Improved Indifferentiability Security Bound for the JH Mode

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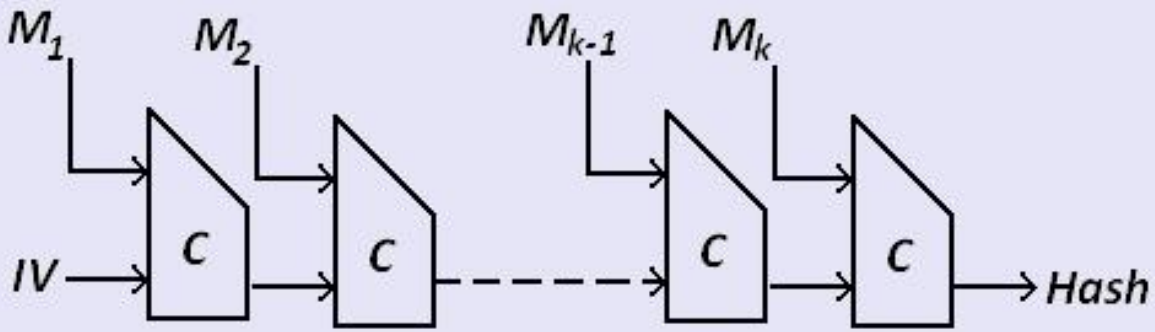
Hash function design

- Iterative hash functions are usually composed of two parts:
  - A compression function $C : \{0,1\}^{m+n} \to \{0,1\}^n$
  - A mode of operation H to extend C
- We denote the hash function by $H^C : \{0,1\}^* \to \{0,1\}^n$.

Example: classical Merkle-Damgård mode *MD*



- If C is collision-resistant then so is MD^C.

New modes of operation

- Flaws in Merkle-Damgård
  - Length-extension attack
  - Joux's multi-collision attack
  - Herding attack
  - Kelsey-Schneier 2nd preimage attack
- # of SHA-3 submissions using *MD* mode: 0
- Improved ideas
  - Additional postprocessing
  - Adding counters
  - Widening output length of C
  - Multiple applications of C on the same message block

Indifferentiability framework (I)

- The *indifferentiability framework* was introduced in 2004 by Maurer et al. and applied to hash modes of operation by Coron et al. in 2005.
- Indifferentiability focuses on the mode H and not C.
- C is assumed to be ideal, i.e. a random oracle, ideal permutation, or an ideal cipher.
- If H^C is indifferentiable from a random oracle RO, then we can replace RO by H^C in any cryptosystem and the result is as least as secure in the ideal compression model as in the random oracle model.
- **Indifferentiability guarantees resistance against all generic attacks, including the ones given in the previous slide. (Refer to our FSE 2012 Rump session talk.)**

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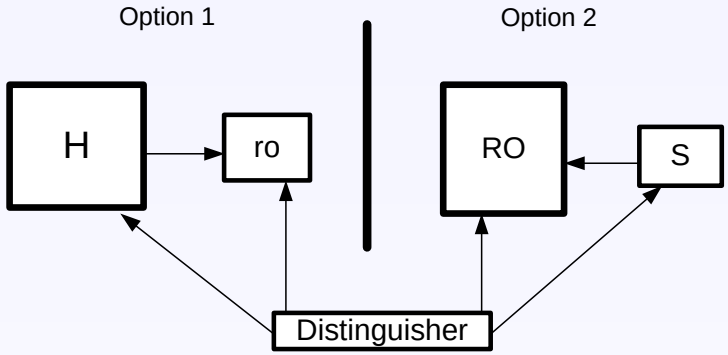
Indifferentiability framework (II)



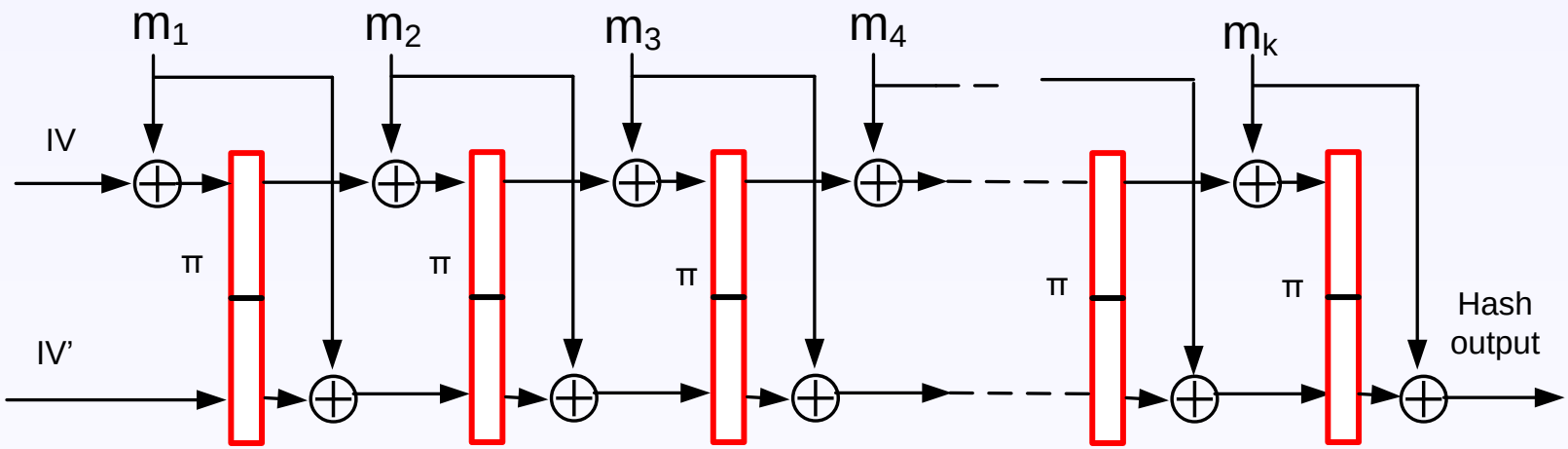
- Indifferentiability measures the extent to which a Distinguisher can tell the difference between Option 1 and Option 2
- Option 1 implements a hash mode *H* with primitive *ro*, a random oracle
- Option 2 consists of a random oracle *RO*, and a simulator *s*

Bounds for popular hash modes of operation

Mode of operation	Message block-length per call (b)	Primitive input-length per call (a)	Rate = $\frac{b}{a-b}$	Primitive Output-length per call	Indiff. bound
MD	n	$2n$	1	n	0
MDP	n	$2n$	1	n	$n/2^*$
EMD	n	$2n$	1	n	$n/2^*$
JH	n	$2n$	1	$2n$	$\boldsymbol{n/3}$
Sponge	n	$2n$	1	$2n$	$\boldsymbol{n/2^*}$
Grøstl	n	$2n$	1	$2n$	$\boldsymbol{n/2}$
Parazoa	n	$2n$	1	$2n$	up to $n/2$
FWP	n	$2n$	1	$2n$	$2n/3$
Skein	n	$2.25n$	4/5	n	$\boldsymbol{n/2^*}$
HAIFA	n	$3n$	1/2	n	$n/2^*$
WP,chopMD	n	$3n$	1/2	$2n$	$n - \log n^{**}$
Shabal	n	$4n$	1/3	$2n$	n^*
BLAKE	$2n$	$4n$	1	$2n$	$\boldsymbol{n/2^*}$

- For each case the hash-output is n-bit.
- The symbols * and ** denote optimal and close to optimal.

The JH mode



- $M \xrightarrow{pad} m_1 m_2 m_3 \cdots m_k$
- π is a permutation
- All wires are n bits
- Variants: Chop n output bits to hash-size h
- Value $n = 512$, $h = 512, 384, 256$ and 224 bits

Previous results on the JH mode of operation

Mode of operation	Message block-length per call	Primitive input-length per call	1st preimage resistance	2nd preimage resistance	Collision bound	Indiff. bound
JH-n	n	$2n$	$n/2$	$n/2$	$n/2^*$	$n/3$
JH-512	512	1024	256	256	256^*	170
JH-256	512	1024	256^*	256^*	128^*	170

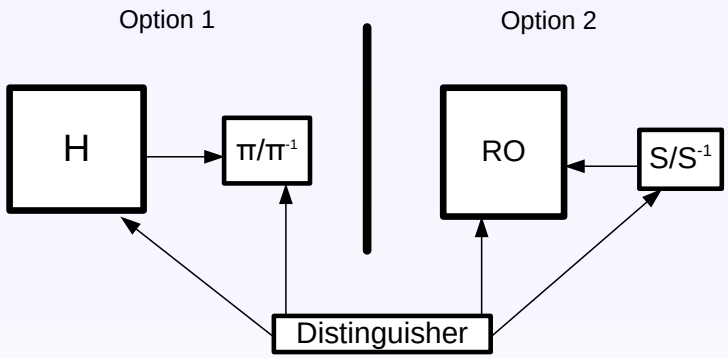
- Our indifferentiability results subsume all previous results
- In addition, it pushes bounds for all known and unknown attacks to $n/2$ bits

Mode of operation	Message block-length per call	Primitive input-length per call	1st preimage resistance	2nd preimage resistance	Collision bound	Indiff. bound
JH-n	n	$2n$	$n/2$	$n/2$	$n/2^*$	$\mathbf{n/2}$
JH-512	512	1024	256	256	256^*	**256**
JH-256	512	1024	256^*	256^*	128^*	$\mathbf{256^*}$

Concrete security of JH-256 and JH-512

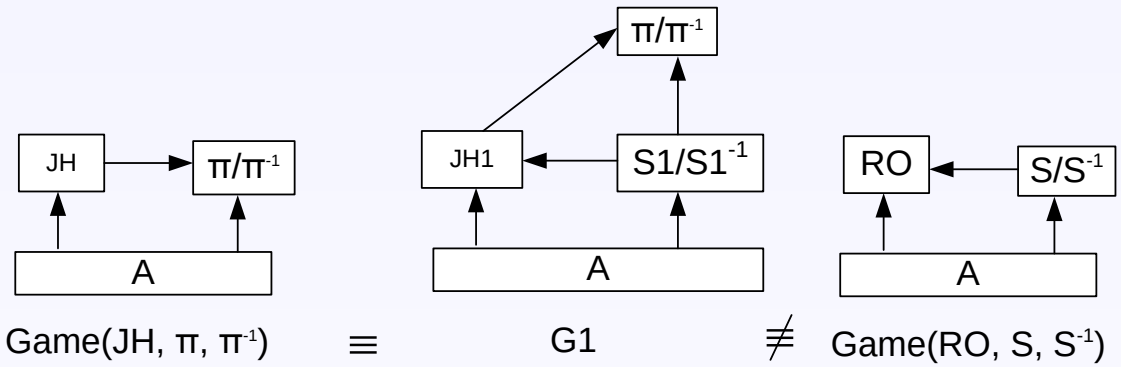
- We have improved the indifferentiability security bound for JH-512 (and JH-256) from **170 to 256 bits**: increase of **86 bits** of security
- This bound is the **optimal** for JH-256, and the **best** for JH-512
- This means JH-512 (and JH-256) **cannot be attacked** nontrivially with work less than 2^{256}, unless there are weaknesses in the underlying permutation

Indifferentiability framework (with permutations)



- Indifferentiability measures the extent to which a Distinguisher can tell the difference between Option 1 and Option 2
- Option 1 implements the JH hash mode with primitive π, an ideal permutation
- Option 2 consists of a random oracle *RO*, and a pair of simulators s and s^{-1}

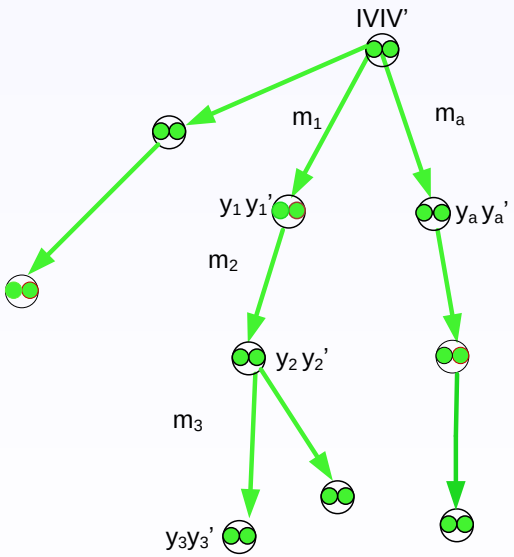
Security games



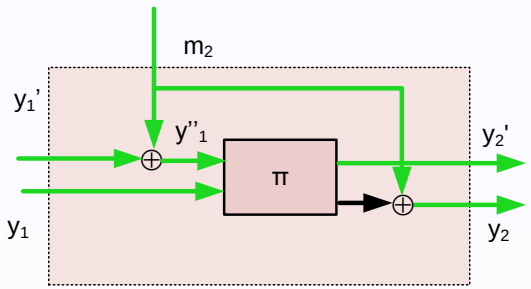
- Game G1 is an intermediate step, allowing us to more easily compare the games
- Game G1 is *equivalent* to Game(JH, π, π^{-1})
- Game G1 has the same code as Game(RO,S, S^{-1}), except when certain *BAD* events occur

The simulator

- (1) The simulator outputs as if it is a random permutation. (2) The simulator “tries her level best” to ensure that the RO output is indistinguishable from JH output.
- To accomplish the above, it builds a graph from the queries and responses. See the paper for complete description.



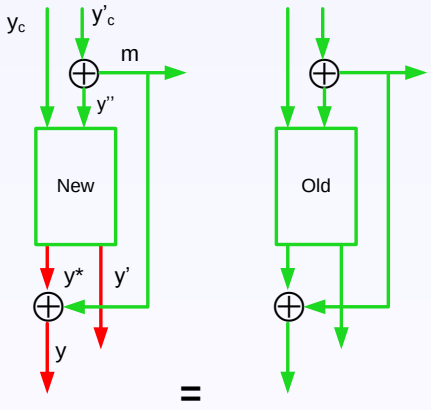
(i)



(ii)

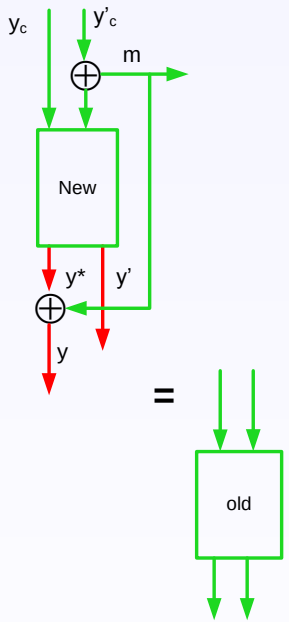
The *BAD* events

- The simulator fails on *BAD* events.



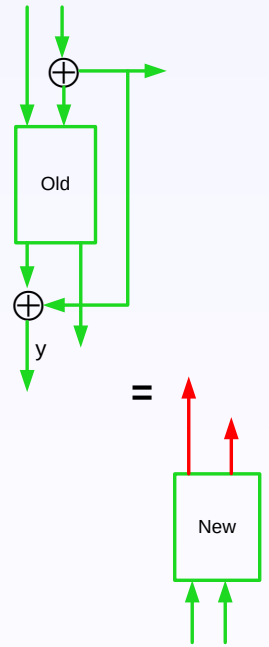
Node-collision
(n bits)

Type1-a



Forward-query-collision
(n bits)

Type1-b



Reverse-query-collision
(n bits)

Type1-c

Conclusion

- We extended the indifferentiability security bound for JH from $n/3$ to $n/2$ bits: concretely from **170 to 256 bits** for both JH-256 and JH-512.
- It seems possible to improve the bound even to $2n/3$ bits, that is, **to 342 bits.** We are presently working on it (refer to our FSE 2012 rump session presentation).