc==1024) || (x<conn[sid].dat->in_ContentLength));
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```

Interaction: request-method="POST", content-length = -1000, data= a string > 24 bytes

```
1.  if (strcmp(conn[sid].dat->in_RequestMethod, "POST")==0) {
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.....
3.      conn[sid].PostData=calloc(conn[sid].dat->in_ContentLength+1024,
sizeof(char));
Allocate -1000 + 1024 bytes = 24 bytes
.....
4.      pPostData=conn[sid].PostData;
5.      do {
6.          rc=recv(conn[sid].socket, pPostData, 1024, 0) Boom!
.....
7.          pPostData+=rc;
8.          x+=rc;
9.      } while ((rc==1024) || (x<conn[sid].dat->in_ContentLength));
10.  conn[sid].PostData[conn[sid].dat->in_ContentLength]='\0';
11.  }
```



Combinatorial Sequence Testing

- Suppose we want to see if a system works correctly regardless of the order of events. How can this be done efficiently?
- Failure reports often say something like: 'failure occurred when A started if B is not already connected'.
- Can we produce compact tests such that all t-way sequences covered (possibly with interleaving events)?

Event	Description
<i>a</i>	connect flow meter
<i>b</i>	connect pressure gauge
<i>c</i>	connect satellite link
<i>d</i>	connect pressure readout
<i>e</i>	start comm link
<i>f</i>	boot system



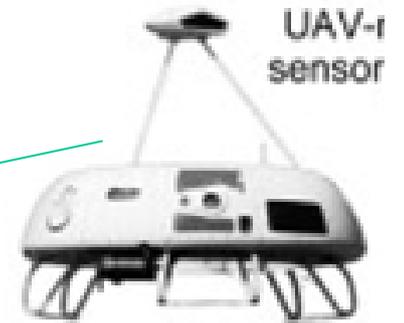
Sequence Covering Array

- With 6 events, all sequences = $6! = 720$ tests
- Only 10 tests needed for all 3-way sequences, results even better for larger numbers of events
- Example: `.*c.*f.*b.*` covered. Any such 3-way seq covered.



Test	Sequence					
1	a	b	c	d	e	f
2	f	e	d	c	b	a
3	d	e	f	a	b	c
4	c	b	a	f	e	d
5	b	f	a	d	c	e
6	e	c	d	a	f	b
7	a	e	f	c	b	d
8	d	b	c	f	e	a
9	c	e	a	d	b	f
10	f	b	d	a	e	c

Example: USAF laptop application



Connection Sequences

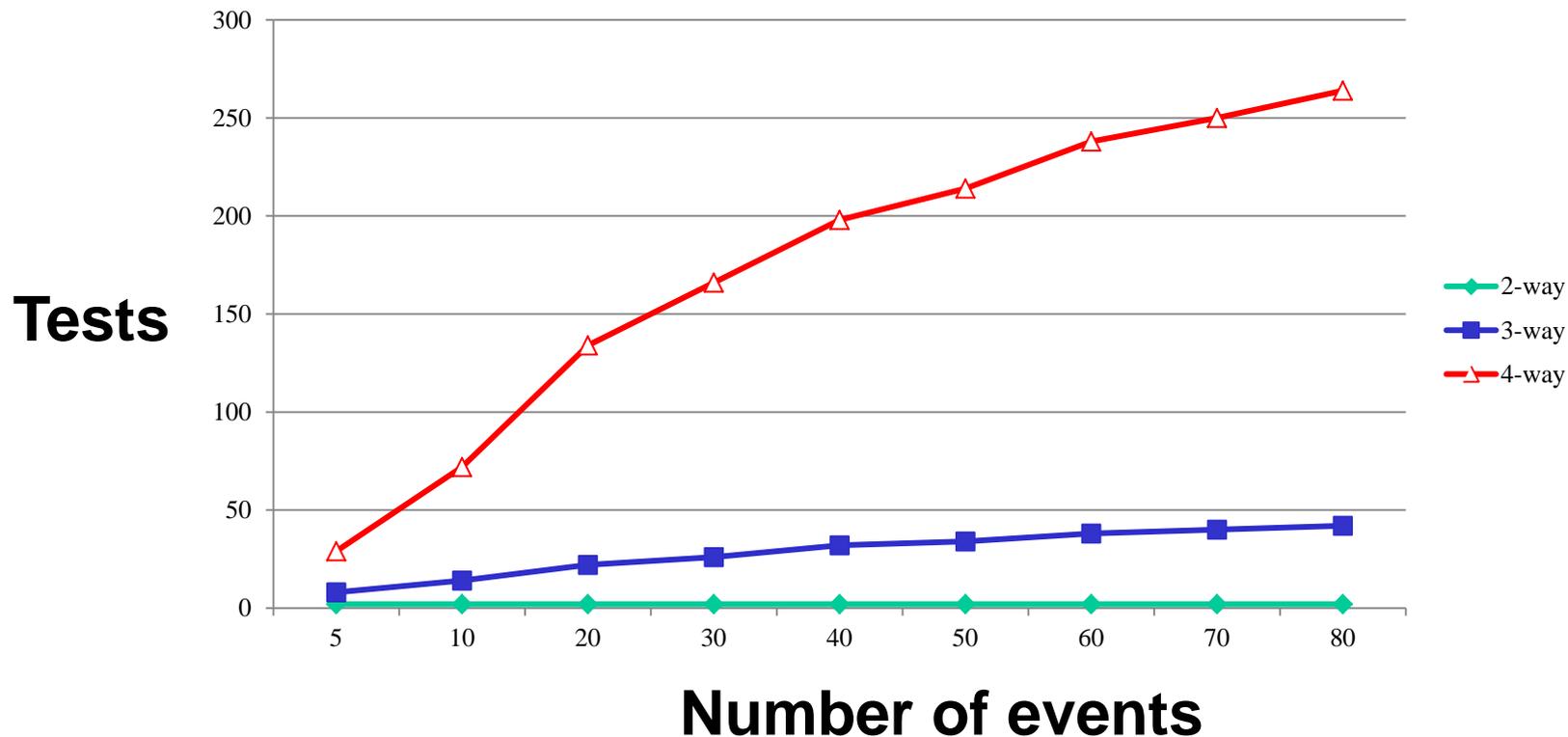
1	Boot	P-1 (USB-RIGHT)	P-2 (USB-BACK)	P-3 (USB-LEFT)	P-4	P-5	App	Scan
2	Boot	App	Scan	P-5	P-4	P-3 (USB-RIGHT)	P-2 (USB-BACK)	P-1 (USB-LEFT)
3	Boot	P-3 (USB-RIGHT)	P-2 (USB-LEFT)	P-1 (USB-BACK)	App	Scan	P-5	P-4
	etc...							

Results

- Tested peripheral connection for 3-way sequences
- Some faults detected that would not have been found with 2-way sequence testing; may not have been found with random
 - Example:
 - If P2-P1-P3 sequence triggers a failure, then a full 2-way sequence covering array would not have found it
(because 1-2-3-4-5-6-7 and 7-6-5-4-3-2-1 is a 2-way sequence covering array)

Sequence Covering Array Properties

- 2-way sequences require only 2 tests
(write events in any order, then reverse)
- For > 2 -way, number of tests grows with $\log n$, for n events
- Simple greedy algorithm produces compact test set
- Not previously described in CS or math literature



Combinatorial Coverage Measurement

Tests	Variables			
	a	b	c	d
1	0	0	0	0
2	0	1	1	0
3	1	0	0	1
4	0	1	1	1

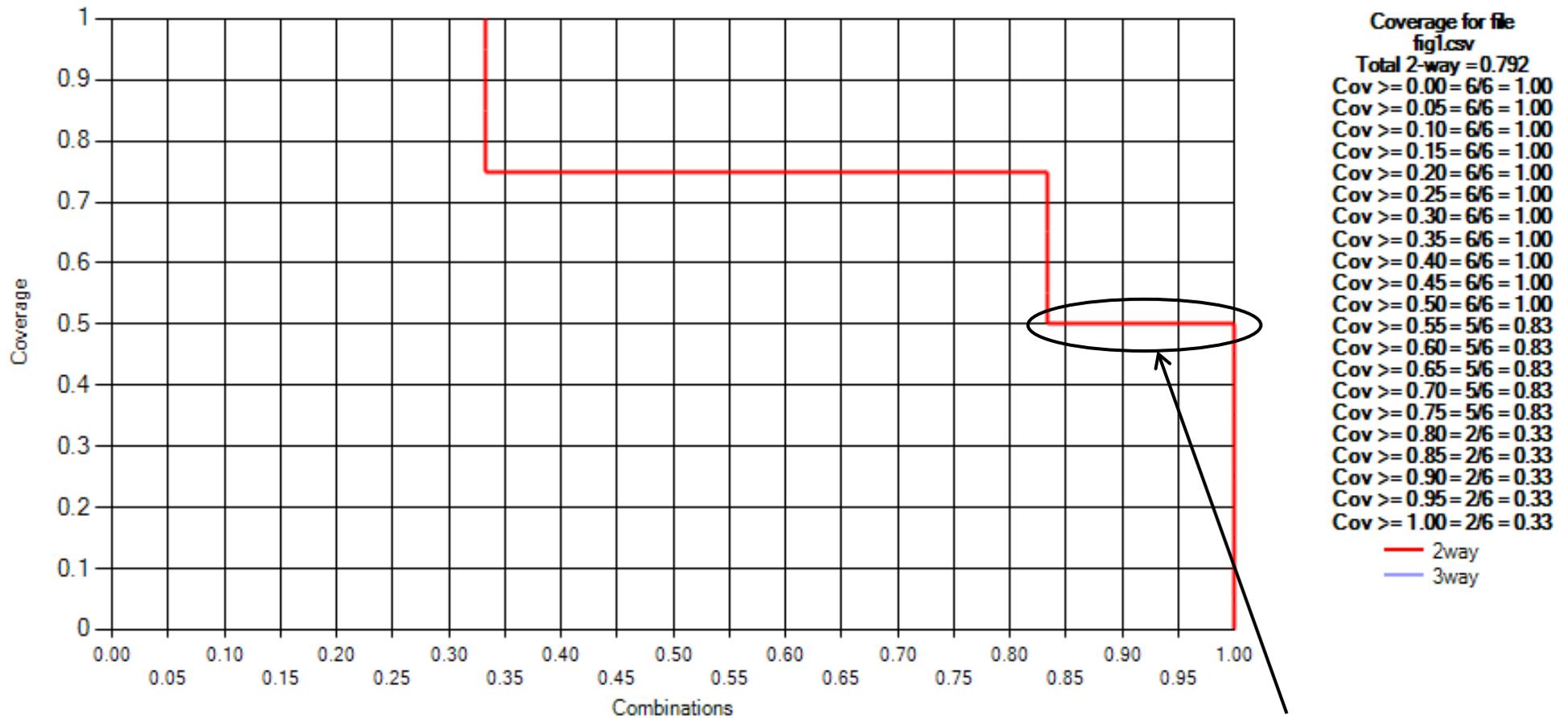
Variable pairs	Variable-value combinations covered	Coverage
<i>ab</i>	00, 01, 10	.75
<i>ac</i>	00, 01, 10	.75
<i>ad</i>	00, 01, 11	.75
<i>bc</i>	00, 11	.50
<i>bd</i>	00, 01, 10, 11	1.0
<i>cd</i>	00, 01, 10, 11	1.0

100% coverage of 33% of combinations

75% coverage of half of combinations

50% coverage of 16% of combinations

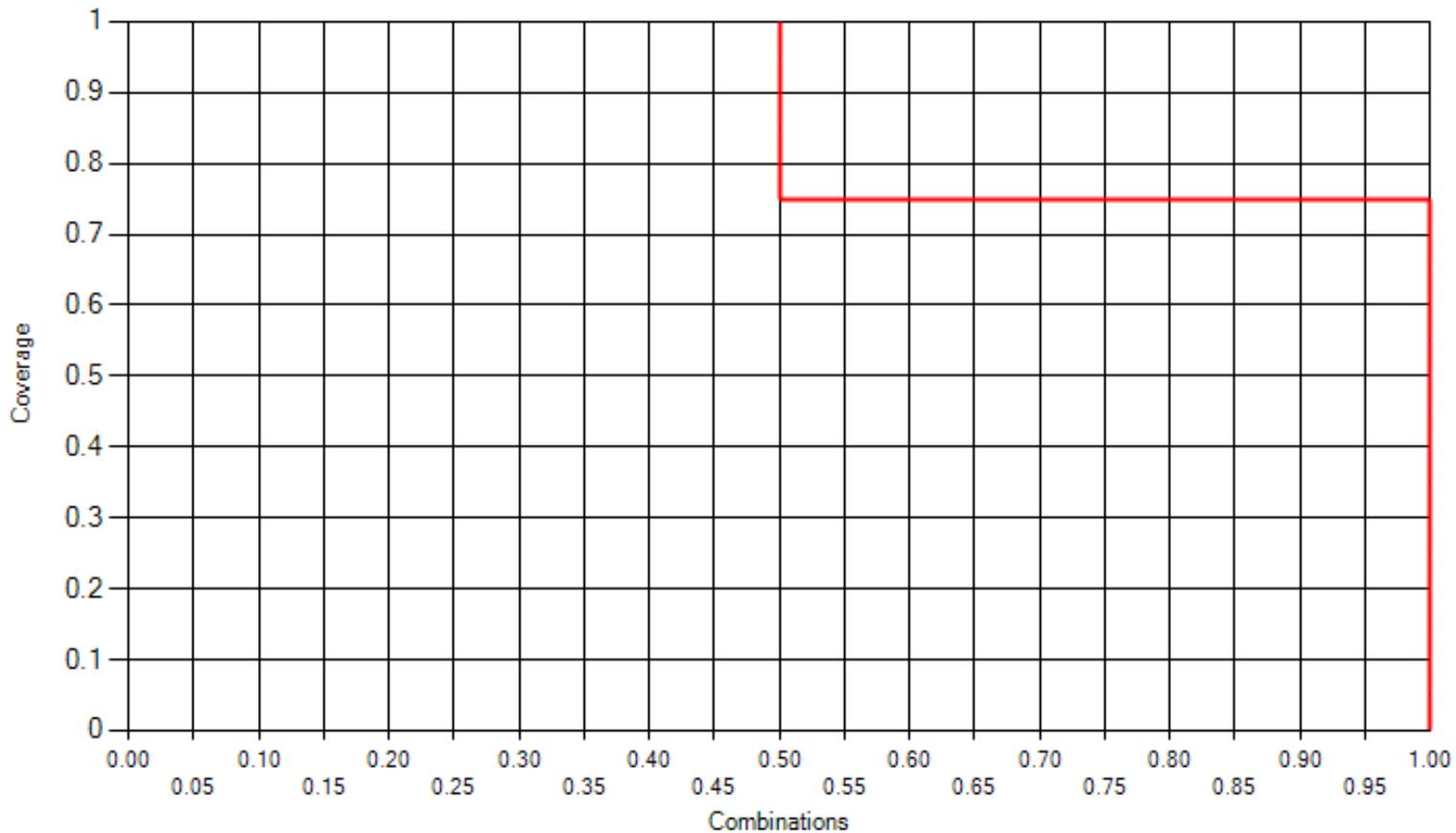
Graphing Coverage Measurement



100% coverage of 33% of combinations
75% coverage of half of combinations
50% coverage of 16% of combinations

Bottom line:
All combinations
covered to at least 50%

Adding a test

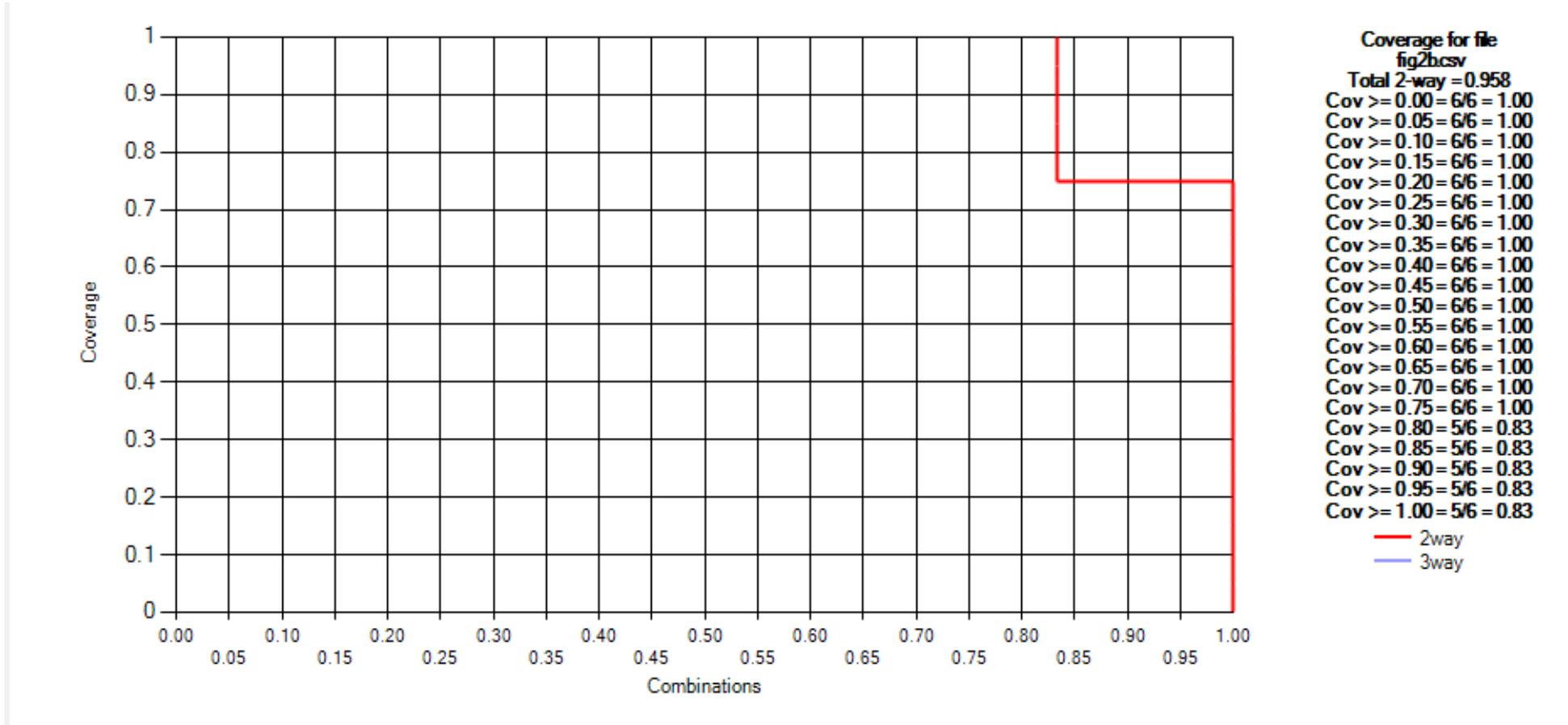


Coverage for file
fig2acsv
Total 2-way = 0.875
Cov >= 0.00 = 6/6 = 1.00
Cov >= 0.05 = 6/6 = 1.00
Cov >= 0.10 = 6/6 = 1.00
Cov >= 0.15 = 6/6 = 1.00
Cov >= 0.20 = 6/6 = 1.00
Cov >= 0.25 = 6/6 = 1.00
Cov >= 0.30 = 6/6 = 1.00
Cov >= 0.35 = 6/6 = 1.00
Cov >= 0.40 = 6/6 = 1.00
Cov >= 0.45 = 6/6 = 1.00
Cov >= 0.50 = 6/6 = 1.00
Cov >= 0.55 = 6/6 = 1.00
Cov >= 0.60 = 6/6 = 1.00
Cov >= 0.65 = 6/6 = 1.00
Cov >= 0.70 = 6/6 = 1.00
Cov >= 0.75 = 6/6 = 1.00
Cov >= 0.80 = 3/6 = 0.50
Cov >= 0.85 = 3/6 = 0.50
Cov >= 0.90 = 3/6 = 0.50
Cov >= 0.95 = 3/6 = 0.50
Cov >= 1.00 = 3/6 = 0.50

— 2way
— 3way

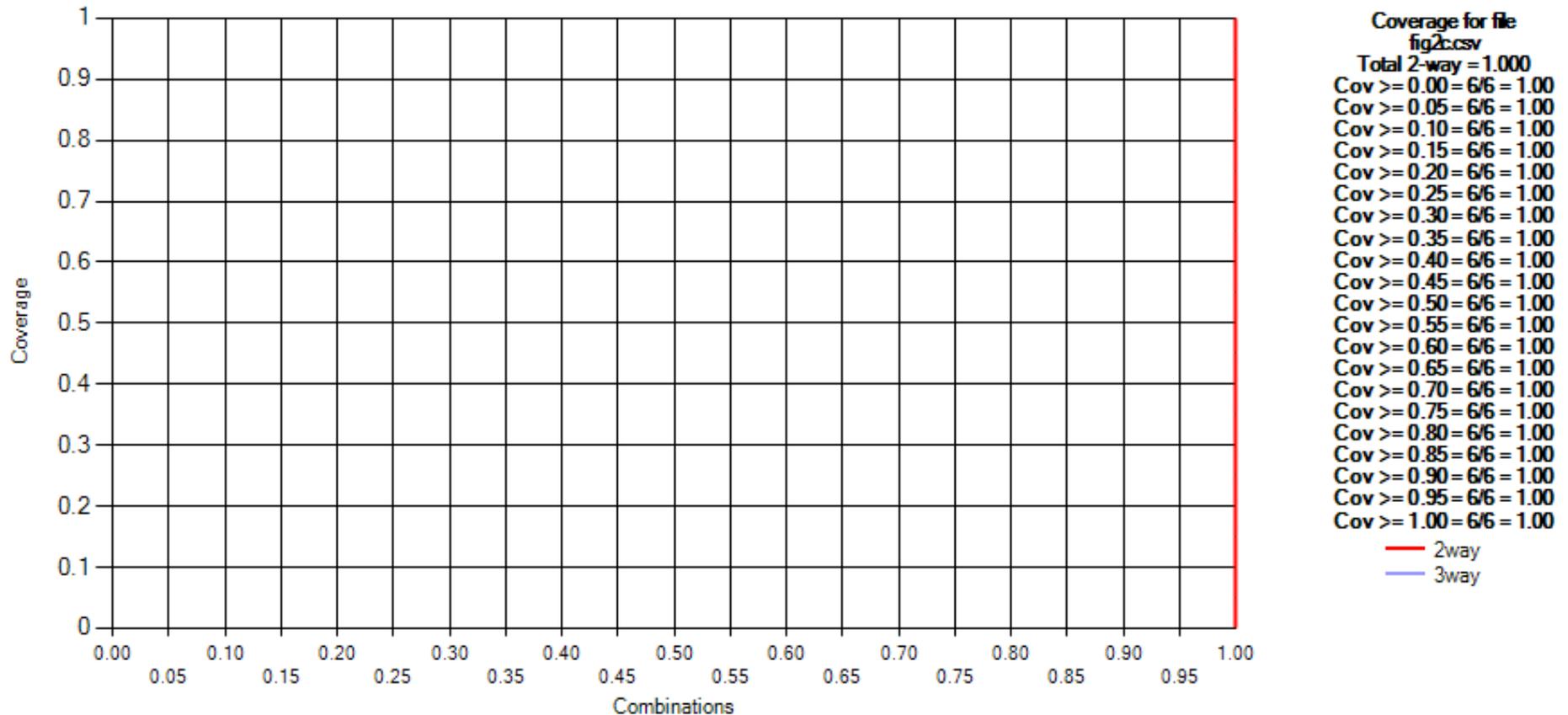
Coverage after adding test [1,1,0,1]

Adding another test



Coverage after adding test [1,0,1,1]

Additional test completes coverage



Coverage after adding test [1,0,1,0]
All combinations covered to 100% level,
so this is a covering array.

Combinatorial Coverage Measurement

NIST

Auto-detect N tests, N parms

Number of tests: 7489

Number of parameters: 82

Set number of tests and parameters

Load input file | Show input file

7489 tests, 82 parameters loaded

Compute 2-way coverage

Compute 3-way coverage

Clear chart | Save chart

Exit

Chart
X = proportion of combinations
Y = combination variable-value coverage

2 way stats:
Combinations: 3,321
Var/val coms: 14,761
Total coverage: 0.940

3 way stats:
Combinations: 88,560
Var/val coms: 828,135
Total coverage: 0.831

Combinatorial Coverage Measurement

Detect all values automatically Set boundaries for equivalence classes

Parameter 0 Detect Prev Next N classes 2 Set Boundary 0 = Save bound

Values for this parameter:
0.1

Cov >= 0.00	=	88560/88560	=	1.000
Cov >= 0.05	=	88560/88560	=	1.000
Cov >= 0.10	=	88560/88560	=	1.000
Cov >= 0.15	=	88560/88560	=	1.000
Cov >= 0.20	=	88560/88560	=	1.000
Cov >= 0.25	=	88559/88560	=	1.000
Cov >= 0.30	=	88547/88560	=	1.000
Cov >= 0.35	=	88505/88560	=	0.999
Cov >= 0.40	=	88380/88560	=	0.998
Cov >= 0.45	=	88041/88560	=	0.994
Cov >= 0.50	=	87762/88560	=	0.991
Cov >= 0.55	=	85766/88560	=	0.968
Cov >= 0.60	=	84969/88560	=	0.959
Cov >= 0.65	=	73116/88560	=	0.826
Cov >= 0.70	=	71208/88560	=	0.804
Cov >= 0.75	=	70391/88560	=	0.795
Cov >= 0.80	=	60191/88560	=	0.680
Cov >= 0.85	=	59154/88560	=	0.668
Cov >= 0.90	=	47532/88560	=	0.537
Cov >= 0.95	=	46880/88560	=	0.529
Cov >= 1.00	=	46869/88560	=	0.529

How do we automate checking correctness of output?



- **Creating test data is the easy part!**
- How do we check that the code worked correctly on the test input?
 - **Crash testing** server or other code to ensure it does not crash for any test input (like 'fuzz testing')
 - Easy but limited value
 - **Built-in self test with embedded assertions** – incorporate assertions in code to check critical states at different points in the code, or print out important values during execution
 - **Full scale model-checking** using mathematical model of system and model checker to generate expected results for each input - expensive but tractable

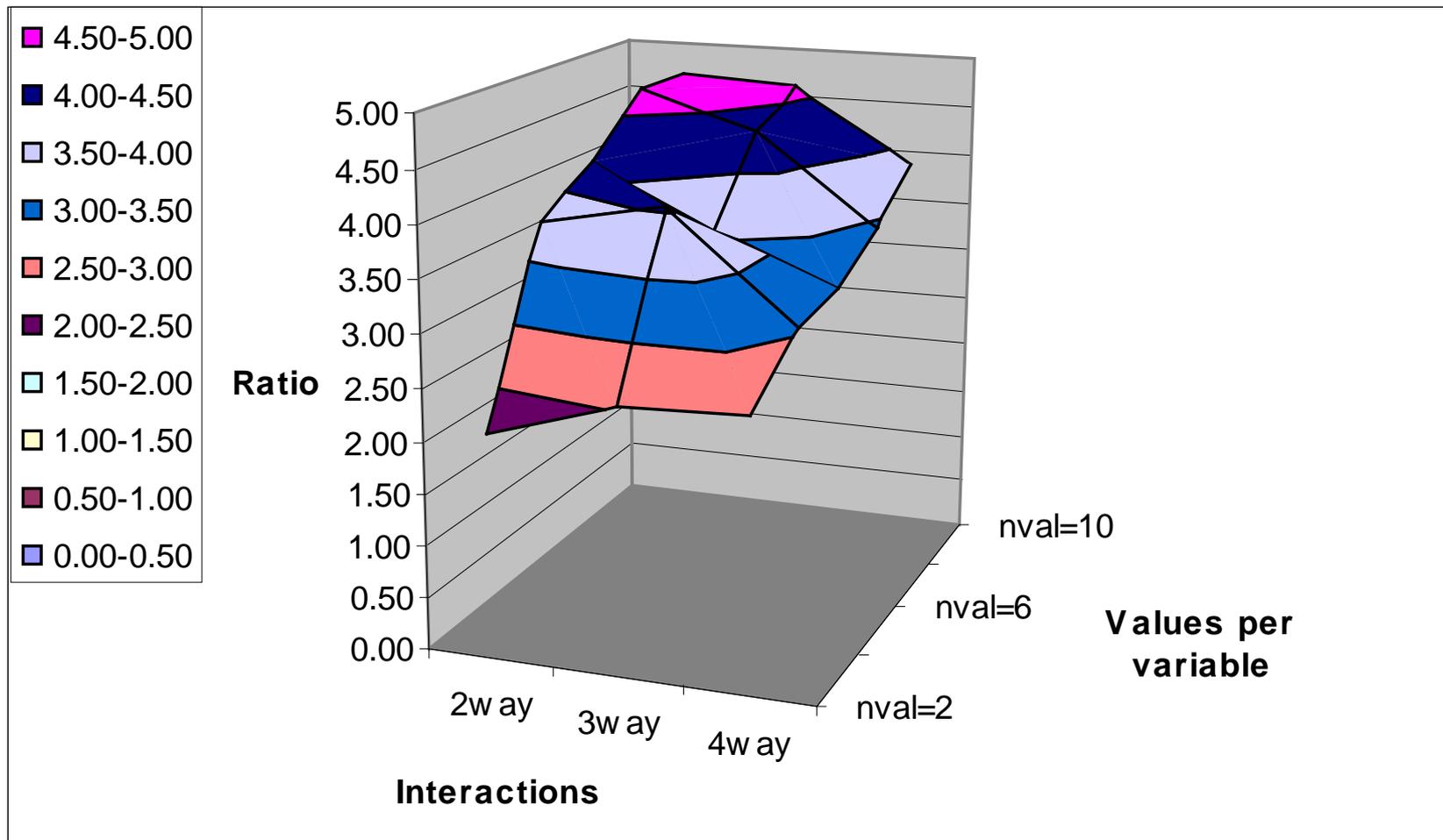
Crash Testing

- Like “fuzz testing” - send packets or other input to application, watch for crashes
- Unlike fuzz testing, input is non-random; cover all t-way combinations
- May be more efficient - random input generation requires several times as many tests to cover the t-way combinations in a covering array

Limited utility, but can detect high-risk problems such as:

- buffer overflows
- server crashes

Ratio of Random/Combinatorial Test Set Required to Provide t-way Coverage



Embedded Assertions

Simple example:

```
assert( x != 0); // ensure divisor is not zero
```

Or pre and post-conditions:

```
/requires amount >= 0;
```

```
/ensures balance == \old(balance) - amount &&  
\result == balance;
```

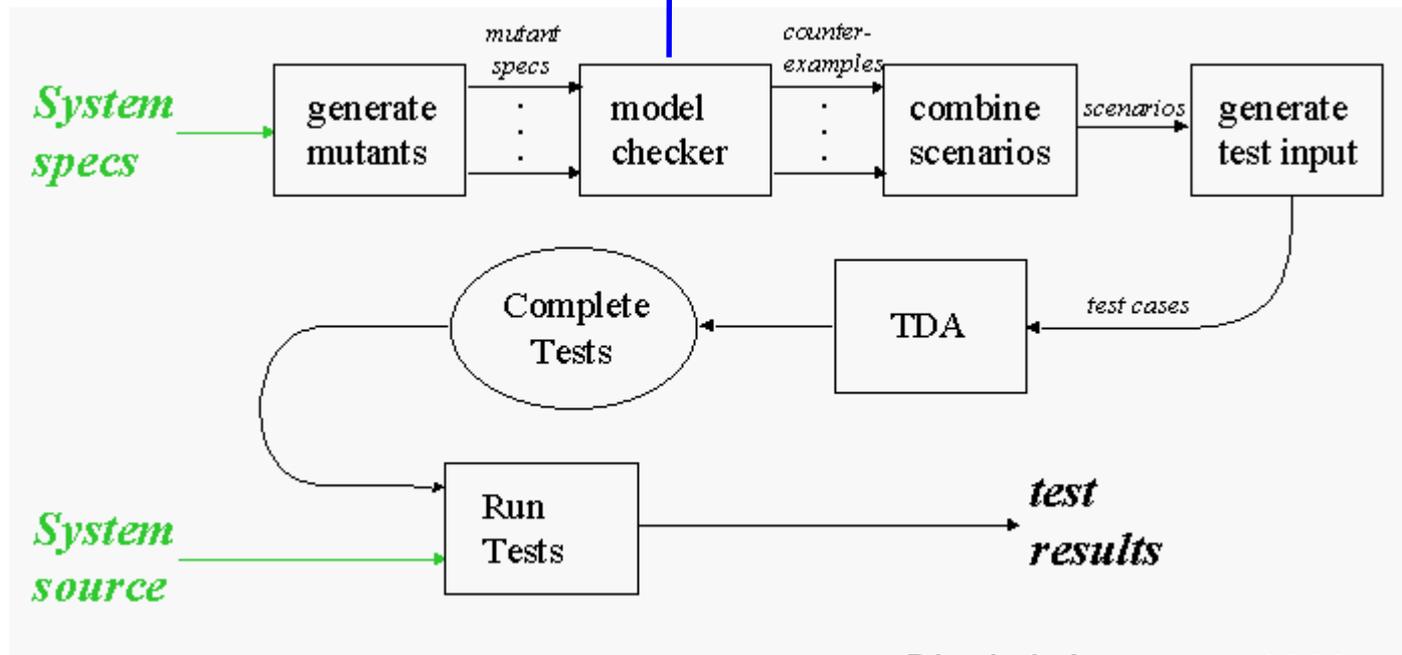
Embedded Assertions

Assertions check properties of expected result:

```
ensures balance == \old(balance) - amount  
&& \result == balance;
```

- Reasonable assurance that code works correctly across the range of expected inputs
- May identify problems with handling unanticipated inputs
- Example: Smart card testing
 - Used Java Modeling Language (JML) assertions
 - Detected 80% to 90% of flaws

Using model checking to produce tests



- Model-checker test production: if assertion is not true, then a counterexample is generated.

- This can be converted to a test case.

Model checking example

```
-- specification for a portion of tcas - altitude separation.
-- The corresponding C code is originally from Siemens Corp. Research
-- Vadim Okun 02/2002
MODULE main
VAR
  Cur_Vertical_Sep : { 299, 300, 601 };
  High_Confidence : boolean;
  ...
init(alt_sep) := START_;
next(alt_sep) := case
  enabled & (intent_not_known | !tcas_equipped) : case
    need_upward_RA & need_downward_RA : UNRESOLVED;
    need_upward_RA : UPWARD_RA;
    need_downward_RA : DOWNWARD_RA;
    1 : UNRESOLVED;
  esac;
  1 : UNRESOLVED;
esac;
...
SPEC AG ((enabled & (intent_not_known | !tcas_equipped) &
!need_downward_RA & need_upward_RA) -> AX (alt_sep = UPWARD_RA))
-- "FOR ALL executions,
-- IF enabled & (intent_not_known ....
-- THEN in the next state alt_sep = UPWARD_RA"
```

The usual logic operators, plus temporal logic

“FOR ALL executions,
IF enabled & (intent_not_known
THEN in the next state alt_sep = UPWARD_RA”

execution paths

states on the execution paths

**SPEC AG ((enabled & (intent_not_known | !tcas_equipped) &
!need_downward_RA & need_upward_RA)
-> AX (alt_sep = UPWARD_RA))**

(step-by-step explanation in combinatorial testing tutorial)

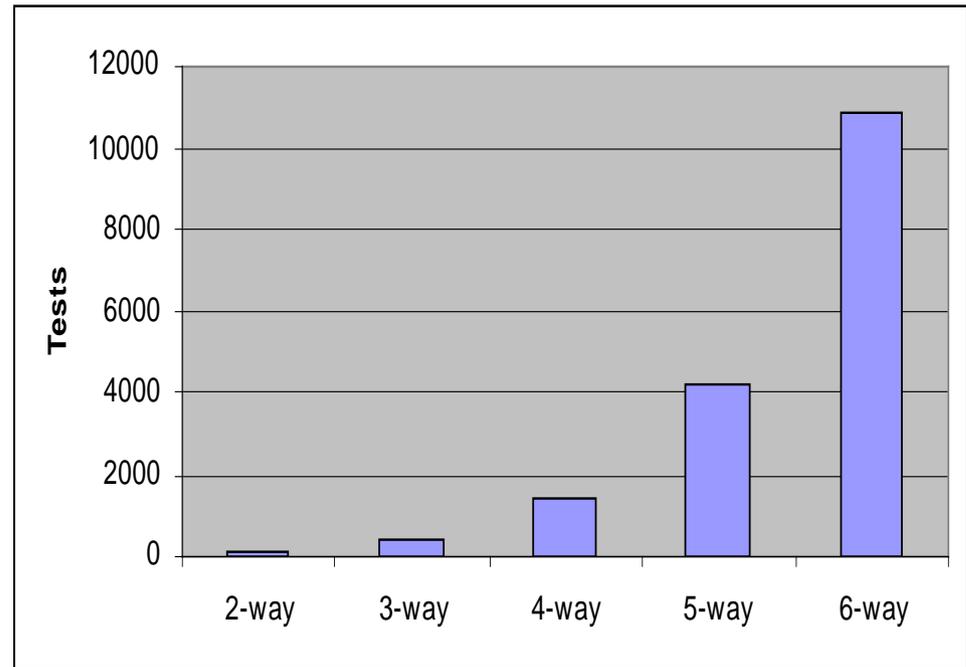
Testing inputs

- ✿ Traffic Collision Avoidance System (TCAS) module
 - Used in previous testing research
 - 41 versions seeded with errors
 - 12 variables: 7 boolean, two 3-value, one 4-value, two 10-value
 - All flaws found with 5-way coverage
 - Thousands of tests - generated by model checker in a few minutes



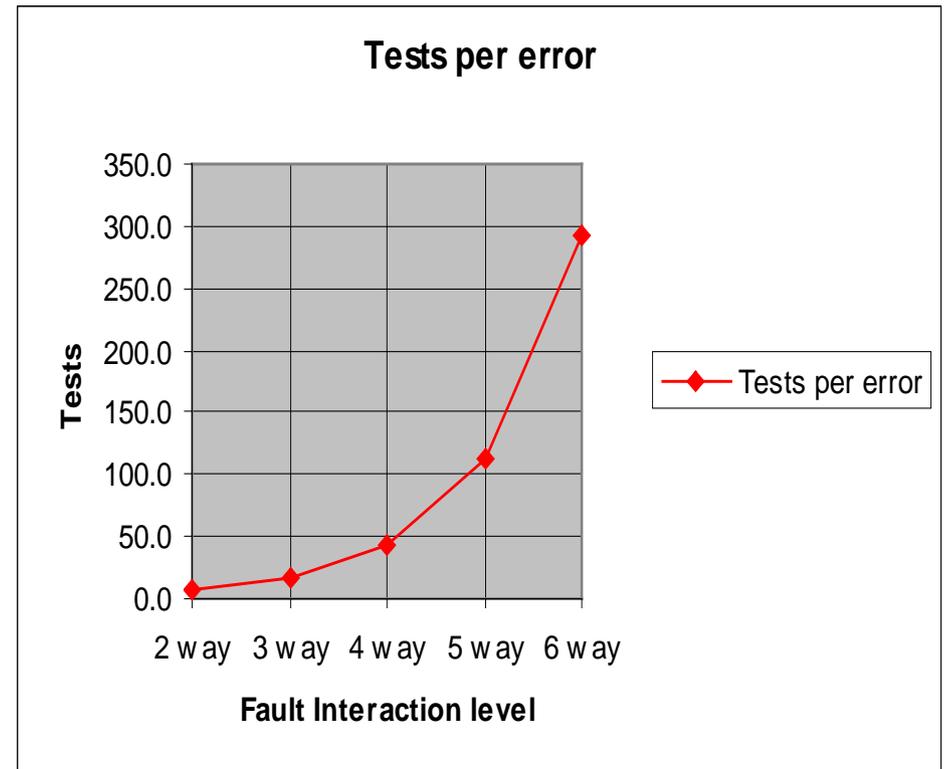
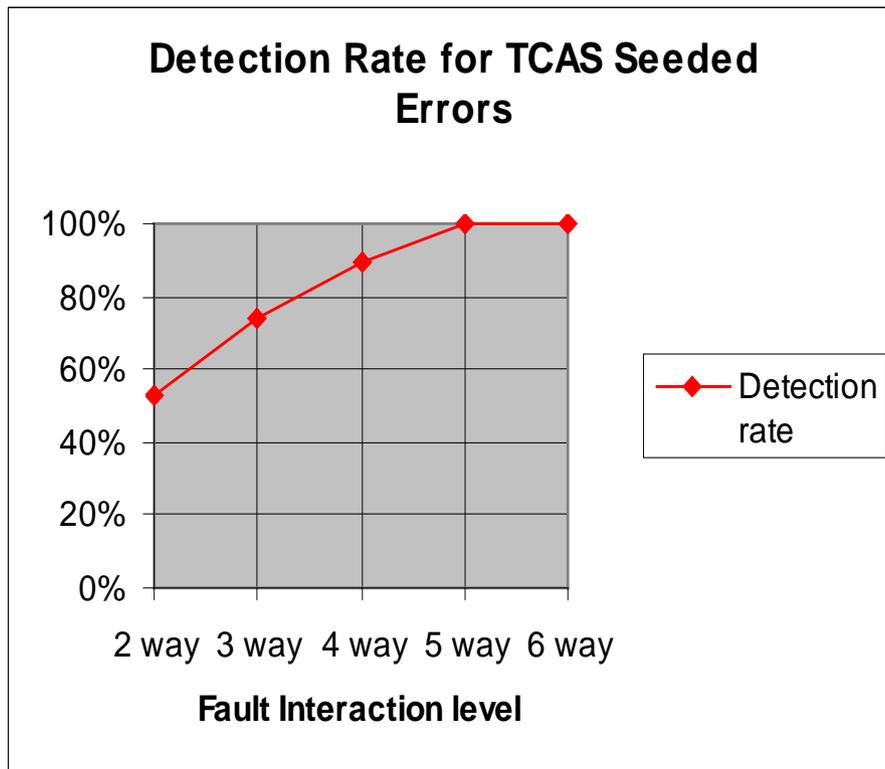
Tests generated

<i>t</i>	Test cases
2-way:	156
3-way:	461
4-way:	1,450
5-way:	4,309
6-way:	11,094



Results

- Roughly consistent with data on large systems
- But errors harder to detect than real-world examples



Bottom line for model checking based combinatorial testing:
Expensive but can be highly effective

Integrating into Testing Program

- Test suite development
 - Generate covering arrays for tests
OR
 - Measure coverage of existing tests
and supplement
- Training
 - Testing textbooks – Ammann & Offutt, Mathur
 - Combinatorial testing tutorial →
 - User manuals
 - Worked examples
 - **Coming soon – *Introduction to Combinatorial Testing* textbook**

NIST Special Publication 800-142

NIST
National Institute of
Standards and Technology
Technology Administration
U.S. Department of Commerce

INFORMATION SECURITY

PRACTICAL COMBINATORIAL TESTING

D. Richard Kuhn, Raghu N. Kacker, Yu Lei

October, 2010



U.S. Department of Commerce
Gary Locke, Secretary

National Institute of Standards and Technology
Patrick Gallagher, Director

Industrial Usage Reports

- Coverage measurement – Johns Hopkins Applied Physics Lab
- Sequence covering arrays, with US Air Force
- Cooperative Research & Development Agreement with Lockheed Martin - report 2012
- DOM Level 3 events conformance test – NIST
- New work with NASA IV&V



- NIST offers research opportunities for undergraduate students
- 11 week program
- Students paid \$5,500 stipend, plus housing and travel allotments as needed
- Competitive program supports approximately 100 students NIST-wide (approx. 20 in ITL)
- Open to US citizens who are undergraduate students or graduating seniors
- Closed for 2012, but apply for next year! (February)

<http://www.nist.gov/itl-surf-program.cfm>

Please contact us if you are interested.

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<http://csrc.nist.gov/acts>



NIST



NC STATE



Utah State
University



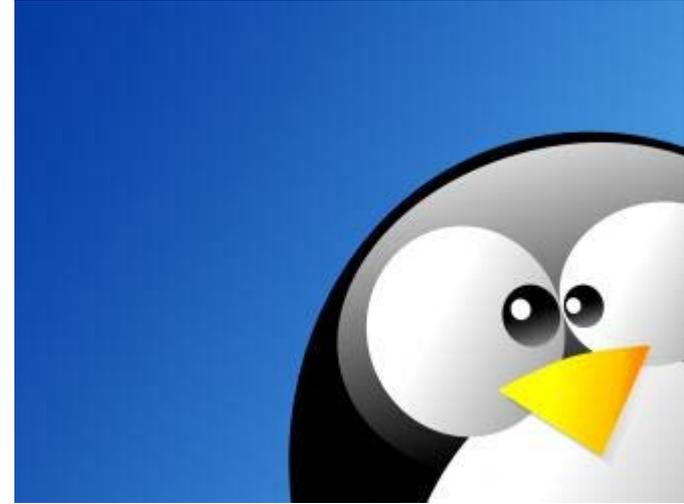
UMBC

APL
The Johns Hopkins University
Applied Physics Laboratory

U.S. AIR FORCE

LOCKHEED MARTIN
We never forget who we're working for™

Extra stuff



Example: GPS system

plug in GPS; ignition off; ignition on; boot screen; unplug GPS -> **screen locks**



What is NIST and why are we doing this?

- US Government agency, whose mission is to support US industry through developing better measurement and test methods
- 3,000 scientists, engineers, and support staff including 3 Nobel laureates
- Research in physics, chemistry, materials, manufacturing, computer science
- Trivia: NIST is one of the only federal agencies chartered in the Constitution (also DoD, Treasury, Census)



NIST

National Institute of
Standards and Technology

Four eras of evolution of DOE

Era 1:(1920's ...): Beginning in agricultural then animal science, clinical trials, medicine

Era 2:(1940's ...): Industrial productivity – new field, same basics

Era 3:(1980's ...): Designing robust products – new field, same basics

Then things begin to change . . .

Era 4:(2000's ...): Combinatorial Testing of Software

Tutorial Overview

1. Why are we doing this?
2. What is combinatorial testing?
3. What tools are available?
4. Is this stuff really useful in the real world?
- 5. What's next?**

Tradeoffs

- Advantages

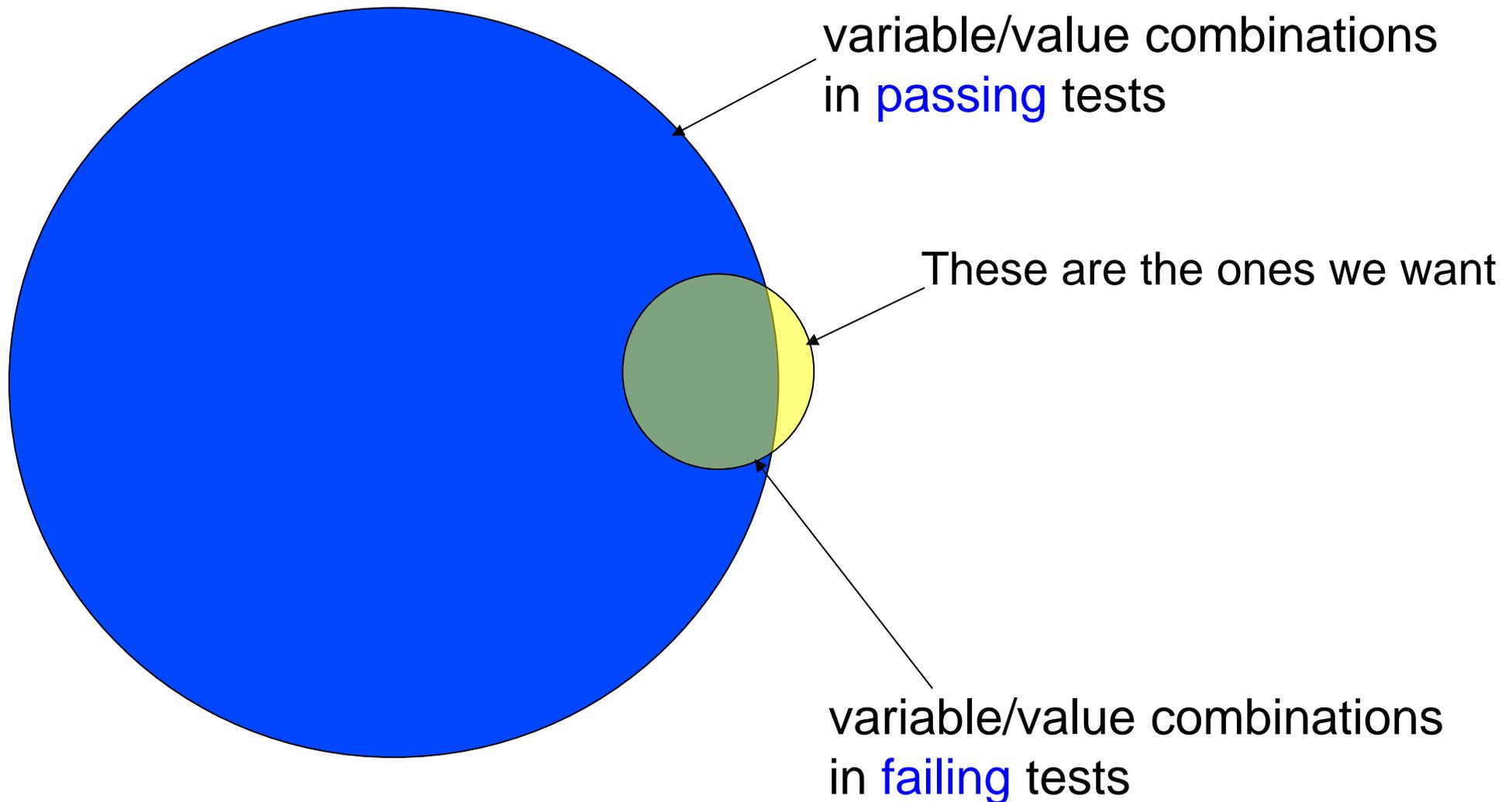
- Tests rare conditions
- Produces high code coverage
- Finds faults faster
- May be lower overall testing cost

- Disadvantages

- Expensive at higher strength interactions (>4-way)
- May require high skill level in some cases (if formal models are being used)

Fault location

Given: a set of tests that the SUT fails, which combinations of variables/values triggered the failure?

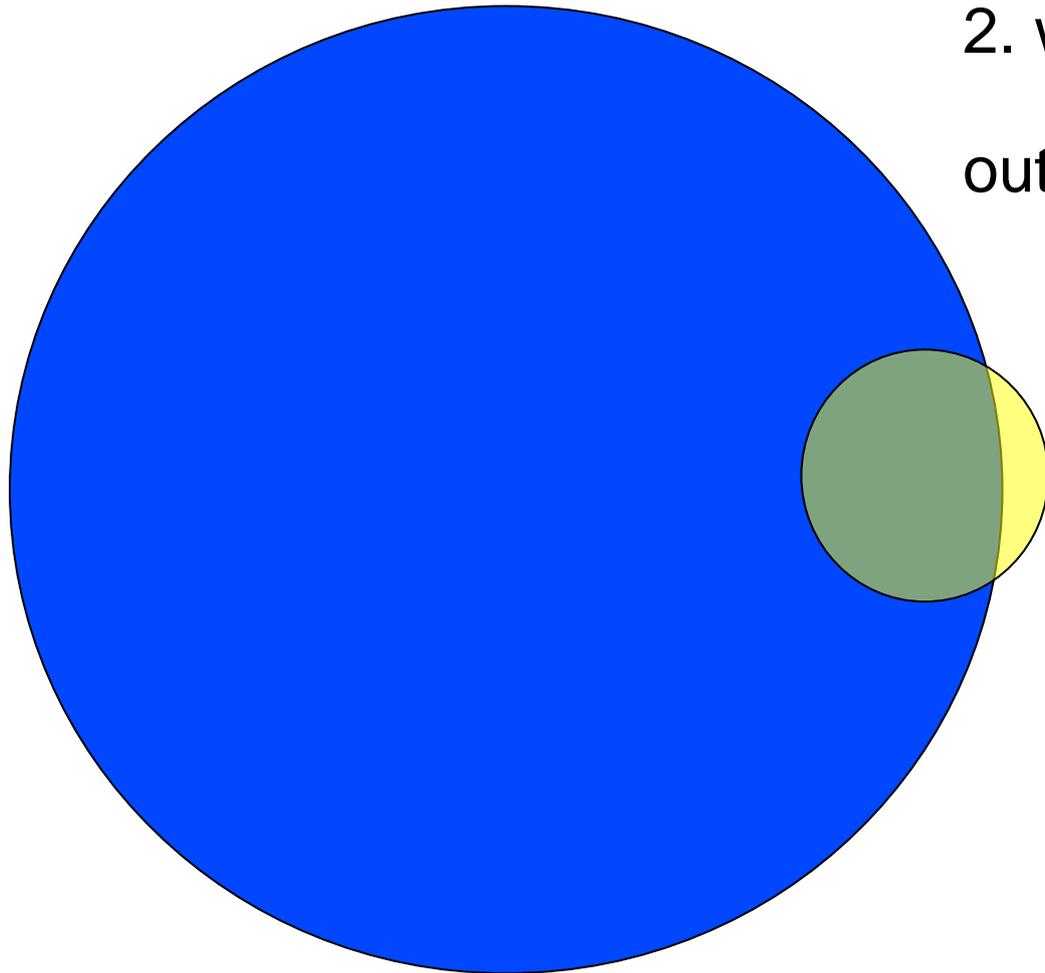


Fault location – what's the problem?

If they're in failing set but not in passing set:

1. which ones triggered the failure?
2. which ones don't matter?

out of $v^t \binom{n}{t}$ combinations



Example:

30 variables, 5 values each
= 445,331,250

5-way combinations

142,506 combinations
in each test

Background: Interaction Testing and Design of Experiments (DOE)

Complete sequence of steps to ensure appropriate data will be obtained, which permit objective analysis that lead to valid conclusions about cause-effect systems

Objectives stated ahead of time

Opposed to observational studies of nature, society ...

Minimal expense of time and cost

Multi-factor, not one-factor-at-a-time

DOE implies design and associated data analysis

Validity of inferences depends on design

A DOE plan can be expressed as matrix

Rows: tests, columns: variables, entries: test values or treatment allocations to experimental units

Agriculture and biological investigations-1

System under investigation

Crop growing, effectiveness of drugs or other treatments

Mechanistic (cause-effect) process; predictability limited

Variable Types

Primary test factors (farmer can adjust, drugs)

Held constant

Background factors (controlled in experiment, not in field)

Uncontrolled factors (Fisher's genius idea; randomization)

Numbers of treatments

Generally less than 10

Objectives: compare treatments to find better

Treatments: qualitative or discrete levels of continuous

Agriculture and biological investigations-2

Scope of investigation:

Treatments actually tested, direction for improvement

Key principles

Replication: minimize experimental error (which may be large)
replicate each test run; averages less variable than raw data

Randomization: allocate treatments to experimental units at
random; then error treated as draws from normal distribution

Blocking (homogeneous grouping of units): systematic effects
of background factors eliminated from comparisons

Designs: Allocate treatments to experimental units

Randomized Block designs, Balanced Incomplete Block
Designs, Partially balanced Incomplete Block Designs

Robust products-1

System under investigation

Design of product (or design of manufacturing process)

Variable Types

Control Factors: levels can be adjusted

Noise factors: surrogates for down stream conditions

AT&T-BL 1985 experiment with 17 factors was large

Objectives:

Find settings for robust product performance: product lifespan under different operating conditions across different units

Environmental variable, deterioration, manufacturing variation

Robust products-2

Scope of investigation:

Optimum levels of control factors at which variation from noise factors is minimum

Key principles

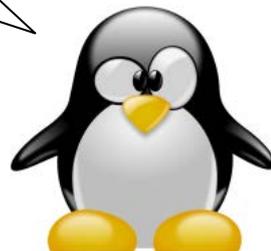
Variation from noise factors

Efficiency in testing; accommodate constraints

Designs: Based on Orthogonal arrays (OAs)

Taguchi designs (balanced 2-way covering arrays)

This stuff is great!
Let's use it for software!



What is the most effective way to integrate combinatorial testing with model checking?

- Given $AG(P \rightarrow AX(R))$
“for all paths, in every state,
if P then in the next state, R holds”
- For k-way variable combinations, $v1 \ \& \ v2 \ \& \ \dots \ \& \ v_k$
- v_i abbreviates “var1 = val1”
- Now combine this constraint with assertion to produce counterexamples. Some possibilities:

1. $AG(v1 \ \& \ v2 \ \& \ \dots \ \& \ v_k \ \& \ P \rightarrow AX \ !(R))$

2. $AG(v1 \ \& \ v2 \ \& \ \dots \ \& \ v_k \rightarrow AX \ !(1))$

3. $AG(v1 \ \& \ v2 \ \& \ \dots \ \& \ v_k \rightarrow AX \ !(R))$

What happens with these assertions?

1. $AG(v1 \ \& \ v2 \ \& \ \dots \ \& \ vk \ \& \ P \ \rightarrow \ AX \ !(R))$

P may have a negation of one of the v_i , so we get

$0 \ \rightarrow \ AX \ !(R)$

always true, so no counterexample, no test.

This is too restrictive!

1. $AG(v1 \ \& \ v2 \ \& \ \dots \ \& \ vk \ \rightarrow \ AX \ !(1))$

The model checker makes non-deterministic choices for variables not in $v1..vk$, so all R values may not be covered by a counterexample.

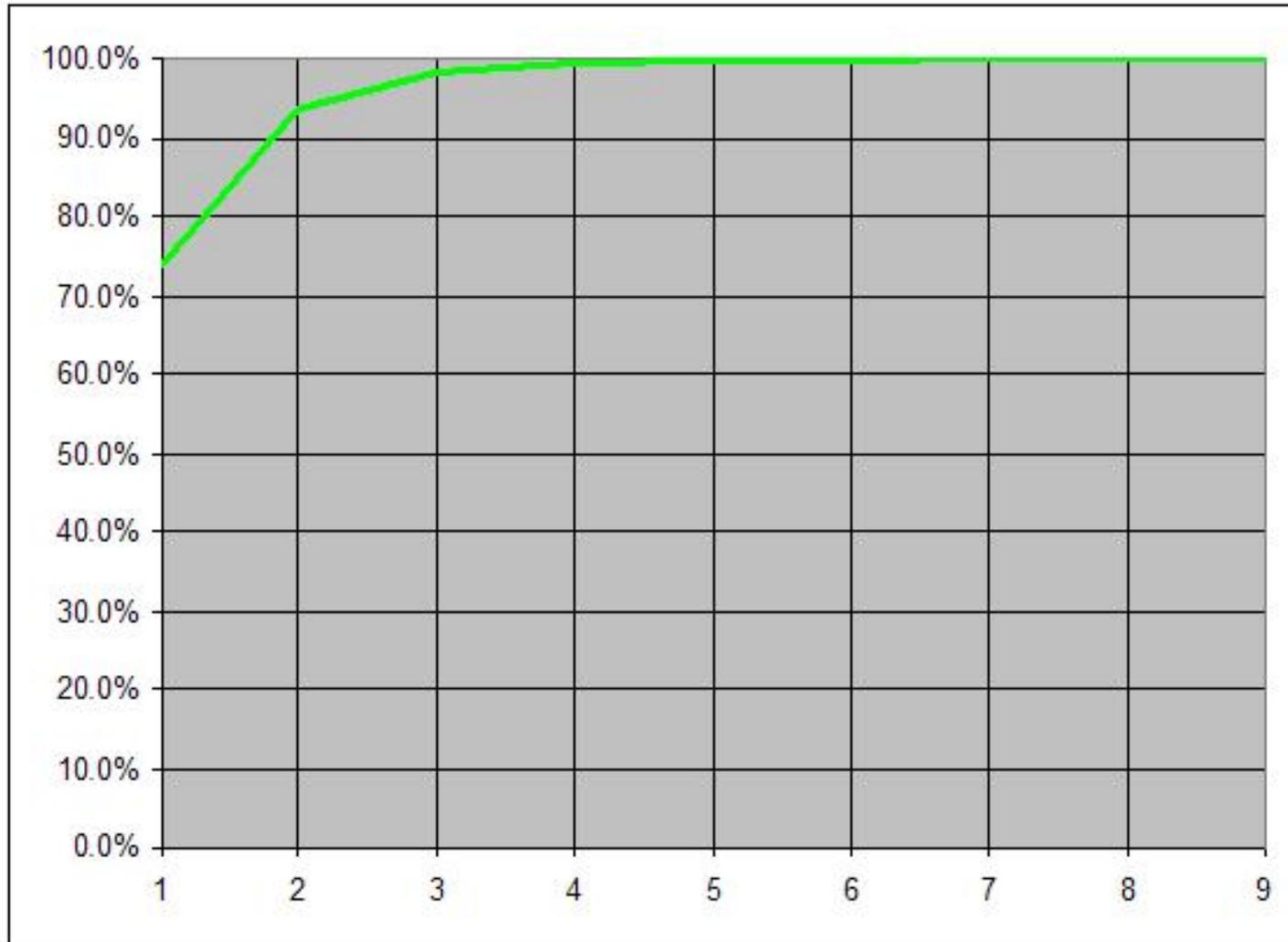
This is too loose!

2. $AG(v1 \ \& \ v2 \ \& \ \dots \ \& \ vk \ \rightarrow \ AX \ !(R))$

Forces production of a counterexample for each R.

This is just right!

What causes this distribution?



One clue: branches in avionics software.
7,685 expressions from *if* and *while* statements

Evolution of
design of experiments (DOE)
to
combinatorial testing of
software and systems using
covering arrays

Design of Experiments (DOE)

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Early history

Scottish physician James Lind determined cure of scurvy

Ship HM Bark Salisbury in 1747

12 sailors “were as similar as I could have them”

6 treatments 2 each

Principles used (blocking, replication, randomization)

Theoretical contributor of basic ideas: Charles S Peirce

American logician, philosopher, mathematician

1839-1914, Cambridge, MA

Father of DOE: R A Fisher, 1890-1962, British geneticist

Rothamsted Experiment Station, Hertfordshire, England

Four eras of evolution of DOE

Era 1:(1920's ...): Beginning in agricultural then animal science, clinical trials, medicine

Era 2:(1940's ...): Use for industrial productivity

Era 3:(1980's ...): Use for designing robust products

Era 4:(2000's ...): Combinatorial Testing of Software

Hardware-Software systems, computer security, assurance of access control policy implementation (health care records), verification and validations of simulations, optimization of models, testing of cloud computing applications, platform, and infrastructure

Features of DOE

1. System under investigation
2. Variables (input, output and other), test settings
3. Objectives
4. Scope of investigation
5. Key principles
6. Experiment plans
7. Analysis method from data to conclusions
8. Some leaders (subjective, hundreds of contributors)

Agriculture and biological investigations-1

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Agriculture and biological investigations-3

Analysis method from data to conclusions

Simple statistical model for treatment effects

ANOVA (Analysis of Variance)

Significant factors among primary factors; better test settings

Some of the leaders

R A Fisher, F Yates, ...

G W Snedecor, C R Henderson*, Gertrude Cox, ...

W G Cochran*, Oscar Kempthorne*, D R Cox*, ...

Other: Double-blind clinical trials, biostatistics and medical application at forefront

Industrial productivity-1

System under investigation

Chemical production process, manufacturing processes

Mechanistic (cause-effect) process; predictability medium

Variable Types:

Not allocation of treatments to units

Primary test factors: process variables levels can be adjusted

Held constant

Continue to use terminology from agriculture

Generally less than 10

Objectives:

Identify important factors, predict their optimum levels

Estimate response function for important factors

Industrial productivity-2

Scope of investigation:

Optimum levels in range of possible values (beyond levels actually used)

Key principles

Replication: Necessary

Randomization of test runs: Necessary

Blocking (homogeneous grouping): Needed less often

Designs: Test runs for chosen settings

Factorial and Fractional factorial designs

Latin squares, Greco-Latin squares

Central composite designs, Response surface designs

Industrial productivity-3

Analysis method from data to conclusions

Estimation of linear or quadratic statistical models for relation between factor levels and response

Linear ANOVA or regression models

Quadratic response surface models

Factor levels

Chosen for better estimation of model parameters

Main effect: average effect over level of all other factors

2-way interaction effect: how effect changes with level of another

3-way interaction effect: how 2-way interaction effect changes; often regarded as error

Estimation requires balanced DOE

Some of the leaders

G. E. P. Box*, G. J. Hahn*, C. Daniel, C. Eisenhart*,...

Robust products-1

System under investigation

Design of product (or design of manufacturing process)

Variable Types

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Designs: Based on Orthogonal arrays (OAs)

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Analysis method from data to conclusions

Pseudo-statistical analysis

Signal-to-noise ratios, measures of variability

Some of the leaders: Genichi Taguchi

Use of OAs for software testing

Functional (black-box) testing

- Hardware-software systems

- Identify single and 2-way combination faults

Early papers

- Taguchi followers (mid1980's)

- Mandl (1985) Compiler testing

- Tatsumi et al (1987) Fujitsu

- Sacks et al (1989) Computer experiments

- Brownlie et al (1992) AT&T

Generation of test suites using OAs

- OATS (Phadke*, AT&T-BL)

Combinatorial Testing of Software and Systems -1

System under investigation

Hardware-software systems combined or separately

Mechanistic (cause-effect) process; predictability full (high)

Output unchanged (or little changed) in repeats

Configurations of system or inputs to system

Variable Types: test-factors and held constant

Inputs and configuration variables having more than one option

No limit on variables and test setting

Identification of factors and test settings

Which could trigger malfunction, boundary conditions

Understand functionality, possible modes of malfunction

Objectives: Identify t -way combinations of test setting of any t out of k factors in tests actually conducted which trigger malfunction;

$t \ll k$

Combinatorial Testing of Software and Systems -2

Scope of investigation:

Actual t -way (and higher) combinations tested; no prediction

Key principles: no background no uncontrolled factors

No need of blocking and randomization

No need of replication; greatly decrease number of test runs

Investigation of actual faults suggests: $1 < t < 7$

Complex constraints between test settings (depending on possible paths software can go through)

Designs: Covering arrays cover all t -way combinations

Allow for complex constraints

Other DOE can be used; CAs require fewer tests (exception when OA of index one is available which is best CA)

'Interaction' means number of variables in combination (not estimate of parameter of statistical model as in other DOE)

Combinatorial Testing of Software and Systems -3

Analysis method from data to conclusions

No statistical model for test setting-output relationship; no prediction

No estimation of statistical parameters (main effects, interaction effects)

Test suite need not be balanced; covering arrays unbalanced

Often output is $\{0,1\}$

Need algorithms to identify fault triggering combinations

Some leaders

AT&T-BL alumni (Neil Sloan*), Charlie Colbourn* (AzSU) ...

NIST alumni/employees (Rick Kuhn*), Jeff Yu Lei* (UTA/NIST)

Other applications

Assurance of access control policy implementations

Computer security, health records

Components of combinatorial testing

Problem set up: identification of factors and settings

Test run: combination of one test setting for each factor

Test suite generation, high strength, constraints

Test execution, integration in testing system

Test evaluation / expected output oracle

Fault localization

Generating test suites based on CAs

CATS (Bell Labs), AETG (BellCore-Telcordia)

IPO (Yu Lei) led to ACTS (IPOG, ...)

Tconfig (Ottawa), CTGS (IBM), TOG (NASA),...

Jenny (Jenkins), TestCover (Sherwood),...

PICT (Microsoft),...

ACTS (NIST/UTA) free, open source intended

Effective efficient for t -way combinations for $t = 2, 3, 4, 5, 6, \dots$

Allow complex constraints

Mathematics underlying DOE/CAs

1829-32 Évariste Galois (French, shot in duel at age 20)

1940's R. C. Bose (father of math underlying DOE)

1947 C. R. Rao* (concept of orthogonal arrays)

Hadamard (1893), RC Bose, KA Bush, Addelman, Taguchi,

1960's G. Taguchi* (catalog of OAs, industrial use)

Covering arrays (Sloan* 1993) as math objects

Renyi (1971, probabilist, died at age 49)

Roux (1987, French, disappeared leaving PhD thesis)

Katona (1973), Kleitman and Spencer (1973), Sloan* (1993),

CAs connection to software testing: key papers

Dalal* and Mallows* (1997), Cohen, Dalal, Fredman, Patton(1997),
Alan Hartman* (2003), ...

Catalog of Orthogonal Arrays (N J A Sloan*, AT&T)

Sizes of Covering Arrays (C J Colbourn*, AzSU)

Concluding remarks

DOE: approach to gain information to improve things

Combinatorial Testing is a special kind of DOE

Chosen input \rightarrow function \rightarrow observe output

Highly predictable system; repeatability high understood

Input space characterized in terms of factors, discrete settings

Critical event when certain t -way comb encountered $t \ll k$

Detect such t -way combinations or assure absence

Exhaustive testing of all k -way combinations not practical

No statistical model assumed

Unbalanced test suites

Smaller size test suites than other DOE plans, which can be used

Many applications