

# Automated Search for Round 1 Differentials for SHA-1

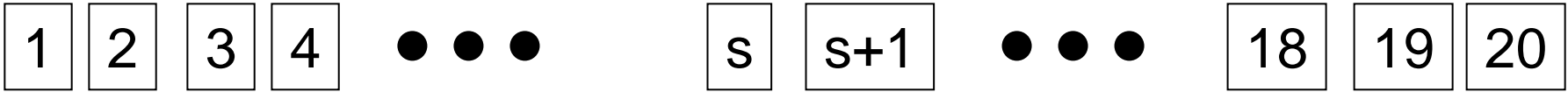
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# Motivation for Research

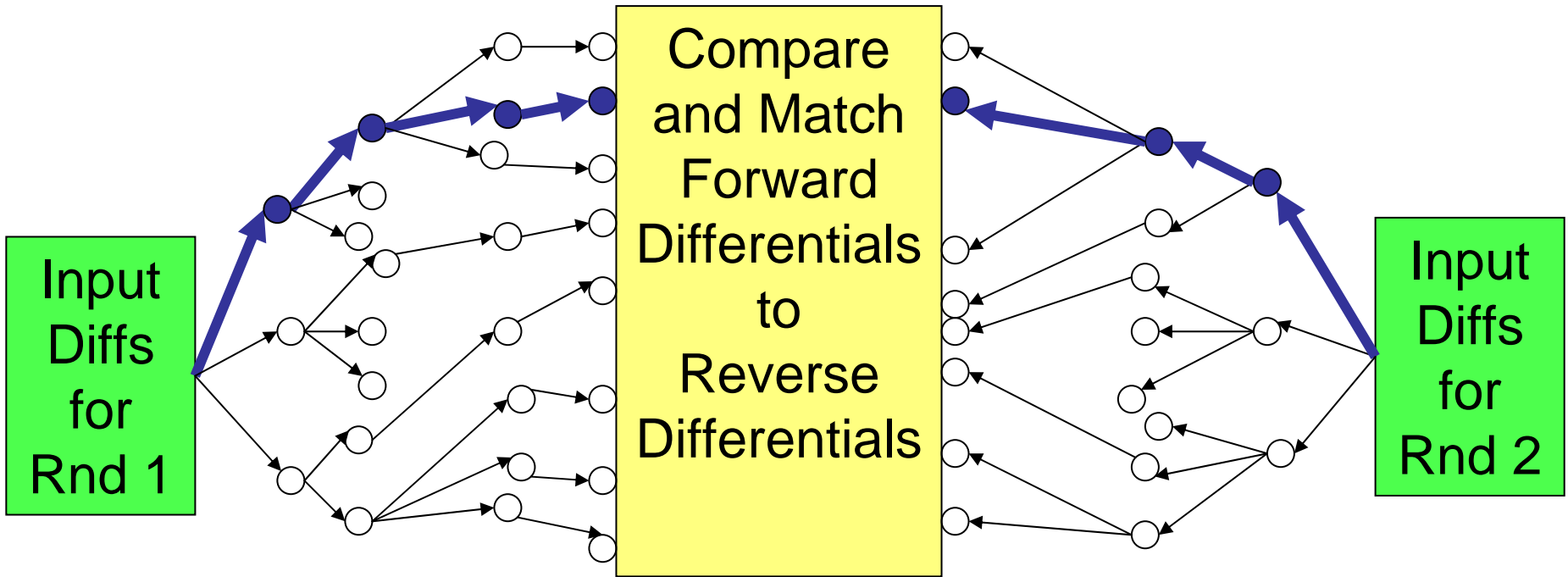
- Given:
  - Disturbance vector (XOR diffs in msg words),
  - Input difference to Round 1,
  - Input difference for Round 2, ...
- ...is there a differential path?
- Which Round 1 differential path is optimal?
  - E.g. improvements to MD5 attacks
- How do we find optimal paths?
  - **Automate search!**

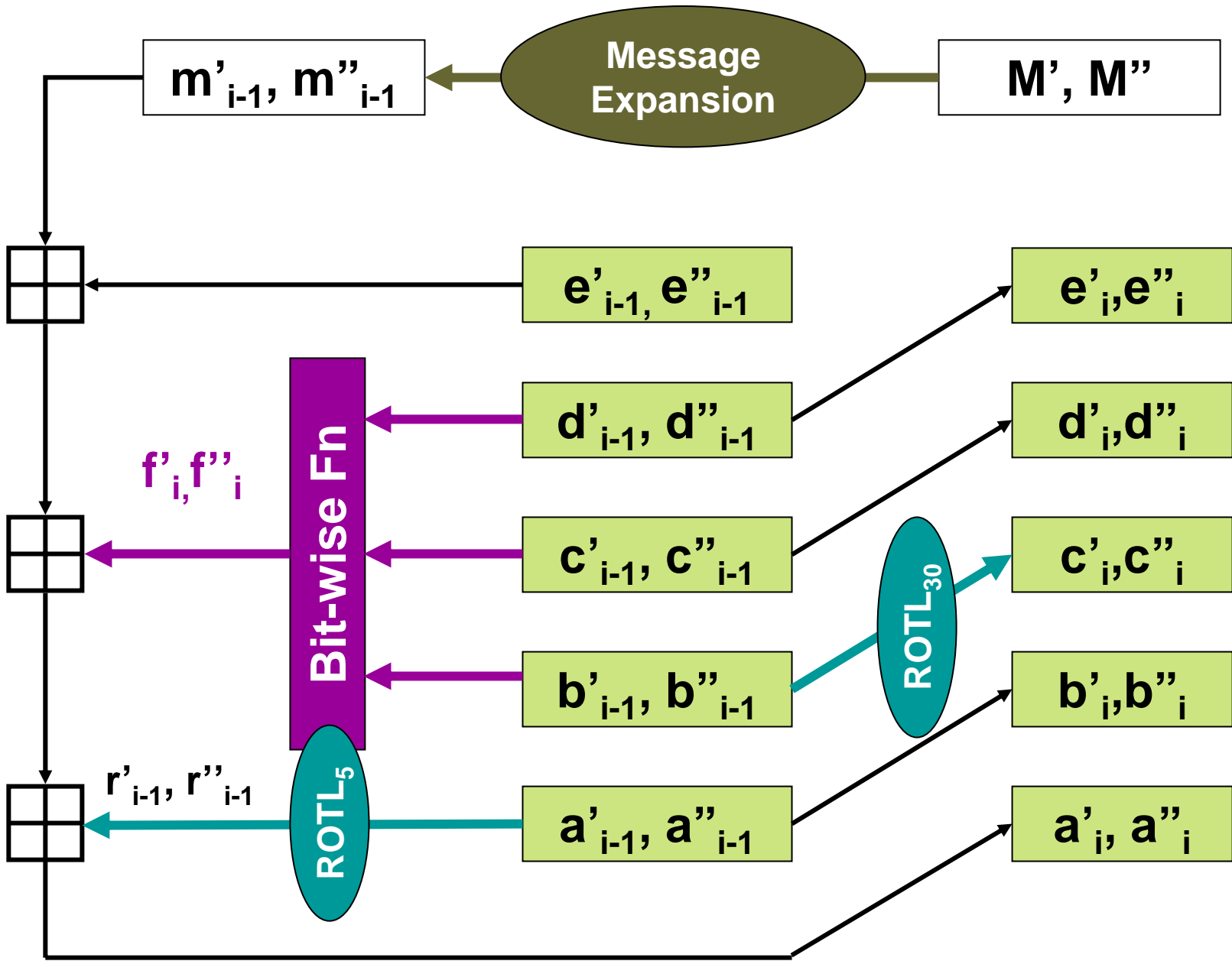


Generate set of FORWARD Differentials Steps 1 to s

Generate set of REVERSE Differentials Steps 20 to (s+1)

Sequence of XOR Diffs  $\Delta_{\oplus m}$  for Steps 1-20





# ADD & XOR Differences

- **ADD difference**

$$-\Delta_+X = X'' - X' \pmod{2^{32}}$$

- **XOR Difference**

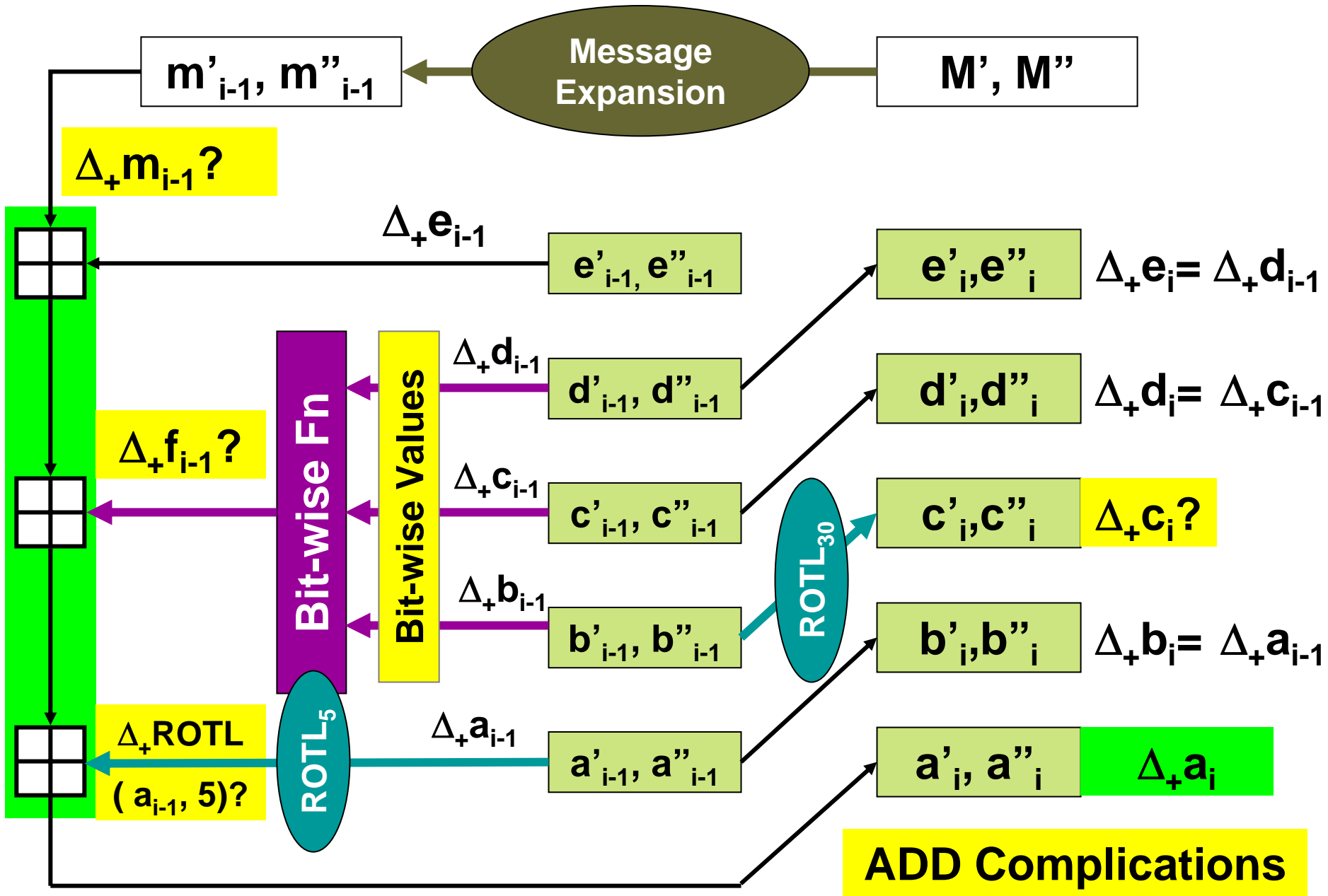
$$-\Delta_{\oplus}X = X'' \oplus X'$$

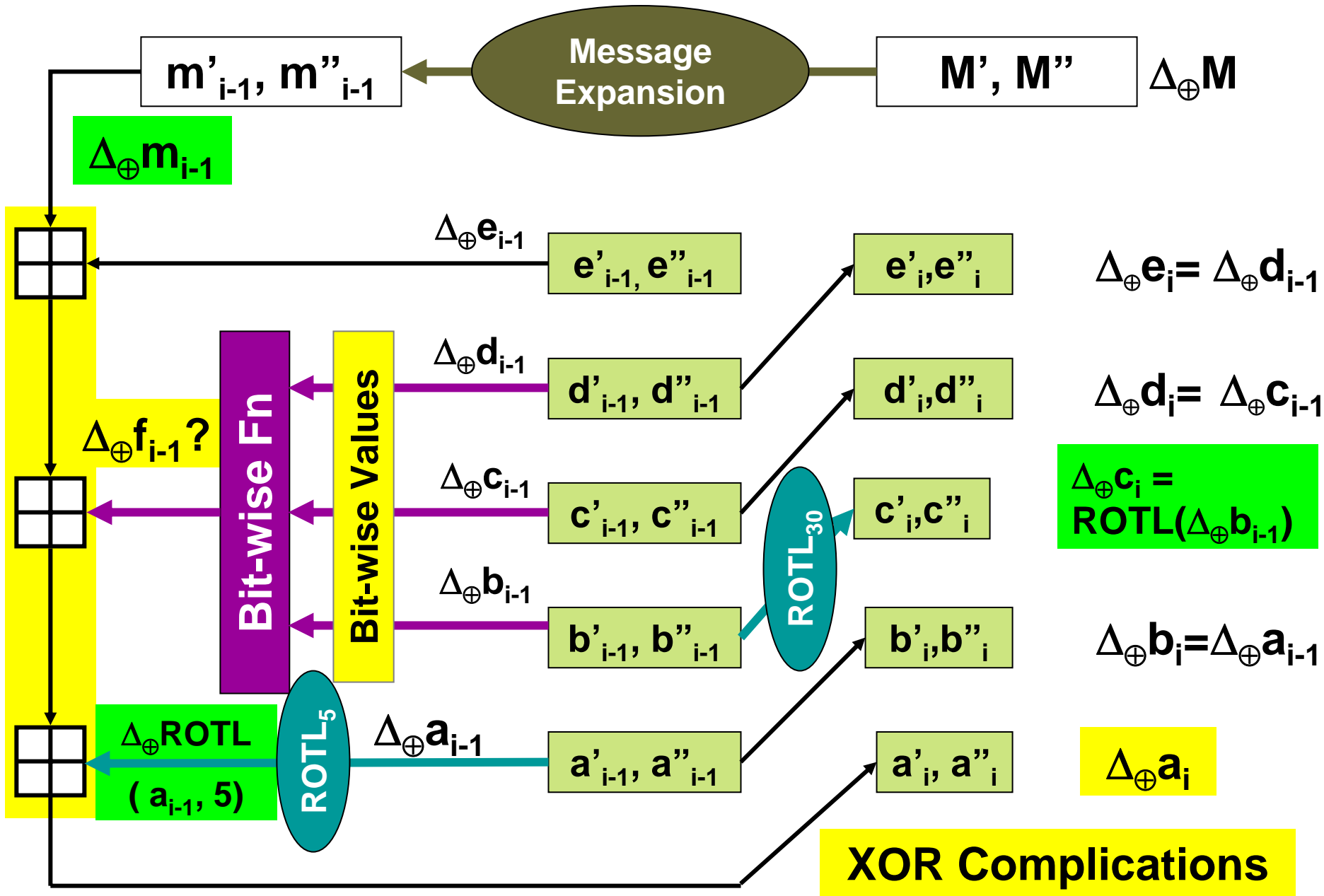
- **Properties**

$$-\Delta_+(X+Y) = \Delta_+X + \Delta_+Y$$

$$-\Delta_{\oplus}(X \oplus Y) = \Delta_{\oplus}X \oplus \Delta_{\oplus}Y$$

$$-\Delta_{\oplus}\text{ROTL}(X, r) = \text{ROTL}(\Delta_{\oplus}X, r): r \text{ fixed}$$





# Nabla representation $\nabla X$

- $\nabla X[j] =$ 
  - @ if  $X''[j] \neq X'[j]$
  - + if  $(X''[j], X'[j]) = (1, 0) \leftrightarrow X''[j] - X'[j] = +1$
  - - if  $(X''[j], X'[j]) = (0, 1) \leftrightarrow X''[j] - X'[j] = -1$
  - \* if  $X''[j] = X'[j]$
  - 0 if  $X''[j] = X'[j] = 0$
  - 1 if  $X''[j] \neq X'[j] = 1$
- $\Delta_+ X = \sum_{+,-} \nabla X[j] 2^j$



# Example

Bit 3322222222111111111  
 10987654321098765432109876543210

$X' = 001110101010101000101011101010101000$

$X'' = 10101010011010100101011100100101000$

$\nabla X = +01-1010-+1010-+010110-+-0101000$

$\Delta_{\oplus} = 10010000110000011000000011100000000$

$\Delta_{+} = +2^{31} -2^{28} -2^{23} +2^{22} -2^{17} +2^{16} -2^9 +2^8 -2^7$

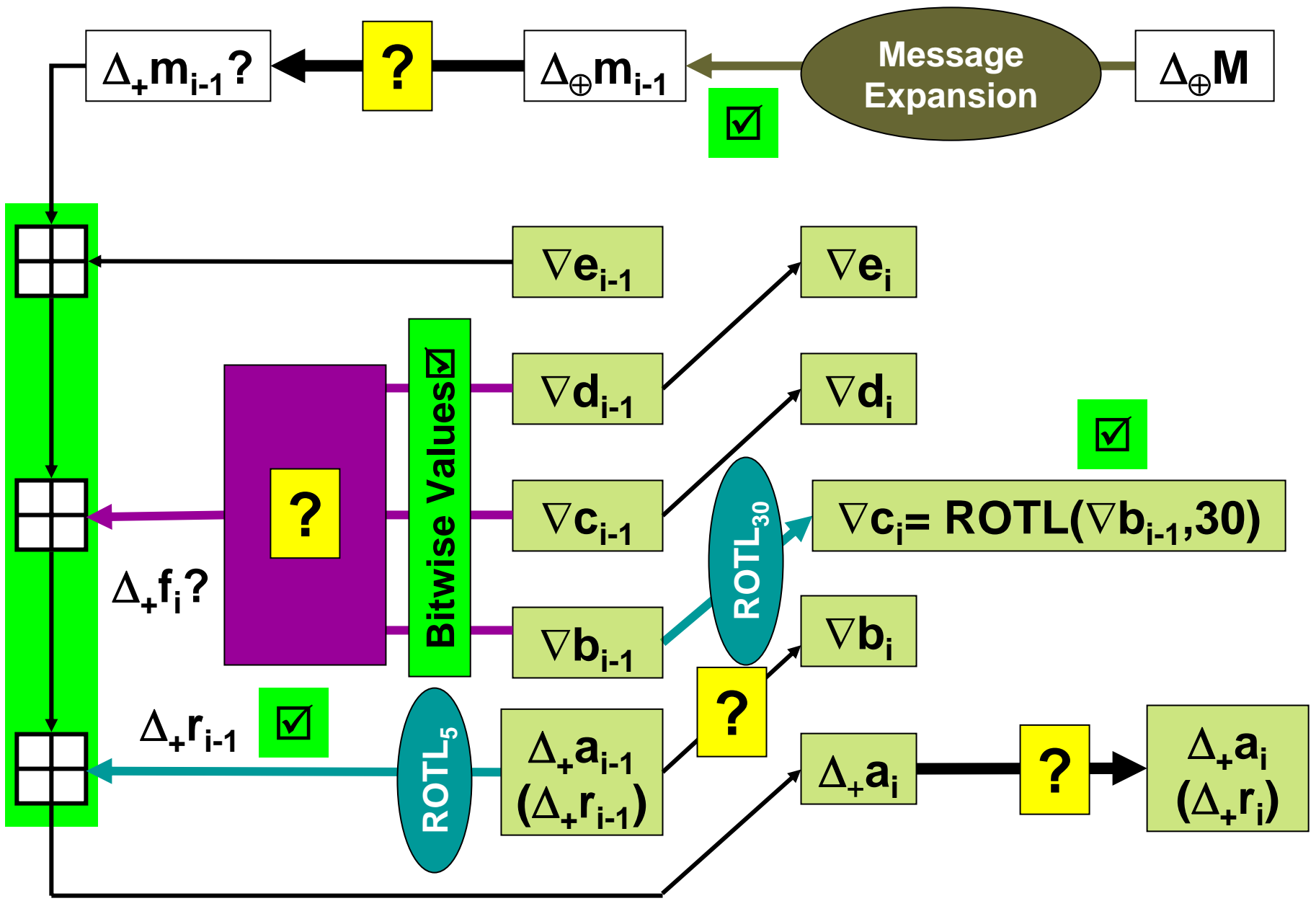
$= 1874787968 = 0x6FBFEFE80$

$= 01101111 10111110 11111110 10000000$

- $X', X'' \rightarrow \nabla X \rightarrow \Delta_{+} X, \Delta_{\oplus} X$

# Observations

- $\nabla \text{ROTL}(X,r) = \text{ROTL}(\nabla X,r)$
- XOR diffs only where @, +, -
  - @, +, - = **dynamic** bits
  - \*, 1, 0 = **static** bits
- ADD diff fully defined by +, - (& @ MSB only)
- Values of static bits don't affect XOR diff or ADD diff
  - Static bits only of interest in IF function



# Branching Points

- Given **XOR** diff,  $\exists$  multiple **ADD** diffs
- Given **ADD** diff,  $\exists$  multiple **ADD** diffs for **ROTL**
- Given **ADD** diff,  $\exists$  multiple **XOR** diffs
- Given **XOR & ADD** diff in,  $\exists$  multiple **ADD** diff out (IF)

Know:	Want:	Fn	Choice:
$\Delta_{\oplus}m$	$\Delta_{+}m$		?
$\Delta_{+}a$	$\Delta_{+}r$	<b>ROTL</b>	?
$\Delta_{+}a, \Delta_{+}r$	$\nabla b$		?
$\nabla b, \nabla c,$ $\nabla d$	$\Delta_{+}f$	<b>IF</b>	?

# Given XOR diffs find ADD diffs

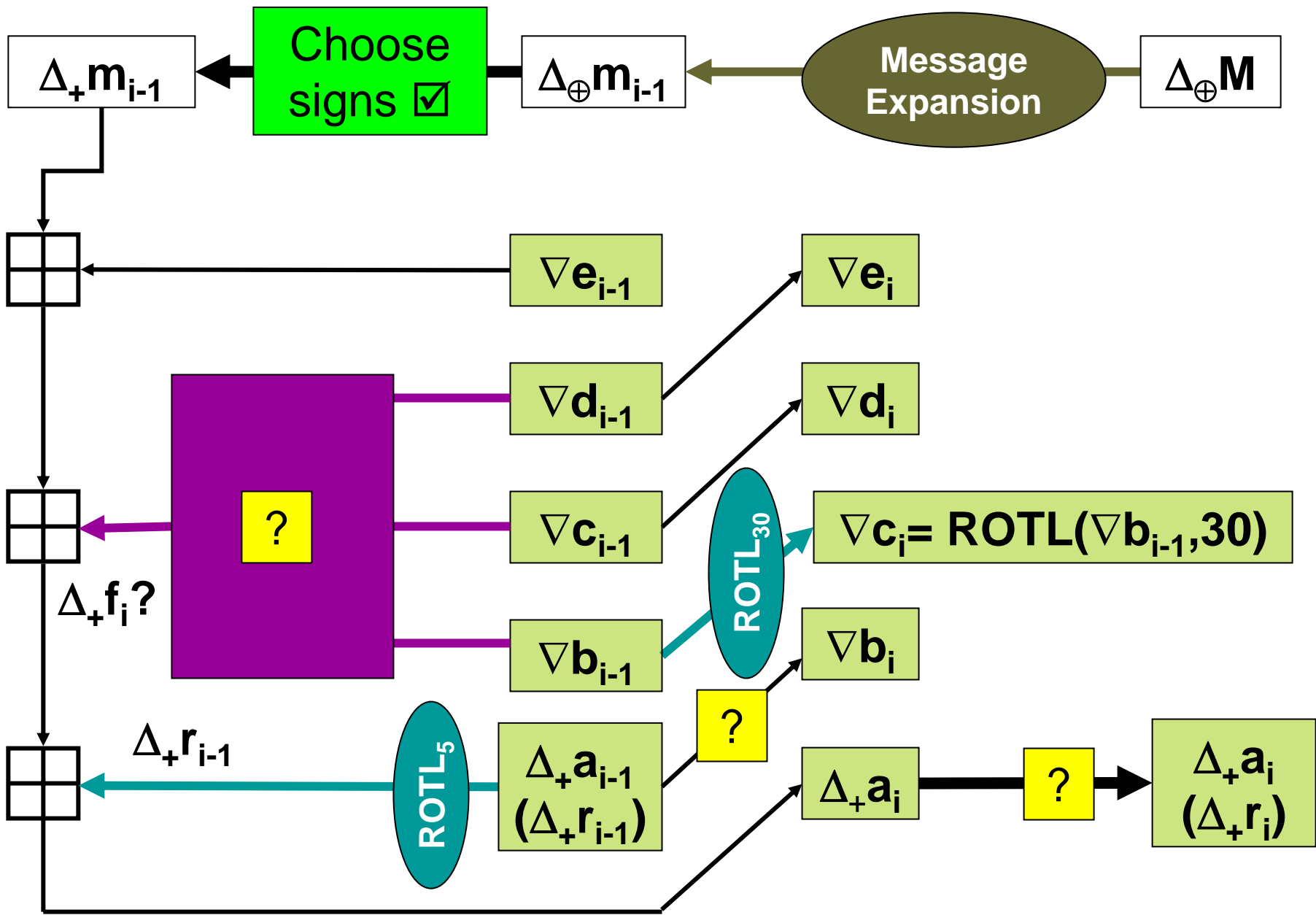
Bit 3322222222...

109876543210...

$$\Delta \oplus = 000100001100...0$$

$\nabla 0 =$	* * * + * * * * ++ * * ...	$\Delta + =$	+2 <sup>28</sup>	+2 <sup>23</sup>	+2 <sup>22</sup>
$\nabla 1 =$	* * * + * * * * +- * * ...	$\Delta + =$	+2 <sup>28</sup>	+2 <sup>23</sup>	-2 <sup>22</sup>
$\nabla 2 =$	* * * + * * * * -+ * * ...	$\Delta + =$	+2 <sup>28</sup>	-2 <sup>23</sup>	+2 <sup>22</sup>
$\nabla 3 =$	* * * + * * * * -- * * ...	$\Delta + =$	+2 <sup>28</sup>	-2 <sup>23</sup>	-2 <sup>22</sup>
$\nabla 4 =$	* * * - * * * * ++ * * ...	$\Delta + =$	-2 <sup>28</sup>	+2 <sup>23</sup>	+2 <sup>22</sup>
$\nabla 5 =$	* * * - * * * * +- * * ...	$\Delta + =$	-2 <sup>28</sup>	+2 <sup>23</sup>	-2 <sup>22</sup>
$\nabla 6 =$	* * * - * * * * -+ * * ...	$\Delta + =$	-2 <sup>28</sup>	-2 <sup>23</sup>	+2 <sup>22</sup>
$\nabla 7 =$	* * * - * * * * -- * * ...	$\Delta + =$	-2 <sup>28</sup>	-2 <sup>23</sup>	-2 <sup>22</sup>

Each is a distinct addition difference



# Given $\Delta+ = 2^{28} + 2^{25}$ find XOR diffs

$\nabla 0 = * * * + * * + * \dots$	$\Delta \oplus = 00010010\dots$
$\nabla 1 = * * * + * + - * \dots$	$\Delta \oplus = 00010110\dots$
$\nabla 2 = * * * + + - - * \dots$	$\Delta \oplus = 00011110\dots$
$\nabla 3 = * * + * - - - * \dots$	$\Delta \oplus = 00101110\dots$
$\nabla 4 = * * + - * * + * \dots$	$\Delta \oplus = 00110010\dots$
$\nabla 5 = * * + - * + - * \dots$	$\Delta \oplus = 00110110\dots$
$\nabla 6 = * * + - + - - * \dots$	$\Delta \oplus = 00111110\dots$
$\nabla 7 = * + - - * * + * \dots$	$\Delta \oplus = 01110010\dots$
$\nabla 8 = * + - - * + - * \dots$	$\Delta \oplus = 01110110\dots$
$\nabla 9 = * + - - + - - * \dots$	$\Delta \oplus = 01111110\dots$
$\nabla A = * + - * - - - * \dots$	$\Delta \oplus = 01101110\dots$
$\nabla B = + - - - * * + * \dots$	$\Delta \oplus = 11110010\dots$
$\nabla C = - - - - * * + * \dots$	$\Delta \oplus = 11110010\dots$
$\nabla D = + - - - * + - * \dots$	$\Delta \oplus = 11110110\dots$
$\nabla E = - - - - * + - * \dots$	$\Delta \oplus = 11110110\dots$

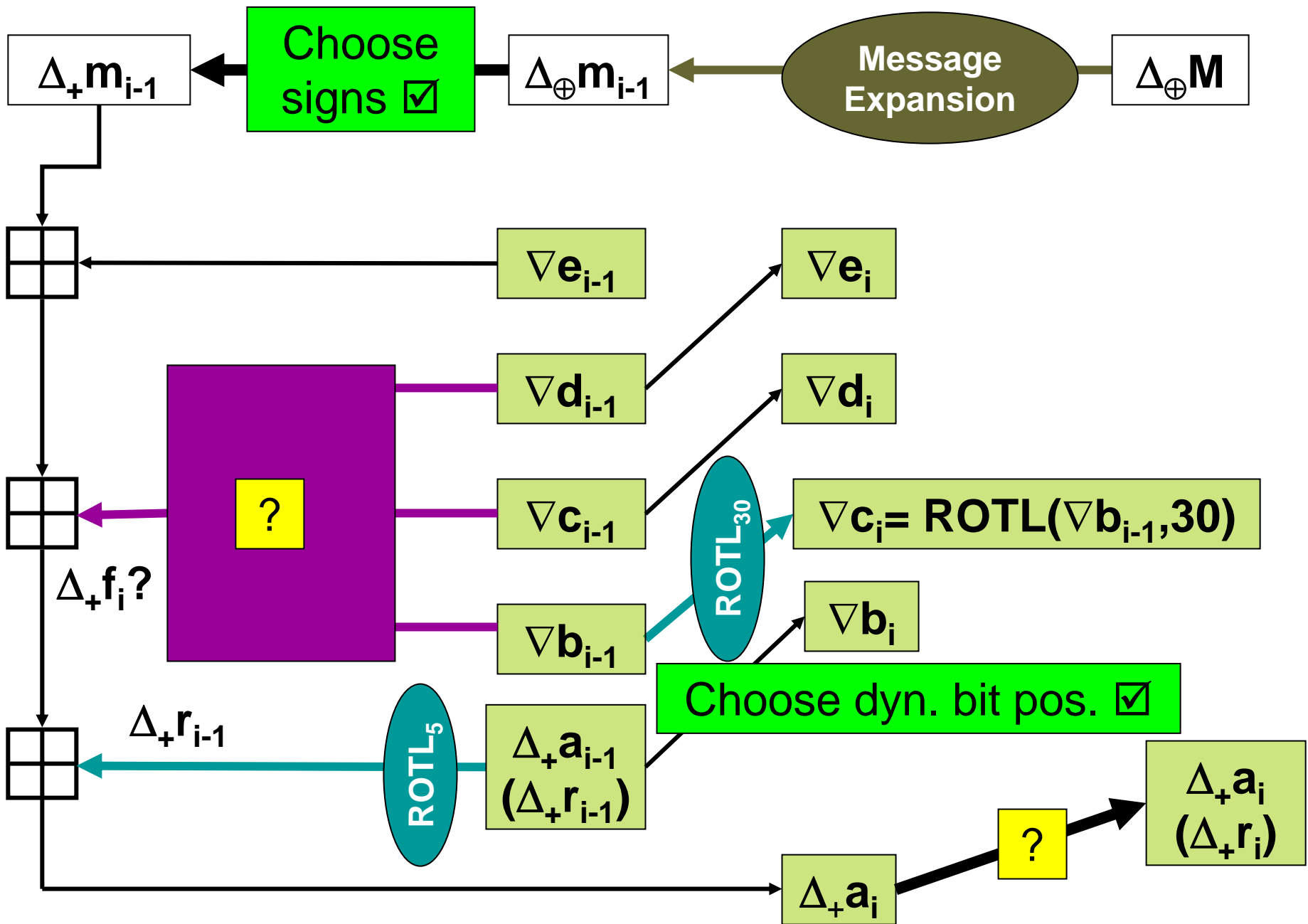
- Carry addition differences up to higher order bits

- Cancel with existing higher order differences

or...

- Add to higher order differences

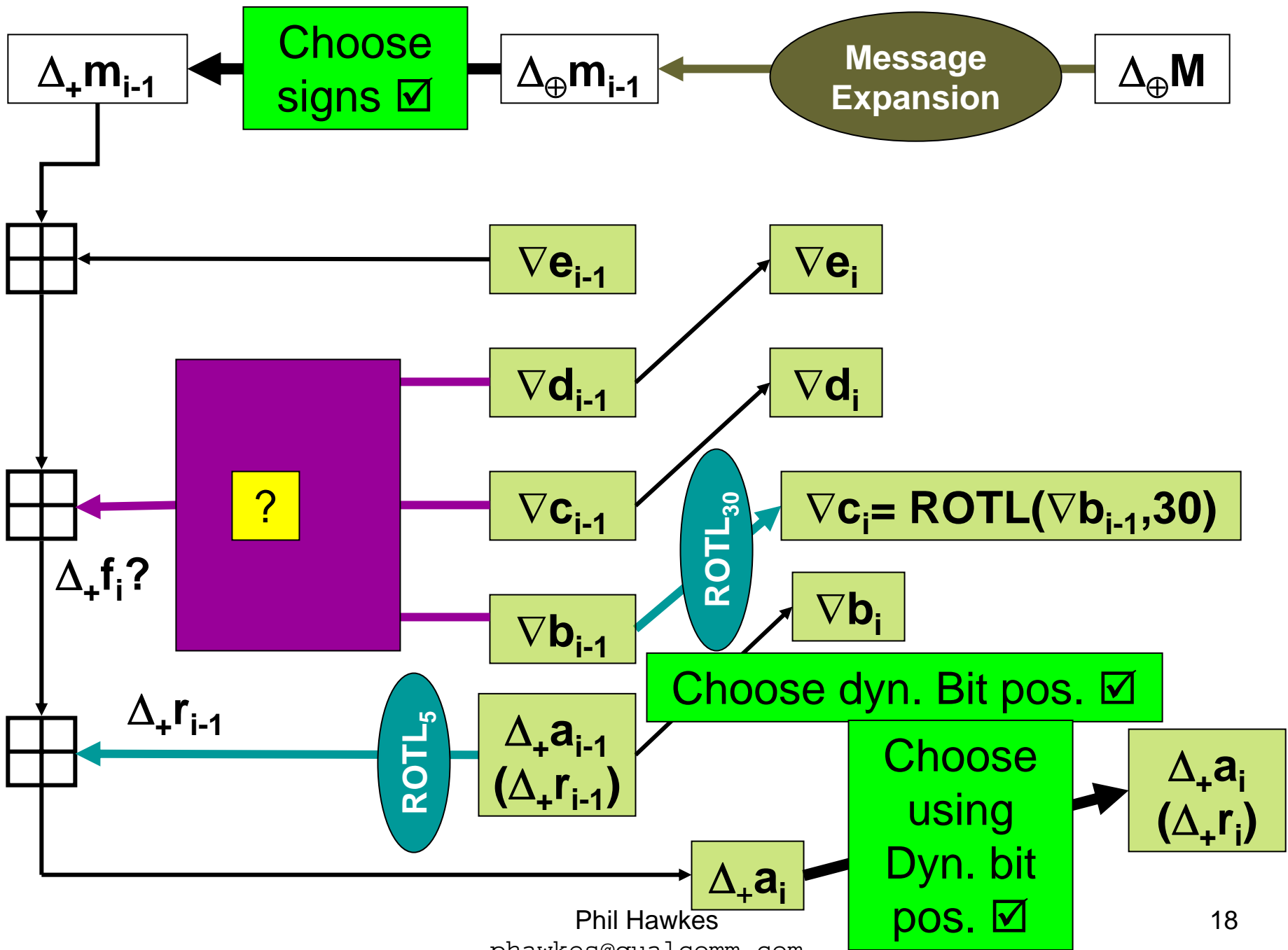
etc





$$\Delta_+ a_{i-1} = 2^{28} + 2^{25}, \Delta_+ \text{ROTL}(a_{i-1}, 5) = ?$$

$\nabla 0 = * * * + * * + * \dots$	$\nabla R0 = * + * \dots * * * + *$	$= 2^{30} + 2^1$
$\nabla 1 = * * * + * + - * \dots$	$\nabla R1 = + - * \dots * * * + *$	$\cong \Delta_+ R0$
$\nabla 2 = * * * + + - - * \dots$	$\nabla R2 = - - * \dots * * * + +$	$= 2^{30} + 2^1 + 2^0$
$\nabla 3 = * * + * - - - * \dots$	$\nabla R3 = - - * \dots * * + * -$	$\cong \Delta_+ R2$
$\nabla 4 = * * + - * * + * \dots$	$\nabla R4 = * + * \dots * * + - *$	$\cong \Delta_+ R0$
$\nabla 5 = * * + - * + - * \dots$	$\nabla R5 = + - * \dots * * + - *$	$\cong \Delta_+ R0$
$\nabla 6 = * * + - + - - * \dots$	$\nabla R6 = - - * \dots * * + - +$	$\cong \Delta_+ R2$
$\nabla 7 = * + - - * * + * \dots$	$\nabla R7 = * + * \dots * + - - *$	$\cong \Delta_+ R0$
$\nabla 8 = * + - - * + - * \dots$	$\nabla R8 = + - * \dots * + - - *$	$\cong \Delta_+ R0$
$\nabla 9 = * + - - + - - * \dots$	$\nabla R9 = - - * \dots * + - - +$	$\cong \Delta_+ R2$
$\nabla A = * + - * - - - * \dots$	$\nabla RA = - - * \dots * + - * -$	$\cong \Delta_+ R2$
$\nabla B = + - - - * * + * \dots$	$\nabla RB = * + * \dots + - - - *$	$\cong \Delta_+ R0$
$\nabla C = - - - - * * + * \dots$	$\nabla RC = * + * \dots - - - - *$	$= 2^{30} - 2^4 - 2^3 - 2^2 - 2^1$
$\nabla D = + - - - * + - * \dots$	$\nabla RD = + - * \dots + - - - *$	$\cong \Delta_+ R0$
$\nabla E = - - - - * + - * \dots$	$\nabla RE = + - * \dots - - - - *$	$\cong \Delta_+ RC$ etc



# IF function

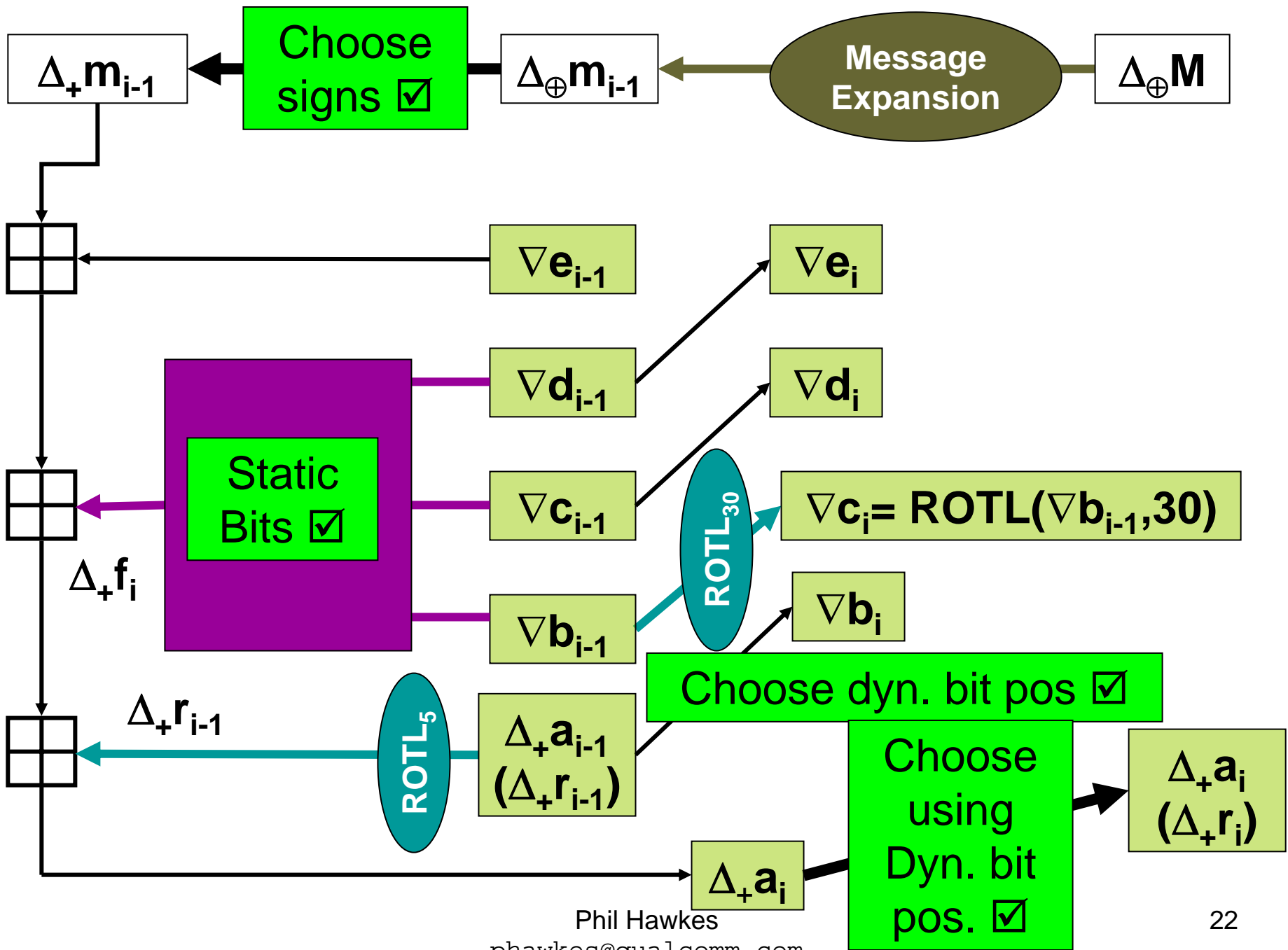
- What is known about inputs
  - Position of Dynamic & Static bits
  - Sign of Dynamic bits
- Static bits are left to specify
  - Initially  $\nabla \mathbf{b}[j] = '*'$
  - Assign values  $\{0, 1\}$  to static bits of  $\mathbf{b}[j], \mathbf{c}[j], \mathbf{d}[j]$
  - static bits of  $\mathbf{c}[j]$  and  $\nabla \mathbf{d}[j]$  may have been assigned earlier

# b[ j ] is Static

<b>b</b>	<b>c</b>	<b>d</b>	<b>f</b>	Options
Static	Static	Static	Static	
Static	Static	Dyn.	Dyn.	<b>b=0</b>
			Static	<b>b=1</b>
Static	Dyn.	Static	Stat	<b>b=0</b>
			Dyn.	<b>b=1</b>
Static	Dyn.	Dyn.	Dyn.	<b>b ∈ { * , 0 , 1 }</b>

# b[ j ] is Dynamic

b	c	d	f	Options	
Dyn.	Static	Static	Static	<b>c = d</b>	
			Dyn.	{+, -}	$(c,d) \in \{ (0,1), (1,0) \}$
				@	<b>c ≠ d</b>
Dyn.	Static	Dyn.	Static/Dyn.	<b>c ∈ {0, 1}</b>	
Dyn.	Dyn.	Static	Static/Dyn.	<b>d ∈ {0, 1}</b>	
Dyn.	Dyn.	Dyn.	Static/Dyn.		



# Options at Branching Points

<b>Know:</b>	<b>Want:</b>	<b>Fn</b>	<b>Choice:</b>
$\Delta_+ \mathbf{a}_{i-1}$	$\Delta_+ \mathbf{r}_{i-1}$	<b>ROTL</b>	Positions of Dynamic bits
$\Delta_{\oplus} \mathbf{m}_{i-1}$	$\Delta_+ \mathbf{m}_{i-1}$		“Sign” of dynamic bits {+,-}
$\Delta_+ \mathbf{a}_{i-2},$ $\Delta_+ \mathbf{r}_{i-2}$	$\nabla \mathbf{b}_{i-1}$		Positions of Dynamic bits
$\nabla \mathbf{b}, \nabla \mathbf{c},$ $\nabla \mathbf{d}$	$\Delta_+ \mathbf{f}$	<b>IF</b>	Values of Static Bits

# Branching within Forward Step

Choose  $\Delta_+ r$

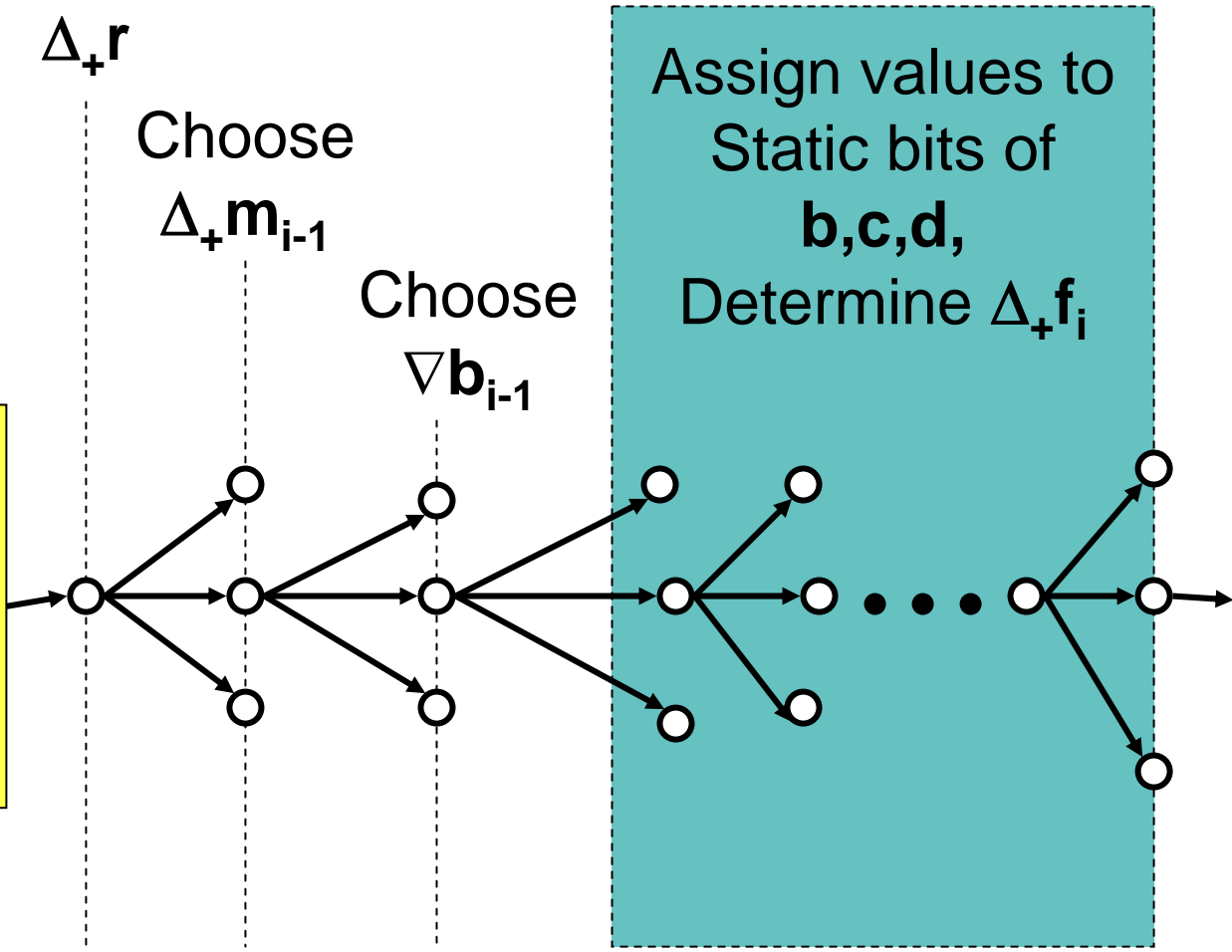
Choose  $\Delta_+ m_{i-1}$

Choose  $\nabla b_{i-1}$

Assign values to Static bits of  $b, c, d$ , Determine  $\Delta_+ f_i$

Existing Condit'ns from prev. steps

Compute  $\Delta_+ a_i$ , Pass to next step





# Progress

- Implemented Forward search and Reverse search
- Designed comparison/matching
  - Not implemented at time of writing