# Caligo, an Extensible Block Cipher -andCHash, a Caligo Based Hash 

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#### Abstract

The Caligo operations are performed on whole blocks only. No subdivision passes through an s-box or a Feistel network. The cipher definition is the same for any block size, allowing exhaustive search for statistical deviations on small block variants. I also propose CHash, a hash function that takes advantage of the cipher extensibility and resists the extension attack.


Keywords: Caligo, extensible, block cipher, symmetric key, CHash, extension attack.

## 1) Introduction

Currently, there is no provably secure block cipher. After withstanding many attempts of cryptanalysis, the algorithm is assumed to be secure. With a single definition for any block size, Caligo permits full inspection of small block variants, simplifying the search for defects and associated cryptanalytic attacks.

In sections 2 and 3, the algorithm is described. In section 4, the definition of CHash is given. In sections 5 and 6, the results of seeking for frequently differentials, impossible differentials and linear correlations are presented. In section 7, competitive performance on two 64-bit processors is shown.

## 2) Encryption and Decryption

Given a natural number $n$, each element of the set $\Omega=\left\{0,1,2, . ., 2^{n}-1\right\}$ have a distinct binary representation in the set $\Psi=\{0,1\}^{\mathrm{n}}$. The term block identifies an integer from $\Omega$ or the correspondent bit-sequence of $\Psi$ ( $n$ is called the block size). Therefore, we can do algebraic or bitwise operations on a block by considering it an element of $\Omega$ or $\Psi$ respectively.

Been $r$ the number of rounds of the cipher, the encryption and decryption round functions $\mathrm{f}_{i}, \mathrm{~g}_{i}$ : $\Omega^{3 r} \times \Omega \rightarrow \Omega$ are defined, respectively, by

$$
\begin{array}{lll}
\mathrm{f}_{\mathrm{i}}(\mathrm{~K}, \mathrm{X})=(\mathrm{K}[3 i] \times(\mathrm{X} \oplus \mathrm{~K}[3 i+1]))^{\prime}+\mathrm{K}[3 i+2] & \left(\bmod 2^{n}\right) & (0 \leq i<r) \\
\mathrm{g}_{i}(\mathrm{~K}, \mathrm{X})=\left(\mathrm{K}[3 i]^{-1} \times(\mathrm{X}-\mathrm{K}[3 i+2])^{\prime \prime}\right) \oplus \mathrm{K}[3 i+1] & \left(\bmod 2^{n}\right) & (0 \leq i<r) \tag{2.2}
\end{array}
$$

where
a) The subkey vector $\mathrm{K}=\langle\mathrm{K}[0], \mathrm{K}[1], . . \mathrm{K}[3 \mathrm{r}-1]\rangle$ have 3 r odd-blocks derived from a
secret block called masterkey (see section 3). $\mathrm{K}[3 \mathrm{i}]^{-1}$ is the multiplicative inverse (mod $2^{\mathrm{n}}$ ) of $\mathrm{K}[3 i$ ] (these elements should be calculated during the subkey setup and stored together with K ).
b) X is an input block.
c) $"$ is the bit-reversal of a block. Example for $n=8:(01001101) "=10110010$.
d) $\oplus$ is the bitwise xor of two blocks.
e) $\times,+$ and - are the integer multiplication, addition and subtraction, reduced $\bmod 2^{\mathrm{n}}$.

A composition (in the second parameter) of the round functions is performed to construct the encryption and decryption functions :

$$
\begin{align*}
& \mathrm{f}(\mathrm{r}, \mathrm{~K}, \mathrm{X})=\mathrm{f}_{\mathrm{r}-1} \circ \mathrm{f}_{\mathrm{r}-2} \circ \ldots \circ \mathrm{f}_{1} \circ \mathrm{f}_{0}(\mathrm{~K}, \mathrm{X})  \tag{2.3}\\
& \mathrm{g}(\mathrm{r}, \mathrm{~K}, \mathrm{X})=\mathrm{g}_{0} \circ \mathrm{~g}_{1} \circ \ldots \circ \mathrm{~g}_{\mathrm{r}-2} \circ \mathrm{~g}_{\mathrm{r}-1}(\mathrm{~K}, \mathrm{X}) \tag{2.4}
\end{align*}
$$

The above $\mathbf{n}$-bit block round functions (2.1 and 2.2) can embed the encryption/decryption of smaller blocks with $\mathbf{n}-\mathbf{m}$ bit length ( $0 \leq \mathrm{m}<\mathrm{n}$ ). The input/output $\mathrm{n}-\mathrm{m}$ least significant bits represent the smaller block been processed. The most significant $n-m$ bits must be shifted to the least significant $n-m$ positions after the bit-reversal (it's equivalent to shift the least significant $\mathrm{n}-\mathrm{m}$ bits to the most significant $\mathrm{n}-\mathrm{m}$ positions before the bit-reversal). The modified (and more general) round functions are

$$
\begin{array}{lll}
f_{i}(\mathrm{~K}, \mathrm{X})=\left(2^{\mathrm{m}} \times \mathrm{K}[3 i] \times(\mathrm{X} \oplus \mathrm{~K}[3 i+1])\right)^{\prime \prime}+\mathrm{K}[3 i+2] & \left(\bmod 2^{n}\right) & (0 \leq i<r) \\
\mathrm{g}_{i}(\mathrm{~K}, \mathrm{X})=\left(\mathrm{K}[3 i]^{-1} \times\left(2^{\mathrm{m}} \times(\mathrm{X}-\mathrm{K}[3 i+2])\right)^{\prime \prime}\right) \oplus \mathrm{K}[3 i+1] & \left(\bmod 2^{\mathrm{n}}\right) & (0 \leq i<r) \tag{2.6}
\end{array}
$$

The "user" interacts with the cipher by passing and receiving n-bit strings, which are interpreted in accord with the little endian convention (least significant bits in lower addresses).

## 3) Subkey Setup

First consider the sequence of blocks based on a T-function [4] :

$$
\begin{aligned}
& R_{0}=0 \\
& R_{i+1}=R_{i}+\left(\left(R_{i} \times R_{i}\right) \vee 5\right) \quad(i \geq 0)
\end{aligned}
$$

where $\vee$ is the bitwise or operation. Note that $\mathrm{R}_{\mathrm{i}}$ have the same parity of i. Let $\mathrm{C}_{e}$ and $\mathrm{C}_{o}$ be constant vectors formed by 3 r even-blocks and 3 r odd-blocks respectively:

$$
\begin{array}{ll}
\mathrm{C} e[i]=\mathrm{R}_{20+2 \mathrm{i}} & (0 \leq i<3 r) \\
\mathrm{Co} o[i]=\mathrm{R}_{20+2 i+1} & (0 \leq i<3 r)
\end{array}
$$

The cipher masterkey is a single block M provided by the user. From M we derive the "weak" subkey vector W , formed by 3 r odd-blocks:

$$
\mathrm{W}[\mathrm{i}]=\mathrm{M} \oplus \mathrm{C} e[\mathrm{i}] \quad(0 \leq \mathrm{i}<3 \mathrm{r}) \quad \text { when } \mathrm{M} \text { is odd }
$$

or

$$
\mathrm{W}[\mathrm{i}]=\mathrm{M} \oplus \mathrm{Co}[\mathrm{i}] \quad(0 \leq i<3 r) \quad \text { when } \mathrm{M} \text { is even }
$$

The subkey vector K, used for encryption and decryption, have 3 r odd-blocks. Is derived from M , $\mathrm{C} o$ and W in the following way :

$$
\begin{aligned}
& \mathrm{M}^{\prime}=\mathrm{f}(\mathrm{r}, \mathrm{C} o, \mathrm{M}) \\
& \mathrm{K}[\mathrm{i}]=\mathrm{f}\left(\mathrm{r}, \mathrm{~W}, \mathrm{M}^{\prime}+\mathrm{i}\right) \vee 1 \quad(0 \leq i<3 r)
\end{aligned}
$$

The or operation forces K [i] to be odd. Since W is derived from M in a very simple way, it's prudent to avoid a direct interaction between them to compute $\mathrm{K}[\mathrm{i}$. The combination of W and $\mathrm{M}^{\prime}$ guarantees that even related masterkeys $(\mathrm{M})$ will produce uncorrelated subkey vectors $(\mathrm{K})$. Furthermore, it's hard for an attacker to find a subkey K[i] based on the knowledge of other subkeys of K.

## 4) CHash, a Caligo Based Hash

Forced by the birthday paradox, cryptographic hashes must generate very large digests. Due to the block extensibility, Caligo can be used in hash modes with any suitable block size, without redefining the algorithm. A particular hash mode to be used with the cipher is proposed in this section. The resulting function is called CHash.

By using the weak vector W (instead of K ), taking into account that W depends on the masterkey $(\mathrm{W}=\omega(\mathrm{M})$ ) and choosing $\mathrm{r}=6$, we define the encryption function $w$ as

$$
\mathrm{w}(\mathrm{M}, \mathrm{X})=\mathrm{f}(6, \mathrm{~W}, \mathrm{X})=\mathrm{f}(6, \omega(\mathrm{M}), \mathrm{X})
$$

The L-bit string to be hashed, $S=S_{1}\left\|S_{2}\right\| . . \| S_{m}$, is viewed as a concatenation of n-bit substrings or blocks. $\mathrm{S}_{\mathrm{m}}$ must be end-padded with zeros if L is not a multiple of $n$. The implementor may choose any suitable block size (generally $160 \leq n \leq 512$ ). The hash construction is

$$
\begin{array}{ll}
\mathrm{H}_{0}=0 \\
\mathrm{H}_{\mathrm{i}}=\mathrm{w}\left(\mathrm{H}_{\mathrm{i}-1}, \mathrm{~S}_{\mathrm{i}}\right) \oplus \mathrm{H}_{\mathrm{i}-1} \oplus \mathrm{~S}_{\mathrm{i}} & (1 \leq \mathrm{i} \leq \mathrm{m}) \\
\mathrm{H}_{\mathrm{m}+1}=\mathrm{w}\left(1, \mathrm{H}_{\mathrm{m}} \oplus \mathrm{~L}\right) \oplus \mathrm{H}_{\mathrm{m}} \oplus \mathrm{~L} & (\mathrm{~m} \geq 0) \tag{4.2}
\end{array}
$$

where
a) The Preneel-Miyaguchi mode is applied to compute $\mathrm{H}_{1} . . \mathrm{H}_{\mathrm{m}}$. This scheme compensates the use of W (instead of K), since an attacker can't control the masterkey $\left(\mathrm{H}_{\mathrm{i}-1}\right)$ directly. For a discussion on Preneel-Miyaguchi security, see ref. [6] and [7].
b) $\mathrm{H}_{m+1}$ is the hash of S .
c) The use of L in $\mathrm{H}_{\mathrm{m}+1}$ is equivalent to the Merkle-Damgård strengthening.
d) $\mathrm{H}_{\mathrm{m}+1}$ breaks the chain on the first parameter by using the fixed masterkey 1. Moreover, it doesn't depend on S directly. Therefore, $\mathrm{H}_{\mathrm{m}+1}$ is unlikely to appear in (or used to calculate)
an intermediate state of another string. This avoids the extension attack, in which the adversary can compute the hash of $S \| S^{\prime}$ without knowing $S$ (or a part of it), but only the hash of S (for Merkle-Damgård strengthened strings, given only the hash of S and the block $L$ representing the length of $S$, he can calculate the hash of $S\|L\| S^{\prime}$ ) (see [5, section 6.3.1]).
e) $m=\lfloor(L+n-1) / n\rfloor$. For the empty string, $L=0 \Rightarrow m=0 \Rightarrow H_{m+1}=H_{1}=w(1,0)$.
f) The input size limitation is $0 \leq \mathrm{L}<2^{\mathrm{n} / 2}$.

Since the W setup is faster than a block encryption, CHash is not supposed to be much slower than an encryption mode like CBC. The compression function (4.1) timings are given in section 7.

## 5) Differential Properties

For the cipher, the relation between an input block $X$, an input difference $U$ and the resulting output difference V is given by

$$
\begin{equation*}
\mathrm{V}=\mathrm{f}(\mathrm{r}, \mathrm{~K}, \mathrm{X}) \oplus \mathrm{f}(\mathrm{r}, \mathrm{~K}, \mathrm{X} \oplus \mathrm{U}) \tag{5.1}
\end{equation*}
$$

In the encryption and decryption functions, the addition and subtraction of a constant operand $\mathrm{K}[3 i+2]$ protects the cipher against differentials $\langle\mathrm{U}, \mathrm{V}\rangle$ when $\mathrm{U} \oplus \mathrm{V}$ have high Hamming weight or $\mathrm{U} \wedge \mathrm{V}$ (bitwise and) have long runs of 1's (see [2]). The multiplication by K[3i] and K[3i] ${ }^{-1}$ gives little protection against differentials $\langle\mathrm{U}, \mathrm{V}\rangle$ when U and V have low Hamming weight concentrated in the most significant bits. The bit-reversal swaps these bits to the least significant positions, and the next multiplication can provide a good diffusion.

For $\mathrm{M}=0$, all possible 16 -bit input blocks ( X ) and input differences $(\mathrm{U})$ had been tested to count the occurrences of all difference pairs or differentials $\langle\mathrm{U}, \mathrm{V}\rangle$. The most frequent differential, $\left\langle\mathrm{U}_{\max }, \mathrm{V}_{\max }\right\rangle$, for up to 10 rounds was acquired.

For a round function where the addition operation is replaced by $\operatorname{xor}$ (table 5.1), the best pair found is formed by the iterative palindromic difference $U_{p}=U=V=2^{15}-2=0111 \ldots 1110$. The pair $\left\langle U_{p}, U_{p}\right\rangle$ occurs with probability $1 / 2$ in a round. Analyzing this experimental result, we can see that the multiplication have no effect against $\mathrm{U}_{\mathrm{p}}=2^{\mathrm{n}-1}-2$, for all $n$, when X is odd (hence probability $1 / 2$ for any X ). In fact, for an odd subkey $k$ and an odd block $X$, the congruences $\mathrm{X} \oplus \mathrm{U}_{\mathrm{p}} \equiv \mathrm{U}_{\mathrm{p}}+2-\mathrm{X}\left(\bmod 2^{\mathrm{n}}\right)$ and $\mathrm{k}\left(\mathrm{U}_{\mathrm{p}}+2\right) \equiv \mathrm{U}_{\mathrm{p}}+2\left(\bmod 2^{\mathrm{n}}\right)$ implies $\mathrm{k}\left(\mathrm{X} \oplus \mathrm{U}_{\mathrm{p}}\right) \oplus \mathrm{kX} \equiv \mathrm{U}_{\mathrm{p}}(\bmod$ $2^{\mathrm{n}}$ ). The difference $\mathrm{U}_{\mathrm{p}}$ can be concatenated to built a high probability $\left(1 / 2^{r}\right) r$-round differential characteristic [1].

For the addition operation, the probability of the above pair $\left\langle U_{p}, U_{p}\right\rangle$ falls drastically to $1 / 2^{n-2}$, since $U_{p} \wedge U_{p}$ have a run of $n-2$ binary 1's [2]. Accordingly, tables 5.2 and 5.3 show how the distribution of differentials $\langle\mathrm{U}, \mathrm{V}\rangle$ flattens fast as the number of rounds increases, but stops at round four. This same behavior was observed on up to 28 -bit blocks. It's reasonable to conjecture that we gain no additional protection against conventional differential cryptanalysis
with more than four rounds.

| Rounds | $\mathrm{U}_{\max }$ | $\mathrm{V}_{\max }$ | $\left\langle\mathrm{U}_{\max }, \mathrm{V}_{\max }\right\rangle$ <br> occurrences |
| :--- | :---: | :---: | :---: |
| 1 | $0 \times 8000$ | $0 \times 0001$ | $0 \times 10000$ |
| 2 | $0 \times 7 \mathrm{FFE}$ | $0 \times 7 \mathrm{FFE}$ | $0 \times 4000$ |
| 3 | $0 \times 7 \mathrm{FFE}$ | $0 \times 7 \mathrm{FFE}$ | $0 \times 1 \mathrm{FF} 0$ |
| 4 | $0 \times 7 \mathrm{FFE}$ | $0 \times 7 \mathrm{FFE}$ | $0 \times 1028$ |
| 5 | $0 \times 7 \mathrm{FFE}$ | $0 \times 7 \mathrm{FFE}$ | $0 \times 07 \mathrm{~F} 2$ |
| 6 | $0 \times 7 \mathrm{FFE}$ | $0 \times 7 \mathrm{FFE}$ | $0 \times 03 \mathrm{AE}$ |
| 7 | $0 \times 7 \mathrm{FFE}$ | $0 \times 7 \mathrm{FFE}$ | $0 \times 01 \mathrm{E} 0$ |
| 8 | $0 \times 7 \mathrm{FFE}$ | $0 \times 7 \mathrm{FFE}$ | $0 \times 00 \mathrm{E} 4$ |
| 9 | $0 \times 7 \mathrm{FFE}$ | $0 \times 7 \mathrm{FFE}$ | $0 \times 0072$ |
| 10 | $0 \times 7 \mathrm{FFE}$ | $0 \times 7 \mathrm{FFE}$ | $0 \times 0040$ |

Table 5.1: Max. differential occurrences for $\mathrm{n}=16$ and addition replaced by xor

| Round <br> S | $U_{\max }$ | $V_{\max }$ | $\left\langle U_{\max }, V_{\max }\right\rangle$ <br> occurrences |
| :--- | :---: | :---: | :---: |
| 1 | $0 \times 8000$ | $0 \times 0003$ | $0 \times 8000$ |
| 2 | $0 \times 0824$ | $0 \times 0004$ | $0 \times 180 A$ |
| 3 | $0 \times 2000$ | $0 \times 0120$ | $0 \times 0050$ |
| 4 | $0 \times 5200$ | $0 \times 0003$ | $0 \times 002 A$ |
| 5 | $0 \times 3 B 82$ | $0 \times 1 A F B$ | $0 \times 0012$ |
| 6 | $0 \times F 048$ | $0 \times 1$ B39 | $0 \times 0014$ |
| 7 | $0 \times C 454$ | $0 \times F F 89$ | $0 \times 0014$ |
| 8 | $0 \times D A 97$ | $0 \times 5774$ | $0 \times 0014$ |
| 9 | $0 \times 6 E C 6$ | $0 \times A E E 4$ | $0 \times 0014$ |
| 10 | $0 \times 0823$ | $0 \times 28 E B$ | $0 \times 0012$ |

Table 5.2: Max. differential occurrences for $\mathrm{n}=16$ and correct round functions

| Rounds | $U_{\max }$ | $V_{\max }$ | $\left\langle U_{\max }, V_{\max }\right\rangle$ <br> occurrences |
| :--- | :--- | :--- | :--- |
| 1 | $0 \times 20000$ | $0 \times 00003$ | $0 \times 20000$ |
| 2 | $0 \times 29091$ | $0 \times 00003$ | $0 \times 01 \mathrm{~F} 7 \mathrm{C}$ |
| 3 | $0 \times 02440$ | $0 \times 010 \mathrm{A0}$ | $0 \times 000 \mathrm{DE}$ |
| 4 | $0 \times 3 \mathrm{~B} 236$ | $0 \times 2 \mathrm{DB} 86$ | $0 \times 00016$ |
| 5 | $0 \times 08 \mathrm{A1E}$ | $0 \times 1 \mathrm{C} 2 \mathrm{C} 7$ | $0 \times 00014$ |
| 6 | $0 \times 17672$ | $0 \times 0165 \mathrm{E}$ | $0 \times 00016$ |
| 7 | $0 \times 3147 \mathrm{~B}$ | $0 \times 12 \mathrm{CB} 1$ | $0 \times 00016$ |
| 8 | $0 \times 02 \mathrm{AB} 5$ | $0 \times 1 \mathrm{~A} 822$ | $0 \times 00014$ |
| 9 | $0 \times 00 \mathrm{A10}$ | $0 \times 133 \mathrm{BA}$ | $0 \times 00014$ |
| 10 | $0 \times 01089$ | $0 \times 31860$ | $0 \times 00014$ |

Table 5.3: Max. differential occurrences for $\mathrm{n}=18$ and correct round functions

In the tables 5.2 and $5.3, \mathrm{U}=2^{\mathrm{n}-1}(0 \times 8000$ and $0 \times 20000)$ seems to be good input differences (they pass through multiplication with probability 1 and the addition transforms the (bit-reversed) difference $0 \times 01$ into $0 \times 03$ with probability $1 / 2$ ). Therefore, it's interesting to verify what the fixed $U=2^{24-1}(0 x 800000)$ and $U=2^{28-1}(0 x 8000000)$ can do with 24 and 28 -bit blocks respectively (tables 5.4 and 5.5).

| Round <br> s | U | $\mathrm{V}_{\text {max }}$ | $\left\langle\mathrm{U}, V_{\text {max }}\right\rangle$ <br> occurrences |
| :--- | :---: | :---: | :---: |
| 1 | $0 \times 800000$ | $0 \times 000003$ | $0 \times 800000$ |
| 2 | $0 \times 800000$ | $0 \times A 92160$ | $0 \times 002280$ |
| 3 | $0 \times 800000$ | $0 \times 1 \mathrm{~F} 0210$ | $0 \times 00011 \mathrm{~A}$ |
| 4 | $0 \times 800000$ | $0 \times 82897 \mathrm{~F}$ | $0 \times 000010$ |
| 5 | $0 \times 800000$ | $0 \times 9 \mathrm{CE} 780$ | $0 \times 00000 \mathrm{E}$ |
| 6 | $0 \times 800000$ | $0 \times E C D 8 F 7$ | $0 \times 000010$ |
| 7 | $0 \times 800000$ | $0 \times E 99$ DFD | $0 \times 000010$ |
| 8 | $0 \times 800000$ | $0 \times 8$ A7D68 | $0 \times 000012$ |
| 9 | $0 \times 800000$ | $0 \times 555$ D5A | $0 \times 000010$ |
| 10 | $0 \times 800000$ | $0 \times 3 F E 0 F A$ | $0 \times 000010$ |

Table 5.4 : Max. differential occurrences for $\mathrm{n}=24$ and $\mathrm{U}=0 \mathrm{x} 800000$

| Rounds | U | $\mathrm{V}_{\max }$ | $\left\langle\mathrm{U}, V_{\max }\right\rangle$ <br> occurrences |
| :--- | :---: | :---: | :---: |
| 1 | $0 \times 8000000$ | $0 \times 0000003$ | $0 \times 8000000$ |
| 2 | $0 \times 8000000$ | $0 \times 58221 \mathrm{E} 2$ | $0 \times 0003624$ |
| 3 | $0 \times 8000000$ | $0 \times 9400128$ | $0 \times 0000430$ |
| 4 | $0 \times 8000000$ | $0 \times 0000030$ | $0 \times 000002 \mathrm{E}$ |
| 5 | $0 \times 8000000$ | $0 \times 17$ A3DCA | $0 \times 0000010$ |
| 6 | $0 \times 8000000$ | $0 \times 36$ A3B96 | $0 \times 0000010$ |
| 7 | $0 \times 8000000$ | $0 \times$ D41C966 | $0 \times 0000010$ |
| 8 | $0 \times 8000000$ | $0 \times 5$ F69B6C | $0 \times 0000010$ |
| 9 | $0 \times 8000000$ | $0 \times 06$ CBC79 | $0 \times 0000012$ |
| 10 | $0 \times 8000000$ | $0 \times 8$ EFE2DE | $0 \times 0000014$ |

Table 5.5 : Max. differential occurrences for $\mathrm{n}=28$ and $\mathrm{U}=0 \times 8000000$

A noteworthy detail in round four of tables 5.2 to 5.5 , is how close are the occurrences of the most frequent differential, despite the block size variation.

## Impossible Differentials

For a given non-zero input difference $U_{0}$, we see in equation 5.1 that $X$ and $X^{\prime}=X \oplus U_{0}$ give the same V. So there are at most $2^{\mathrm{n}-1}$ possible differentials $\left\langle\mathrm{U}_{0}, \mathrm{~V}\right\rangle$ and $2^{\mathrm{n}-1}$ impossible ones. But after changing the masterkey we may get differentials that could not be found with the previous one.

The following tests had been done with 16 and 18 -bit blocks and up to 10 rounds. In equation 5.1, for each value of $U$, the keys $M=0 . .63$ had been combined with all values of $X$ to compute the number N of not found values of V . Tables 5.6 and 5.7 show the larger value of $\mathrm{N}\left(\mathrm{N}_{\max }\right)$, the associated input difference $U_{\max }$ and the probability $\mathrm{P}_{\max }=\mathrm{N}_{\max } /\left(2^{\mathrm{n}}-1\right)(\mathrm{V}=0$ excluded $)$ of finding impossible differentials under these keys.

| Rounds | $U_{\max }$ | $N_{\max }$ | $P_{\max }$ |
| :---: | :---: | :---: | :---: |
| 1 | $0 \times 8000$ | $0 \times F F F 0$ | 0.999771 |
| 2 | $0 \times 8000$ | $0 \times 1 A 68$ | 0.103151 |
| 3 | $0 \times 0800$ | $0 \times 0005$ | 0.000076 |
| 4 | $0 \times 0001$ | $0 \times 0000$ | 0.000000 |
| 5 | $0 \times 0001$ | $0 \times 0000$ | 0.000000 |
| 6 | $0 \times 0001$ | $0 \times 0000$ | 0.000000 |
| 7 | $0 \times 0001$ | $0 \times 0000$ | 0.000000 |
| 8 | $0 \times 0001$ | $0 \times 0000$ | 0.000000 |
| 9 | $0 \times 0001$ | $0 \times 0000$ | 0.000000 |
| 10 | $0 \times 0001$ | $0 \times 0000$ | 0.000000 |

Table 5.6: Max. impossible differentials for $n=16$ and $\mathrm{M}=0 . .63$

| Rounds | $U_{\max }$ | $N_{\max }$ | $P_{\max }$ |
| :--- | :---: | :---: | :---: |
| 1 | $0 \times 20000$ | $0 \times 3 F F E E$ | 0.999935 |
| 2 | $0 \times 20000$ | $0 \times 0 B 0 B C$ | 0.172593 |
| 3 | $0 \times 10000$ | $0 \times 00044$ | 0.000259 |
| 4 | $0 \times 00001$ | $0 \times 00000$ | 0.000000 |
| 5 | $0 \times 00001$ | $0 \times 00000$ | 0.000000 |
| 6 | $0 \times 00001$ | $0 \times 00000$ | 0.000000 |
| 7 | $0 \times 00001$ | $0 \times 00000$ | 0.000000 |
| 8 | $0 \times 00001$ | $0 \times 00000$ | 0.000000 |
| 9 | $0 \times 00001$ | $0 \times 00000$ | 0.000000 |
| 10 | $0 \times 00001$ | $0 \times 00000$ | 0.000000 |

Table 5.7: Max. impossible differentials for $n=18$ and $\mathrm{M}=0 . .63$

These tables prove that, in 16 and 18-bit cases, there are no impossible differentials from 4 to 10 rounds of the cipher. This is coherent with the fact that, for round 4 an beyond, the differential distributions are equally closer to a normal distribution, as tables 5.2 to 5.5 suggest.

## 6) Linear Properties

To verify linear dependencies in the cipher, fixed bit groups from the input (X) and output (Y) blocks are xored together. This is equivalent to xor the bits of the number

$$
\mathrm{Z}=\left(\mathrm{m}_{\mathrm{x}} \wedge \mathrm{X}\right) \oplus\left(\mathrm{m}_{\mathrm{y}} \wedge \mathrm{Y}\right)
$$

where
a) $m_{x}$ and $m_{y}$ are the bit group selecting masks
b) $\wedge$ is the bitwise and operator
c) $\oplus$ is the bitwise exclusive-or (xor) operator
d) $Y=f(r, K, X)$

Ideally, the resulting bit should be odd with probability $1 / 2$ for a randomly chosen X .
An exhaustive search was done with the 16 -bit variant $(\mathrm{n}=16)$ and $\mathrm{M}=0$. For each mask pair $\langle\mathrm{mx}, \mathrm{my}\rangle$, all X and Y was generated. The bits of each Z was xored and the number of odd results accumulated in N . The odd parity probability was approximated by $\mathrm{p}=\mathrm{N} / 2^{16}$ and the bias (deviation from $1 / 2$ ) by $|p-1 / 2|$. The mask pairs with higher bias, for up to 10 rounds, are in table 6.1. The bias for 18 and 20-bit blocks with fixed $m_{x}(0 x 00001)$ can bee seen in tables 6.2 and 6.3.

These tables suggest that no additional protection against a linear attack is achieved with more than four rounds.

| Rounds | mx | my | N | p | Bias |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | $0 \times 0001$ | $0 \times 8000$ | $0 \times 26 \mathrm{EA}$ | 0.1520 | 0.3480 |
| 2 | $0 \times 400 \mathrm{~F}$ | $0 \times \mathrm{B} 000$ | $0 \times 5 \mathrm{E} 18$ | 0.3676 | 0.1324 |
| 3 | $0 \times 0 \mathrm{C} 01$ | $0 \times 8 \mathrm{D} 30$ | $0 \times 865 \mathrm{C}$ | 0.5248 | 0.0248 |
| 4 | $0 \times 07 \mathrm{E} 3$ | $0 \times \mathrm{B} 000$ | $0 \times 83 \mathrm{AE}$ | 0.5144 | 0.0144 |
| 5 | $0 \times B D 75$ | $0 \times 57 \mathrm{E} 4$ | $0 \times 7 \mathrm{CD} 6$ | 0.4876 | 0.0124 |
| 6 | $0 \times \mathrm{DDB} 5$ | $0 \times \mathrm{DC} 68$ | $0 \times 7 \mathrm{CB} 0$ | 0.4871 | 0.0129 |
| 7 | $0 \times 1 \mathrm{C} 6 \mathrm{E}$ | $0 \times 5 \mathrm{D} 44$ | $0 \times 7 \mathrm{CBA}$ | 0.4872 | 0.0128 |
| 8 | $0 \times F D 1 \mathrm{~A}$ | $0 \times \mathrm{BD} 9 \mathrm{~A}$ | $0 \times 7 \mathrm{CD} 4$ | 0.4876 | 0.0124 |
| 9 | $0 \times B 9 \mathrm{~F} 2$ | $0 \times 0492$ | $0 \times 7 \mathrm{CD} 0$ | 0.4875 | 0.0125 |
| 10 | $0 \times 4106$ | $0 \times 0 \mathrm{D98}$ | $0 \times 8304$ | 0.5118 | 0.0118 |

Table 6.1: Higher bias for $\mathrm{n}=16$ and $\mathrm{M}=0$

| Rounds | mx | my | N | Bias |
| :--- | :---: | :---: | :---: | :---: |
| 1 | $0 \times 00001$ | $0 \times 20000$ | $0 \times 01 \mathrm{~F} 66$ | 0.4693 |
| 2 | $0 \times 00001$ | $0 \times 1 \mathrm{~A} 67 \mathrm{E}$ | $0 \times 21922$ | 0.0245 |
| 3 | $0 \times 00001$ | $0 \times 3 \mathrm{CF} 63$ | $0 \times 20816$ | 0.0079 |
| 4 | $0 \times 00001$ | $0 \times 147 \mathrm{~F} 9$ | $0 \times 1 \mathrm{FB} 42$ | 0.0046 |
| 5 | $0 \times 00001$ | $0 \times 1206 \mathrm{~A}$ | $0 \times 1 \mathrm{FAFC}$ | 0.0049 |
| 6 | $0 \times 00001$ | $0 \times 384 \mathrm{DA}$ | $0 \times 1 \mathrm{FA} 82$ | 0.0054 |
| 7 | $0 \times 00001$ | $0 \times 12 \mathrm{~A} 2 \mathrm{C}$ | $0 \times 204 \mathrm{~B} 8$ | 0.0046 |
| 8 | $0 \times 00001$ | $0 \times 0 \mathrm{EE} 66$ | $0 \times 1 \mathrm{FB} 0 \mathrm{C}$ | 0.0048 |
| 9 | $0 \times 00001$ | $0 \times 347 \mathrm{C} 0$ | $0 \times 2051 \mathrm{E}$ | 0.0050 |
| 10 | $0 \times 00001$ | $0 \times 1 \mathrm{~A} 54 \mathrm{E}$ | $0 \times 2048 \mathrm{E}$ | 0.0044 |

Table 6.2: Higher bias for $\mathrm{n}=18, \mathrm{M}=0$ and fixed input mask 0x00001

| Rounds | $\mathrm{mx}_{\mathrm{x}}$ | my | N | Bias |
| :--- | :---: | :---: | :---: | :---: |
| 1 | $0 \times 00001$ | $0 \times 80000$ | $0 \times \mathrm{E} 6296$ | 0.3991 |
| 2 | $0 \times 00001$ | $0 \times \mathrm{A} 3685$ | $0 \times 82 \mathrm{CD} 4$ | 0.0109 |
| 3 | $0 \times 00001$ | $0 \times \mathrm{BDC} 23$ | $0 \times 80 \mathrm{EFC}$ | 0.0037 |
| 4 | $0 \times 00001$ | $0 \times 291 \mathrm{DC}$ | $0 \times 7 \mathrm{~F} 42 \mathrm{E}$ | 0.0029 |
| 5 | $0 \times 00001$ | $0 \times 7 \mathrm{DA} 35$ | $0 \times 80 \mathrm{~A} 5 \mathrm{C}$ | 0.0025 |
| 6 | $0 \times 00001$ | $0 \times 210 \mathrm{E} 1$ | $0 \times 7 \mathrm{~F} 5 \mathrm{FC}$ | 0.0024 |
| 7 | $0 \times 00001$ | $0 \times \mathrm{C} 5 \mathrm{~A} 45$ | $0 \times 7 \mathrm{~F} 5 \mathrm{DC}$ | 0.0025 |
| 8 | $0 \times 00001$ | $0 \times 37 \mathrm{ACF}$ | $0 \times 7 \mathrm{~F} 5 \mathrm{~A} 0$ | 0.0025 |
| 9 | $0 \times 00001$ | $0 \times 27 \mathrm{CDF}$ | $0 \times 7 \mathrm{~F} 5 \mathrm{~F} 8$ | 0.0024 |
| 10 | $0 \times 00001$ | $0 \times B 5 A E 2$ | $0 \times 7 \mathrm{~F} 532$ | 0.0026 |

Table 6.3: Higher bias for $\mathrm{n}=20, \mathrm{M}=0$ and fixed input mask 0x00001

## 7) Performance

To be competitive, this algorithm needs a processor with fast 64-bit multiplication like Alpha 21264, Itanium or Athlon64. Table 7.1 compares Caligo with Rijndael C code running on Alpha 21264 processor (see ref. [3]).

| Cipher | Rounds | Key size | Block size | Encryption <br> cycles/byte |
| :---: | :---: | :---: | :---: | :---: |
| Rijndael | 10 | 128 | 128 | 18 |
| Caligo | 6 | 128 | 128 | 23 |
| Caligo | 6 | 256 | 256 | 20 |
| Caligo | 6 | 320 | 320 | 23 |
| Caligo | 6 | 512 | 512 | 39 |

Table 7.1: Alpha 21264, C code performance of Caligo.
Table 7.2 shows the timings on the AMD Athlon64 processor under Red-Hat Linux. In this case, the mul and bswap 64-bit assembly instructions was embedded in the C code.

| Rounds | Block size | Encryption <br> cycles/byte | Decryption <br> cycles/byte |
| :---: | :---: | :---: | :---: |
| 6 | 128 | 21 | 20 |
| 6 | 256 | 23 | 23 |
| 6 | 320 | 26 | 26 |
| 6 | 512 | 38 | 39 |

Table 7.2: AMD Atlhon64, C+assembly code performance of Caligo.

Table 7.3 gives the CHash compression function (4.1) performance on the Alpha 21264 processor. The code had been written in C.

| Hash | Block size | Cycles/block | Cycles/byte |
| :---: | :---: | :---: | :---: |
| CHash-256 | 256 | 926 | 28 |
| CHash-320 | 320 | 1247 | 31 |
| CHash-512 | 512 | 0 | 45 |

Table 7.3: Alpha 21264, C code performance of the CHash compression function.

## References

[1] V. Furman "Differential Cryptanalysis of Nimbus", Fast Software Encryption: 8th International Workshop
[2] A. Machado "Differential Probability of Modular Addition with a Constant Operand" http://eprint.iacr.org/2001/052
[3] R. Weiss and N. Binkert "A comparison of AES candidates on the Alpha 21264" http://csrc.nist.gov/encryption/aes/round2/conf3/papers/18-rweiss.pdf
[4] A. Klimov "Applications of T-functions in cryptography" http://www.wisdom.weizmann.ac.il/~ask/th.ps.gz
[5] N. Ferguson and B. Schneier "Practical Cryptography". Wiley Publishing, 2003.
[6] B. Preneel, R. Govaerts and J. Vandewalle, "Hash functions based on block ciphers: A synthetic approach" http://www.cosic.esat.kuleuven.be/publications/article-48.pdf
[7] J. Black, P. Rogaway, and T. Shrimpton, "Black-Box Analysis of the Block-Cipher-Based Hash-Function Constructions from PGV" http://www.cs.ucdavis.edu/~rogaway/papers/hash.pdf

## Appendix A: Caligo Test Vectors

The masterkey $(M)$, plaintext $\left(\mathrm{X}_{0}\right)$ and ciphertext $\left(\mathrm{X}_{r}\right)$ blocks are given in hexadecimal.

$$
\begin{aligned}
& \text { A.1) } n=256, r=6 \\
& M=0000000000000000000000000000000000000000000000000000000000000000 \\
& \mathrm{X}_{0}=0000000000000000000000000000000000000000000000000000000000000000 \\
& X_{r}=4 F 3311 B 6 \text { A9B391B2 AD0D74E6 F55296F2 911BA9F7 18833BCC 0FD9FCE4 E134AC7C } \\
& \mathrm{M}=0000000000000000000000000000000000000000000000000000000000000000 \\
& \mathrm{X}_{0}=0100000000000000000000000000000000000000000000000000000000000000 \\
& X_{r}=32 C 66775 \text { 46371E6F 6E124370 D2149A02 9B61096D 5637AAOE D909AC2C 777D5931 }
\end{aligned}
$$

$M=0100000000000000000000000000000000000000000000000000000000000000$
$\mathrm{X}_{0}=0000000000000000000000000000000000000000000000000000000000000000$
$X_{r}=$ DB87C4DE 5314A39B E79F7B65 E294A9B7 30A03BDD 60F98784 3FEE940F B3E38C09
A.2) $n=320, r=6$
$\mathrm{M}=0000000000000000000000000000000000000000000000000000000000000000$ 0000000000000000
$\mathrm{X}_{0}=0000000000000000000000000000000000000000000000000000000000000000$ 0000000000000000
$\mathrm{X}_{\mathrm{r}}=28554800088778 \mathrm{~B} 4$ C7A14690 3876A6EB 6A663BF2 63C4C131 78C9E23C E40ABA5A 97F1976F D5AD179B
$M=0000000000000000000000000000000000000000000000000000000000000000$ 0000000000000000
$\mathrm{X}_{0}=0100000000000000000000000000000000000000000000000000000000000000$ 0000000000000000
$\mathrm{X}_{\mathrm{r}}=5 \mathrm{BCC5EB3}$ E21526C3 774CED6E C5B60448 B7471983 1C7BE9CA 04D2078D 1A543DD4 5C1CAA47 OC46CE01
$M=0100000000000000000000000000000000000000000000000000000000000000$ 0000000000000000
$\mathrm{X}_{0}=0000000000000000000000000000000000000000000000000000000000000000$ 0000000000000000
$\mathrm{X}_{\mathrm{r}}=$ AC701C56 9B31D28B 76D91C02 4AFE4858 FECC054B 5BC877EB BEAA3954 6DE2A95C 3EA44B4E 4B3303F6

## A.3) $n=512, r=6$

$$
\begin{aligned}
& M=0000000000000000000000000000000000000000000000000000000000000000 \\
& 0000000000000000000000000000000000000000000000000000000000000000 \\
& \mathrm{X}_{0}=0000000000000000000000000000000000000000000000000000000000000000 \\
& 0000000000000000000000000000000000000000000000000000000000000000 \\
& \mathrm{X}_{\mathrm{r}}=9129 \mathrm{FD} 59 \text { 3DCB4430 0097D220 7D4F2384 C07B4365 C7226B7E BEB01779 1E8ED80F } \\
& \text { D6807D91 6C253196 2130D365 1A931443 57C4EE1A EE4E2AC7 6022A2D1 B6338DA4 } \\
& M=0000000000000000000000000000000000000000000000000000000000000000 \\
& 0000000000000000000000000000000000000000000000000000000000000000 \\
& \mathrm{X}_{0}=0100000000000000000000000000000000000000000000000000000000000000 \\
& 0000000000000000000000000000000000000000000000000000000000000000 \\
& X_{r}=4 A 72 B 4 B B \text { 14D98F1A C0A61B69 B5ADC92D B141D71C 96A737C6 D97ACEDO 2D175821 } \\
& \text { 19E59037 8689DA08 455ADC00 } 033 E 9671 \text { 36CE374F E7987FA7 59243F47 } 958119 B 7 \\
& M=0100000000000000000000000000000000000000000000000000000000000000 \\
& 0000000000000000000000000000000000000000000000000000000000000000 \\
& X_{0}=00000000000000000000000000000000000000000000000000000000000000000 \\
& 00000000000000000000000000000000000000000000000000000000000000000 \\
& X_{r}=385 B 012 \mathrm{D} \text { E0BCF842 C00BC9CD FD567EE2 B971323A 29DF26CE 37202E83 634B2466 } \\
& \text { E5688E3E C785C42F 870E9D31 17DD6101 617880EF B0AE0231 2A2B8B67 OBBEDODD }
\end{aligned}
$$

## Appendix B: CHash Test Vectors

B.1) The empty string

```
n=256: A78FD14E 92A1B6A2 3CA9B32B F87D1560 908F7241 675C3F33 356B863F 55DB056A
n=320: BB248F5A 4428391F 38BACB08 B7FE21C1 2C3D338A AC865AB2 5366FD74 3AAE2CED
    8F07F6F0 A2D0DDFD
n=512: DBEE2656 D4E48C27 167B59EB 25D596EA B09A36F3 DCDD634A 7975AE99 6ED9D1D1
    09D3C093 685A7687 E08A36BE AEF3CC4F 7FF27288 23A6EBD1 89FEC156 29536EE9
```


## B.2) The string "abc"

```
n=256: 5BB659EE 309766BA C445C26E 943839D2 E833F3BF 343AEA39 449F5AEF B9F2D404
n=320: 72DD6C73 EBF679A2 17086626 C4BBC793 74D5DE6E 576B3D48 E9977AA2 CFE2352D
    C8E4A75F 71CA2B0D
n=512: D89E708A 2EE4537A 801B56CF 5318DC31 F0A134D0 28EDB69A D4645C54 02688769
    49F377E4 F977002F E7F68420 6E3F82D9 58E78FDF 9CA45E69 70D3B0BC C2FEFE02
```

B.2) The string formed by "a" repeated 1000 times

```
n=256: B7B8F460 CB4F5A54 9334CD86 644B49DE 4F4EAB4B 6F9D54DE 75FEA58A E7760566
n=320: 954F57E7 FBAB9AC5 C32815AB 4A1111E7 AE892ED6 66B93DB3 91A23588 4EEC0693
    712A269B A7686CE9
n=512: 47E10EE0 E28613C5 83A35EB9 9CE63B99 E9E5A02A 9DEF59BC 26DD0F4A B389E1CA
    6DBAE7BD 14F7998C 115523D8 D4F2F8FE 3C4C8CA5 DD51363A D53F6F3B F1557BF7
```

